

SUMMER QUIZ 2011 SOLUTIONS – PART 1

Easy 1

A mini-barbecue can cook two hamburgers at once, and it takes five minutes to cook one side of a hamburger. What is the minimum time required to cook both sides of three hamburgers?

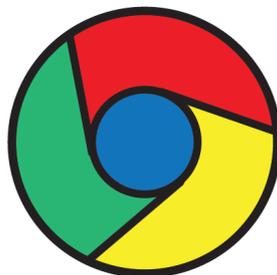


Answer: 15 minutes.

Since we have 6 sides to cook, and we can only cook 2 sides at a time, we definitely need to barbecue in at least 3 stages. So, we require at least $3 \times 5 = 15$ minutes to barbecue the hamburgers. And, 15 minutes is actually enough time if we cook the hamburgers in the following order: first, Burger 1 and Burger 2; then, Burger 1 (second side) and Burger 3; finally, Burger 2 and Burger 3.

Easy 2

We adjust the Google Chrome browser logo so that the four coloured regions all have the same area. If the new logo has a diameter of 6 centimetres, what is the diameter of the blue circle in the middle?

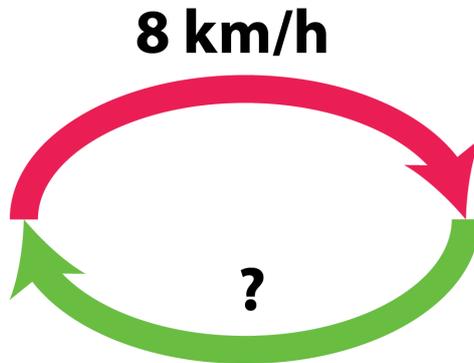


Answer: 3 centimetres.

We want the blue circle to have $\frac{1}{4}$ the area of the whole logo. However, doubling the diameter of a circle multiplies its area by a factor of 4 (since the area is $\frac{1}{4} \pi D^2$). So the blue circle should have $\frac{1}{2}$ the diameter of the logo.

Easy 3

Craig the marathon runner starts the big race badly, averaging 8 kilometres per hour over the first half of the marathon. He wants to average 16 kilometres per hour for the whole race. Can Craig do it?



Answer: No.

A runner averaging 16 km/hr over the whole race would have finished at exactly the same time that Craig actually got to the half-way mark. So, the only way Craig could average 16 km/hr overall would be if he were instantaneously teleported to the finish line.

Easy 4

128 players enter the Australian Open tennis tournament. Half of the players are eliminated in the first round. Then, in the second round, half of the remaining players are eliminated. This continues until only one player remains undefeated, who is then crowned champion. What is the total number of matches played?



Answer: 127.

We can just add up the numbers of matches played, round by round: $64 + 32 + 16 + 8 + 4 + 2 + 1$. However, it's easier to just note that exactly one match must be played for each player eliminated: since 127 players are eliminated, that's the number of matches that must be played.

Easy 5

A set of scales works perfectly, except that the zero point is off. When Tony weighs himself, the scales register 70 kilos, and when Julia weighs herself the scales show 50 kilos. With both of them on the scales, it registers 115 kilos. How much are the scales off, and how much do Tony and Julia really weigh?



Answer: 5 kg, 65 kg (excluding Tony's ego) and 45 kg.

Playing around, it's not hard to guess that the offset is 5 kg, and then it's easy to find Tony's and Julia's weights. To solve the problem methodically, let's write T and J for the actual weights of Tony and Julia, and we'll also write E for the unknown error in the scales. Then we have the three equations

$$\begin{aligned}T + E &= 70 \\J + E &= 50 \\J + T + E &= 115\end{aligned}$$

Adding the first two equations, we see that $J + T + 2E = 120$. Comparing to the third equation, it is easy to see that E must be 5 kg.

Easy 6

The three squares below have sides of length 2 centimetres. How large are the green regions?



Answer: All are $4 - \pi$ square centimetres.

The square has area 4 cm^2 , and the blue circle has area π (since its radius is 1 cm). So, the green region in the first square has area $4 - \pi \text{ cm}^2$. In fact, the three blue regions have equal areas, and so the green regions must have equal areas as well, all being $4 - \pi \text{ cm}^2$. (The four quarter-circles in the third diagram combine to make a circle of radius 1 cm, and the quarter-circle in the second diagram has radius 2 cm.)

Easy 7

There are 11 people in a room, shaking hands, but Jerry has failed to shake hands with someone. What is the maximum number of people who could have shaken hands with everyone else?

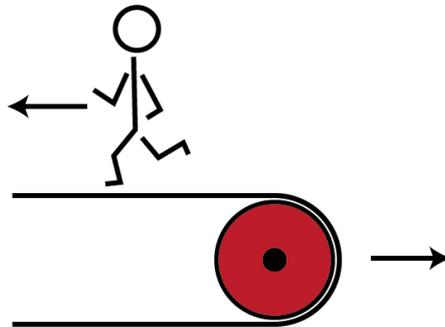


Answer: 9.

Suppose Jerry failed to shake hands with Alice. Then Alice also can't have shaken hands with everyone. So, the best we can do, if all other possible handshakes were made, is to have $11 - 2 = 9$ people shaking hands with everyone.

Easy 8

A very foolish army crew is using the caterpillar track of a tank as their treadmill. The tank moves at 5 kilometres per hour. How fast does a soldier have to run in order to avoid a horrible accident?



Answer: 5 kilometres per hour.

Overall, the soldier wants to be moving at 5 km/hr, the same speed as the tank. However, relative to the ground, the top part of the tread is moving at 10 km/hr. (The top of the wheel, just at the instant it's at the top, is moving at twice the speed of the centre of the wheel). So, to maintain his position on the tread, the soldier must jog at 5 km/hr in the opposite direction.

Easy 9

You want to place ten coins into three cups so that each cup contains an odd number of coins. How do you do it?



Answer: Place one cup inside another.

There are a number of ways to do this. For example, place five coins in the first cup, three coins in the second cup, and two coins in the third cup. Now, place the second cup in the third cup. This has the overall effect that there are five coins in the first cup, three in the second and five in the third.

Easy 10

The local newspaper, the Melbourne Spun, reports the findings of their survey of 1000 readers: 723 of the readers enjoy footy, and 647 of them are keen on cricket. As well, 249 of the readers indicated they actually enjoy both footy and cricket. However, you're suspicious of the Spun's survey. Why?



Answer: More than 1000 people were surveyed.

If the numbers are reliable, then $723 - 249 = 474$ people like footy but not cricket, and $647 - 249 = 398$ like cricket but not footy. Adding in the 249 who like both, that comes to 1121 people (and that's not counting those very unaustralian people who like neither).

