Critique of Victorian VCE Mathematics Exams

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We thank VCAA for arranging an external review and analysis of our concerns with VCE mathematics exams. For this purpose, and as requested, we provide here a critique, including but not limited to examples of serious errors on the 2022 exams and a summary of concerns with the structure, development and vetting processes for these exams.

It is pleasing that there will be an external view. We have significant concerns, however, that the structure and format of the review, and the plans for post-review communication, may hamper the conduct of the review and its potential effectiveness:

- **The reviewer and the manner of their choosing are confidential**

The meaning of an "external provider" who shall conduct this review is unclear. There is no information regarding how that person or body might be chosen, or by whom, or whether the reviewer will be properly qualified and properly independent. Given that, as we argue in Sections 7 and 8, VCAA has had demonstrable problems with engaging qualified personnel, and with openness and candour, this lack of transparency is of serious concern.

- **The terms of reference for the review are confidential**

The review will not be part of a legislatively formal process. Nonetheless, the reviewer will be provided with an introduction and direction. These details matter, and nothing is known of them. As it stands, it is not even clear that this critique would be forwarded in its entirety to any reviewer.

- **The review is limited in scope and input**

There is good reason for limiting the review to VCE exams, and indeed in our recent communication with VCAA we have focused upon the exams. Nonetheless, we have also continually emphasised that the exams are the tip of the iceberg, the clearest sign of broader, systemic problems. In particular, an appreciation of these broader problems is required to understand why VCAA produces such poor exams. At some point, soon, these broader problems must also be addressed.

We also note that the review is seemingly limited only to the concerns raised in this critique. Marty is perhaps the most prominent critic of VCE exams, but we are far from being the only critics, and we stress that we are not claiming that our list of criticisms is exhaustive. Even addressing every one of our listed concerns does not mean that the exams would then be "fixed".

- **The report will be confidential**

That the report will be confidential is of the greatest concern. VCAA's commitment to providing an "overview" does nothing to allay this concern.

In summary, it is pleasing that VCAA has initiated a review of VCE mathematics exams, and VCAA must determine the parameters of that review. In turn, anyone will be free to decide upon their response to the structure of the review, and to any resulting report or "overview".

1 Introduction

In this critique, we shall indicate a number of ways in which VCE mathematics exams are grossly deficient, including in their production, their grading and their reporting. We shall provide many examples from the 2022 exams to illustrate these deficiencies, focusing upon Specialist Mathematics and Mathematical Methods, the subjects with which we are most familiar.

It is important to recognise that we have not attempted to be comprehensive, even in considering the 2022 exams, and it would be almost impossible to be so. Every Specialist Mathematics and Mathematical Methods exam that we have seen is marred by at least minor mathematical problems, and most exams contains at least one major error. Every exam contains a number of poorly conceived questions, more than a few of which exhibit little mathematical sense or purpose. Every exam consists almost entirely of low-mark questions, requiring only shallow answers and effectively guaranteeing an over-weighting of minor matters of form, at the expense of proper concern for the evaluation of proper mathematical

thought. The clear majority of exam questions are poorly written, in a crabbed and convoluted manner; many such questions are so poorly written as to be close to incomprehensible, and more than a few consequently stray into clear error.

Our critique should thus be seen as the first step, not the last, in a comprehensive documentation and subsequent analysis of deficiencies in VCE mathematics exams and in their administration. Any reasonable analysis must include an in-depth comparison to the high-quality NSW HSC exams, as administered by NESA. On this point, it should be noted that the difference in curricula may be a reason for differences in VCE and HSC exams, but it cannot be an excuse for poor VCE exams. If the VCE Study Design (curriculum) does not permit the proper testing of mathematical knowledge and skills and reasoning, then the necessary conclusion is that the Study Design requires significant reform.

2 Major Mathematical Errors on the 2022 Exams

Here, we list major mathematical errors on the 2022 Specialist Mathematics (SM) and Mathematical Methods (MM) exams. We consider an error to be "major" if as a consequence the question has no proper answer or, even if somehow answerable, the question is so mathematically flawed as to very likely to have caused students considerable confusion. For the sake of brevity, we shall not reproduce questions in full: the complete questions should be read in conjunction with our comments.

(a) SM Exam 1, Q3(b)

> *Following the service [of a coffee machine], the mean time taken to dispense 25 cups of coffee is found to be 9 seconds.*

This is a blatant error: following on from part (a), the intention was clearly for the mean time to be 9 seconds *per cup*. However this question might have been subsequently graded, and that is an active concern, many students would have lost time and composure.

To be crystal clear, and this should be confirmed, a student who has answered 3(b) in the manner in which it was *unambiguously* written must have been eligible for full marks. For VCAA to have done otherwise would have been unconscionable. We cannot know since, over a month after the exam, VCAA has not even published exam answers. (By comparison, NESA has already released full sample solutions to the 2022 HSC mathematics exams.)

Given the manner in which VCAA tends to excuse itself of error (see Section 7), we provide some preemptive argument on the proper approach to grading this flawed question. Notwithstanding the implausibility of the quoted sentence in the light of part (a), it is not the students' responsibility to be second-guessing a clear statement of fact in a question. Similarly, although the resulting confidence interval straying into the negative may be considered a red flag, this can occur in the correct solution of a well-posed problem. But again, it is simply not the students' responsibility to be concerned with any of this.

(b) SM Exam 2, MCQ4

> *The polynomial* $p(z) = (z-a)(z-b)(z-c)$ *has complex roots* a, b *and* c*, where* $\text{Re}(a) \neq 0, \text{Re}(b) \neq 0, \text{Re}(c) \neq 0$ *and* $\text{Im}(b) = 0$*. When expanded, the polynomial is a cubic with real coefficients.*

The question simply cannot be answered: none of the statements that follow are "necessarily true", as is illustrated by the simple example $a = 1, b = 2, c = 3$.

It is not entirely clear how such an inexcusable error has occurred, but one must note the atrocious wording, which is an open invitation to error – and to students' misinterpretation – and is inexcusable in its own right. This error is also doubly egregious, since an essentially identical error occurred in 2021 (SM Exam 2, QB2(a)). VCAA's deceit following on from their 2021 error is discussed in Section 7.

(c) SM Exam 2, MCQ19

The question provides enough information to determine the mean cost for an item, but then asks students to determine a confidence interval for this mean. That can formally be done, but it is absurd to find a confidence interval for a known mean, and is thus also astonishingly confusing.

(d) SM Exam 2, QB6(f)

The question asks for the probability that the volume of liquid in a can is below a certain amount. The can masses and the total (can + liquid) masses have been given to be normally distributed, with the parameters provided, but no independence assumption has been declared. As such, the students cannot answer the question without making an unwarranted assumption.

It is not remotely the students' responsibility to resolve the issues with this question, but it also must be noted that it is not even clear which variables might reasonably be assumed independent, or why. It should also be noted that the answer to the exam question depends upon the covariance of the variables; as it happens, and even assuming that the liquid mass is a normal random variable, the answer cannot be determined to the requested three decimal places. Of course any such analysis is beyond the concern of a student taking the exam but, given VCAA's past willingness to minimise the effect of their errors, it should be kept in mind.

(e) MM Exam 2, QB4(e)(ii)

> *Explain why the domain of* $A(k)$ *does not include all values of* k*.*

The question is expressed badly, and is effectively meaningless for multiple reasons. First, it has been declared at the outset that $k > 0$. Second, the question has introduced the function

$A(k)$ implicitly for all positive k as if this is not a concern, and before part (i): any querying of the domain should have been done immediately. Third, the question in part (i) only makes sense if one accepts that $A(k) = 0$ also makes sense, implying that all such (positive) k would be in the domain of A, irrespective of whether the region is empty. Fourthly, the conclusion of the question is best though to be false: there is no reason why one cannot consider a region to be empty, and consequently $A(k)$ make sense for all (positive) k:

3 Minor Mathematical Issues with the 2022 Exams

We list here relatively minor mathematical issues with questions on the 2022 exams. Although these may not be as egregious as the major errors listed above, they are not insignificant and are not to be taken lightly: many amount to error, and all exhibit a lack of care or a lack of clear mathematical thought, or both. Their nature and their number is concerning.

(a) SM Exam 1, Q6(b)(i)

The question asks for vectors to be expressed in terms of x and y (and other terms), but is ambiguous: y has already been given as a function of x.

(b) SM Exam 1, Q6(b)(ii)

The question refers to "the vector scalar (dot) product", which is confusing to the point of meaninglessness.

(c) SM Exam 1, Q10(b)

The question asks for the answer to be given in the form $\frac{(a-\sqrt{b})\pi}{c}$, with a, b and c real. This is absurd. The intended answer was $\frac{(3-\sqrt{3})\pi}{6}$, but of course there are multi-infinitely other correct expressions. This type of instruction is common in VCE exams, and is almost always misconceived. It is reasonable to ask for an answer to be stated in a particular form, but the form must be purposeful and the answer in that form unique.

(d) SM Exam 2, QB4(d)

How far does the ball travel during the first four seconds after passing through O?

The "passing through" framing is unnecessary and distracting: the ball can naturally be considered to have begun its travel at $t = 0$. The question is also ambiguous, since it is unclear whether arc length or Euclidean distance is being requested.

(e) MM Exam 1, Q6(c)(ii) and (iii)

Part (ii) asks for "the smallest positive value for a", which is confused: a has already been declared (implicitly) to be positive, and a has a single unknown value, not a set of values. Part (iii) then begins "Hence ...", but this confuses what has been done in part (ii): determining the smallest positive/possible value of a is not the same as declaring a to have that value.

(f) MM Exam 1, Q8(c)

> *Consider the average value of the function f over the interval $x \in [0, k]$, where $k \in [0, 2]$.*

The inclusion of "$x \in$" is meaningless and confusing. The value $k = 0$ should have been excluded from consideration, since this moves the attention from the continuous to the discrete, and it seems that averages of discrete functions are not within the VCE Study Design.

(g) MM Exam 2, MCQ9

The question asks for "the shortest distance" between two specific points, rather than simply asking for the distance, which is very badly misleading, suggesting that some max-min computation might be required.

4 Poorly Conceived Questions on the 2022 Exams

Here, we provide examples of poorly conceived and/or poorly constructed and/or shallow questions from the 2022 exams; there are many more. The issues with these questions may not amount to errors, but in some ways these questions are even worse: they fail to test proper mathematical thought and computation in a clear and coherent manner. We shall not provide many details of the questions here: it is up to any reviewer to undertake a proper analysis of these and all questions, and consider our suggested, and all, shortcomings.

(a) SM Exam 1, Q4

A simple partial fractions question, worth four marks, which can easily be separated by inspection.

(b) SM Exam 1, Q6(b)

An awkwardly constructed and needlessly complicated vector proof question, with too much scaffolding.

(c) SM Exam 2, MCQ8

A ill-considered and inaccurately worded "direction field" (slope field) question, requiring students to look too carefully to determine the correct answer.

(d) SM Exam 2, MCQ9

A badly executed inverse Euler's method question. A question of this form could be good, but would need to be much cleaner, with simpler numbers, enabling a focus on the algebraic.

(e) SM Exam 2, MCQ10

A needlessly fussy and involved implicit differentiation question: poorly worded and poorly constructed, requiring too careful a consideration of signs.

(f) SM Exam 2, MCQ20

In principle a very good question, but much too long a prompt and much too involved for a multiple choice question.

(g) SM Exam 2, QB3

A very poor differential equations question. Mostly routine, if somewhat meaningless, but the wording for (d) and (e) is so heavy, clumsy and opaque as to be seriously misleading. There is no reason for the first particle to pass through O, rather than beginning at O, and there is no reason for the second particle to begin two seconds later. In principle, (e) is a good question, but the interesting algebra is lost in the mess.

(h) SM Exam 2, QB4

A contrived kinematics question, testing simple CAS skills rather than decent algebra.

(i) MM Exam 1, Q4

A poorly conceived, repetitive and very badly worded probability question. The mathematics being tested is simple, the question consisting of little more than an invitation for students to trick themselves into attempting to calculate probabilities for sampling without replacement.

(j) MM Exam 1, Q7

A long, aimless question, testing almost nothing of substance. It would have been almost impossible to have worded clearly, and it was not. The grading for this question should be very closely scrutinised.

(k) MM Exam 2, MCQ6

Too busy a question, made needlessly busier by the function notation. Students have to hunt too carefully for the correct answer.

(l) MM Exam 2, MCQ14

A pointless test of pushing buttons.

(m) MM Exam 2, MCQ16

In principle a reasonable question, but made pointless by CAS.

(n) MM Exam 2, MCQ18

In principle a reasonable question, but made ugly by CAS. Such a question would be significantly improved by using simpler numbers, or being framed algebraically.

(o) MM Exam 2, QB2

An aimless, pseudo-modelling question, investigating and testing nothing of worth or interest. Part (e) is simply absurd.

(p) MM Exam 2, QB3

A long and contrived pseudo-modelling question, to be answered almost entirely with CAS. The story is pointless and distracting. Although the question is very long and worth 14 marks, nothing is tested to any significant depth.

5 Poor Writing on the 2022 Exams

The questions flagged in Sections 3 – 5 already provide many examples of poor writing, but it is important to understand how consistently poor the writing of VCE mathematics exams has become. Most questions could be improved with proper editing, and very many are so poorly written as to be difficult to read or much worse.

Evidently, a low priority has been put on writing questions in as clear and as short and as straightforward a manner as possible. Rather, the aim appears to have been inflexible adherence to a pedantic, overly-prescriptive and hence confusing formalism, and to following established but pointless conventions. Such needlessly confused wording must disadvantage

ESL students. Such wording will also disproportionately disadvantage stronger students, who will potentially spend time looking for nuances that are not there. Hence, such wording indirectly benefits students trained not to read and not to consider the words, but rather trained to pick out key words in the salad and guess the intended question.

We provide a few examples here, but the list could be much, much longer. The very long, multi-part questions are too awkward to quote here, but are worth particular scrutiny. Seldom does the framing and length of these questions offer anything but confusion.

(a) SM Exam 1, Q8

> *A body moves in a straight line so that when its displacement from a fixed origin O is x metres, its acceleration, a, is $-4x\,\text{ms}^{-2}$. The body accelerates from rest and its velocity, v, is equal to $-2\,\text{ms}^{-1}$ as it passes through the origin. The body then comes to rest again.*
>
> *Find v in terms of x for this interval.*

The wording of this very standard kinematics question is not formally wrong, but it is crowded and disordered. More clearly, one could ask,

A body moves in a straight line, and at the origin O has a velocity of $-2\,\text{ms}^{-1}$. The acceleration of the body is $-4x\,\text{ms}^{-2}$, where x is the displacement of the body from O in meters. Find the velocity of the body in terms of x until the body comes to rest.

(b) SM Exam 2, MCQ6

> *Given $z = x + yi$, where $x, y \in R$ and $z \in C$, an equation that has a graph that has two points of intersection with the graph given by $|z - 5| = 2$ is ...*

This is a very common phrasing, where a fussy pile of standard framing comes before the critical information, in this case the equation for the relation. An alternative:

Consider the relation $|z - 5| = 2$, where $z = x + yi$, and $x, y \in R$. A second relation for which the graphs of the two relations intersect at two points is ...

(c) SM Exam 2, MCQ18

> *The time taken, T minutes, for a student to travel to school is normally distributed with a mean of 30 minutes and a standard deviation of 2.5 minutes.*
>
> *Assuming that individual travel times are independent of each other, the probability, correct to four decimal places, that two consecutive travel times differ by more than 6 minutes is*

This is needlessly dense and tangled. The information and instruction should have been arranged so that the final sentence was simply "The probability that two consecutive travel times differ by more than 6 minutes is ...".

(d) MM Exam 1, Q2(a)

> *Find the rule for an antiderivative of* $g(x)$.

The phrase "the rule for" is, always, unnecessary and muddying, and is ubiquitous in VCE mathematics. Also, and astonishingly, it appears that the intention of "an antiderivative" is the literal meaning, that *any* antiderivative would have been accepted as an answer, with or without $+c$, with or without $+59$. *However*, the expression "the antiderivative" in VCE triggers that $+c$ is required in the answer. To concern students with such micro-interpretation is absurd. It could all be avoided by simply, always, asking for the general antiderivative of the function.

(e) MM Exam 1, Q6

> *The graph of* $y = f(x)$*, where* $f : [0, 2\pi] \to R, f(x) = 2\sin(2x) - 1$ *is shown below.*
>
> *On the axes above, draw the graph of* $y = g(x)$*, where* $g(x)$ *is the reflection of* $f(x)$ *in the horizontal axis.*
>
> *Find all values of* k *such that* $f(k) = 0$ *and* $k \in [0, 2\pi]$.

Such convoluted ordering, and the inclusion of fussy and often gratuitous detail is ubiquitous. An alternative:

Consider the function $f(x) = 2\sin(2x) - 1$ *on* $[0, 2\pi]$*. The graph of* f *is shown below.*

Let the function g *be the reflection of* f *across the* x*-axis. Draw the graph of* g *on the axes above.*

Solve the equation $f(x) = 0$.

(f) MM Exam 1, Q8

> *Part of the graph of* $y = f(x)$ *is shown below. The rule* $A(k) = k\sin(k)$ *gives the area bounded by the graph of* f*, the horizontal axis and the line* $x = k$.

The phrase "Part of" is unnecessary and distracting: if there is any other part of the graph, it is never considered. The second sentence is a tangle, including the needless "The rule". An alternative:

The graph of the function f *is shown below. The area bounded by the graph of* f*, the* x*-axis and the line* $x = k$ *is given by the function* $A(k) = k\sin k$.

(g) MM Exam 2, MCQ5

> *The largest value of a such that the function* $f:(-\infty, a] \to R, f(x) = x^2+3x-10$, *where* f *is one-to-one, is ...*

At minimum the "where" should be "for which", but the entire sentence should be restructured, and would be best split into two. In general, preliminary information for a question should be clearly separated from and come prior to the final statement of the question, and this is too rarely done in VCE mathematics exams.

(h) MM Exam 2, QB1(b)

> *State the derivative of* f *with respect to* x.

This is literally using seven words where one would have sufficed: "'Find $f'(x)$" is all that was required. Such verbiage is standard.

6 The Overall Structure of the 2022 Exams

Above, we have provided many examples of flawed questions, but the VCE mathematics exams should be also be reviewed as a whole, noting the weighting and predominance of CAS. Particular comparison should be made to NESA's HSC exams, with consideration given to the extent and depth of examinable material. To this end, we provide here the mark breakdown for each of the four 2022 VCE exams:

Specialist Mathematics Exam 1 (1 hour, tech-free):

$$(3 \times 1) + (5 \times 2) + (5 \times 3) + (3 \times 4)$$

Specialist Exam 2 (2 hours, CAS):

$$(20 \times 1) + (12 \times 1) + (18 \times 2) + (4 \times 3)$$

Methods Exam 1 (1 hour, tech-free):

$$(9 \times 1) + (8 \times 2) + (5 \times 3)$$

Methods Exam 2 (2 hours, CAS):

$$(20 \times 1) + (23 \times 1) + (10 \times 2) + (3 \times 3) + (2 \times 4)$$

The depth, or lack thereof, of the 3-mark and 4-mark questions warrants particular scrutiny.

7 Delay and Dishonesty in the Exams and Reports

It should be clear to any objective and knowledgable observer that VCE mathematics exams are seriously, systemically flawed. It should also be clear to VCAA. If, however, there is a concern within VCAA to improve VCE mathematics exams, it is not apparent.

VCAA's approach to providing public information about the VCE exams lacks transparency, and the timing and manner of the release of their exam reports actively inhibits external feedback. VCAA is extraordinarily slow to publish exams and reports, the reports are less than informative, and by the reports are much less than candid about flaws in the exams.

To summarise and to compare VCAA's timeline:

- It takes over a month for VCAA to publish the exams, whereas it takes NESA a couple of days;
- It takes at least five months for VCAA exam reports to appear, whereas NESA publishes full sample solutions within about a month, and detailed exam feedback by the beginning of the following school year;
- VCAA reports include only answers and sketches of solutions, whereas NESA publishes full sample solutions.

We cannot know whether this delay and lack of full information is conscious tactic or just self-serving bureaucratic incompetence, but it hardly matters, since the effect is the same: the possibility of and interest in important public scrutiny and debate is diffused. By the time the exam reports have come out, not only has the horse bolted, the horse is well on its way to graduating from university. As for VCE teachers, they are by then too way busy coping with the current year to seek battle on the previous year's nonsense. And so the cycle continues.

The possibility of public pressure is also complicated by VCAA's stubborn unwillingness to acknowledge error, much less apologise for it. The willingness of VCAA to obfuscate, or to simply be silent, is legendary. We provide some examples regarding the 2021 exams.

(a) SM 2021 Exam 2, QB2(a)

The error here is almost identical to the error in SM 2022 Exam 2, MCQ4, which we indicated above. The report acknowledges the "alternative solution" to part (ii), and there are words to the effect that working "correct and complete across" parts (i) and (ii) was "accepted". But the fundamental error is not acknowledged, and the consequential meaningless of the question in part (i) is not acknowledged. This failure amounts to deliberate deceit. It is unprofessional, unethical and cowardly.

(b) SM 2021 Exam 2, QB6(c)

The preamble to the question referred to "main daily sales" rather than the intended "mean daily sales". The error was acknowledged in the report, and one can argue how serious the error was, if not how foolish; the report is at pains to assure us that "students were not disadvantaged", although it is unclear what the evidence for that might be. The problem is that VCAA altered the wording for the published exam, without any indication that they had done so. It is astonishing that VCAA considers such a stealth edit to be acceptable.

(c) MM 2021 Exam 2, QB1(f)

The preamble to the question referred to a "box's length" being "still twice its width", rather than this being true of the rectangle from which the box was constructed. It is possible that the error did not affect many students, but that is unclear, and at minimum the error had to be acknowledged. The report is silent.

(d) MM 2021 Exam 2, QB5

The entire question is a mess, but part (g) is incomprehensible and how students were expected to solve it is unknowable. The exam report provides nothing except the intended answer, a "common incorrect answer" and a self-centred complaint that "an exact answer was required".

8 Conclusion: the Writing and Vetting of Exams

We do not know who writes or vets VCE mathematics exams. We do not how the teams and their duties are organised. We do know that the current process is not remotely close to working as it should. Whatever the qualifications and skills of its members, and despite their undoubted good intentions, the writing and vetting teams are functionally incompetent.

Looking from the outside, weighing the evidence provided by the exams that VCAA produces, there appear to be three overlapping reasons for the current failure.

First of all, there is a clear lack of mathematical expertise. There are errors on the exams that no competent and attentive mathematician would miss. There are questions and wordings of questions that no mathematician would permit. There is a shallowness to the exams to which any mathematician would object.

Secondly, there appears to have been a process of cultural ossification. There are types of questions and conventions for the wording of questions that we cannot imagine would be acceptable in any other setting. This culture seems so entrenched that even when it leads

to questions being meaningless or in error, this can too often go uncorrected.

Thirdly, there appears to have developed within VCAA a self-defensive culture of pedantry and certainty. The principle concern appears not to be to write questions as clearly as possible, not to assess questions with reasonable generosity, but to take an overly formal and legalistic approach, in an attempt to preclude the possibility of complaint. Ironically, this culture, which is highly disrespectful of students, naturally becomes a major source of dissatisfaction.

Last year, we met with VCAA representatives, in order to discuss VCE exams. We were grateful for the meeting. We felt we were taken seriously and VCAA's representatives were tolerant of our strong criticism. Two things, however, stood out from the meeting. First of all, VCAA offered no defense of their exams, but also offered no suggestion of how the process of producing VCE exams might be improved. The second thing that stood out was VCAA's response to *our* suggestion of how the process of producing VCE exams might be improved.

During that meeting, we offered to vet VCE exams, for free. Alternatively, we offered to arrange for other mathematicians to vet the exams. (We are aware that such vetting by strong mathematicians occurs in at least two other States.) Our offer was declined on the spot. The reason, which had something to do with the "democratic" nature of VCAA process, made absolutely no sense to us. Whatever the reason, and whatever the sense or otherwise in that reason, we regarded the outcome, the rejection of our offer, as highly regrettable.

Serious change is required in the production of VCE mathematics exams, and in the state of VCE mathematics more generally. We offered to assist in that change, and our offer was declined. That is ok, but only if VCAA is also prepared to acknowledge that such change is required, and develops good and proper and public plans to effect this change.