Major Errors on the 2022 Victorian VCE Mathematics Exams

Prof. Burkard Polster
burkard.polster@monash.edu

Dr. Marty Ross
marty.ross@monash.edu

October 28, 2023

In December 2022, following prior discussions, we submitted a critique of the 2022 VCE Mathematics Exams to the Victorian Curriculum and Assessment Authority. This critique included what we termed major mathematical errors on the exams. Our critique was requested by VCAA, so as to form the basis of an external review. The review was subsequently conducted by Deloitte, and VCAA also sought the opinion of "two independent state Education bodies".

On October 15, 2023, VCAA emailed us a brief summary of their external review. From this summary it appears that the sole concern of Deloitte's review was on policy and conduct; the summary gives no indication that Deloitte evaluated any exam questions in any manner, and in particular for correctness. The only reference in VCAA's summary to possible errors is in reference to the state Education bodies, who reviewed some exam questions (but with no specification in the summary of which questions). VCAA's summary of the Education bodies' conclusions is as follows:

"The independent state bodies found that there were no major mathematical errors within any of the questions, however identified some room for improvement in the language and grammar used and level of marks provided per question."

Below are the comments from our critique on what we termed major mathematical errors (Section 2 of our full critique), followed by screenshots of the relevant exam questions and the exam reports' summary of these questions.[1]

[1] The exam reports only appeared in February 2023, and thus did not form part of our critique. The complete exams and exam reports are available at VCAA's Specialist Mathematics and Mathematical Methods pages.

1 Major Mathematical Errors on the 2022 Exams

Here, we list major mathematical errors on the 2022 Specialist Mathematics (SM) and Mathematical Methods (MM) exams. We consider an error to be "major" if as a consequence the question has no proper answer or, even if somehow answerable, the question is so mathematically flawed as to very likely to have caused students considerable confusion. For the sake of brevity, we shall not reproduce questions in full: the complete questions should be read in conjunction with our comments.

(a) SM Exam 1, Q3(b)

> *Following the service [of a coffee machine], the mean time taken to dispense 25 cups of coffee is found to be 9 seconds.*

This is a blatant error: following on from part (a), the intention was clearly for the mean time to be 9 seconds *per cup*. However this question might have been subsequently graded, and that is an active concern, many students would have lost time and composure.

To be crystal clear, and this should be confirmed, a student who has answered 3(b) in the manner in which it was *unambiguously* written must have been eligible for full marks. For VCAA to have done otherwise would have been unconscionable. We cannot know since, over a month after the exam, VCAA has not even published exam answers. (By comparison, NESA has already released full sample solutions to the 2022 HSC mathematics exams.)

Given the manner in which VCAA tends to excuse itself of error (see Section 7), we provide some preemptive argument on the proper approach to grading this flawed question. Notwithstanding the implausibility of the quoted sentence in the light of part (a), it is not the students' responsibility to be second-guessing a clear statement of fact in a question. Similarly, although the resulting confidence interval straying into the negative may be considered a red flag, this can occur in the correct solution of a well-posed problem. But again, it is simply not the students' responsibility to be concerned with any of this.

(b) SM Exam 2, MCQ4

> *The polynomial* $p(z) = (z - a)(z - b)(z - c)$ *has complex roots* a, b *and* c*, where* $\mathrm{Re}(a) \neq 0, \mathrm{Re}(b) \neq 0, \mathrm{Re}(c) \neq 0$ *and* $\mathrm{Im}(b) = 0$*. When expanded, the polynomial is a cubic with real coefficients.*

The question simply cannot be answered: none of the statements that follow are "necessarily true", as is illustrated by the simple example $a = 1, b = 2, c = 3$.

It is not entirely clear how such an inexcusable error has occurred, but one must note the atrocious wording, which is an open invitation to error – and to students' misinterpretation – and is inexcusable in its own right. This error is also doubly egregious, since an essentially identical error occurred in 2021 (SM Exam 2, QB2(a)). VCAA's deceit following on from their 2021 error is discussed in Section 7.

(c) SM Exam 2, MCQ19

The question provides enough information to determine the mean cost for an item, but then asks students to determine a confidence interval for this mean. That can formally be done, but it is absurd to find a confidence interval for a known mean, and is thus also astonishingly confusing.

(d) SM Exam 2, QB6(f)

The question asks for the probability that the volume of liquid in a can is below a certain amount. The can masses and the total (can + liquid) masses have been given to be normally distributed, with the parameters provided, but no independence assumption has been declared. As such, the students cannot answer the question without making an unwarranted assumption.

It is not remotely the students' responsibility to resolve the issues with this question, but it also must be noted that it is not even clear which variables might reasonably be assumed independent, or why. It should also be noted that the answer to the exam question depends upon the covariance of the variables; as it happens, and even assuming that the liquid mass is a normal random variable, the answer cannot be determined to the requested three decimal places. Of course any such analysis is beyond the concern of a student taking the exam but, given VCAA's past willingness to minimise the effect of their errors, it should be kept in mind.

(e) MM Exam 2, QB4(e)(ii)

> *Explain why the domain of $A(k)$ does not include all values of k.*

The question is expressed badly, and is effectively meaningless for multiple reasons. First, it has been declared at the outset that $k > 0$. Second, the question has introduced the function $A(k)$ implicitly for all positive k as if this is not a concern, and before part (i): any querying of the domain should have been done immediately. Third, the question in part (i) only makes sense if one accepts that $A(k) = 0$ also makes sense, implying that all such (positive) k would be in the domain of A, irrespective of whether the region is empty. Fourthly, the conclusion of the question is best though to be false: there is no reason why one cannot consider a region to be empty, and consequently $A(k)$ make sense for all (positive) k.

2 The 2022 Exam Questions and Their Reporting

The following are screenshots of questions from the 2022 exams, and of the summaries of these questions in the examination reports. The reports were only released by VCAA in February 2023. Prior to that, VCAA gave no public indication of what VCAA regarded as the correct answers or of how the questions were to be assessed.

(a) Specialist Mathematics Exam 1, Q3

Question 3 (4 marks)

The time taken by a coffee machine to dispense a cup of coffee varies normally with a mean of 10 seconds and a standard deviation of 1.5 seconds.

a. Find the probability that more than 34 seconds is needed to dispense a total of four cups of coffee. Give your answer correct to two decimal places. 2 marks

b. The machine is to be serviced. After it is serviced, it is expected that the mean time taken to dispense a cup of coffee will be reduced, but that the standard deviation will remain the same.

Following the service, the mean time taken to dispense 25 cups of coffee is found to be 9 seconds.

Find a 95% confidence interval for the mean time that the machine takes to dispense a cup of coffee following the service. Give your answer in seconds, correct to one decimal place. 2 marks

Question 3b.

Mark	0	1	2	Average
%	16	32	51	1.3

$$\left(9-2\times\frac{1.5}{\sqrt{25}},9+2\times\frac{1.5}{\sqrt{25}}\right)=(9-2\times0.3,9+2\times0.3)$$
$$=(8.4,9.6)$$

Students needed to use the formula from the formula sheet with $z=1.96$ or $z=2$, presenting a confidence interval, using interval notation. Some basic arithmetic errors were observed.

(b) Specialist Mathematics Exam 2, MCQ4

Question 4

The polynomial $p(z) = (z - a)(z - b)(z - c)$ has complex roots a, b and c, where $\text{Re}(a) \neq 0$, $\text{Re}(b) \neq 0$, $\text{Re}(c) \neq 0$ and $\text{Im}(b) = 0$. When expanded, the polynomial is a cubic with real coefficients.

Which one of the following statements is **necessarily** true?

A. $a + c = 0$

B. $|a| = |c|$

C. $a - c = 0$

D. $|a| = |b|$

E. $a + b + c = 0$

Question	Correct answer	% A	% B	% C	% D	% E	Comments
4	B	7	**61**	14	10	7	

(c) Specialist Mathematics Exam 2, MCQ19

Question 19

The cost, $\$C$, of producing a particular item is a function of its mass, m grams, where $C = 0.3m + 0.5$

The masses of the items are normally distributed with a mean of 7 grams and a standard deviation of 0.1 grams.

When 100 items are produced, a 95% confidence interval for the average cost per item is closest to

A. (2.502, 2.698)

B. (2.555, 2.645)

C. (2.584, 2.616)

D. (2.594, 2.606)

E. (2.598, 2.602)

Question	Correct answer	% A	% B	% C	% D	% E	Comments
19	D	7	8	20	**53**	11	

(d) Specialist Mathematics Exam 2, Section B Q6

Question 6 (9 marks)

A company produces soft drinks in aluminium cans.

The company sources empty cans from an external supplier, who claims that the mass of aluminium in each can is normally distributed with a mean of 15 grams and a standard deviation of 0.25 grams.

A random sample of 64 empty cans was taken and the mean mass of the sample was found to be 14.94 grams.

Uncertain about the supplier's claim, the company will conduct a one-tailed test at the 5% level of significance. Assume that the standard deviation for the test is 0.25 grams.

a. Write down suitable hypotheses H_0 and H_1 for this test. 1 mark

b. Find the p value for the test, correct to three decimal places. 1 mark

c. Does the mean mass of the random sample of 64 empty cans support the supplier's claim at the 5% level of significance for a one-tailed test? Justify your answer. 1 mark

d. What is the smallest value of the mean mass of the sample of 64 empty cans for H_0 not to be rejected? Give your answer correct to two decimal places. 1 mark

The equipment used to package the soft drink weighs each can after the can is filled. It is known from past experience that the masses of cans filled with the soft drink produced by the company are normally distributed with a mean of 406 grams and a standard deviation of 5 grams.

e. What is the probability that the masses of two randomly selected cans of soft drink differ by no more than 3 grams? Give your answer correct to three decimal places. 2 marks

f. 1 mL of soft drink has a mass of 1.04 grams. Assume that the empty cans have a mean mass of 15 grams and a standard deviation of 0.25 grams.

What is the probability that a can of soft drink selected at random contains less than 375 mL of soft drink? Give your answer correct to three decimal places. 3 marks

Question 6f.

Mark	0	1	2	3	Average
%	66	10	2	22	0.8

Mass of drink = Mass of full can − mass of empty can

Mass of drink $\sim \mathrm{N}\left(406-15,\ 5^2+0.25^2\right)$

Volume of drink $\sim \mathrm{N}\left(\frac{406-15}{1.04}, \frac{5^2+0.25^2}{1.04^2}\right)$

$\Pr(V<375)=0.421$.

Many correct responses worked directly in terms of mass with $\Pr(\text{Mass}<375\times1.04)=\Pr(\text{Mass}<390)=0.421$.

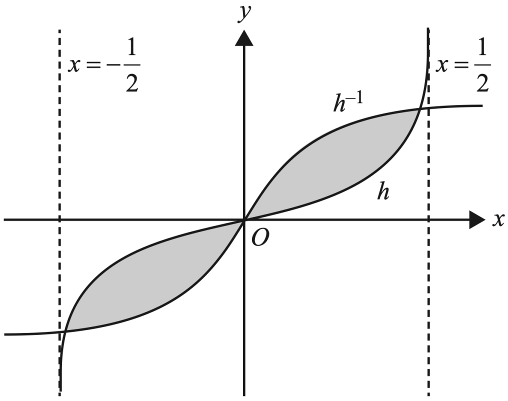
(e) Mathematical Methods Exam 2, Section B Q4(e)

e. Let h be the function $h:\left(-\frac{1}{2}, \frac{1}{2}\right) \to R$, $h(x) = \frac{1}{k}\left(\log_e\left(x+\frac{1}{2}\right) - \log_e\left(\frac{1}{2}-x\right)\right)$, where $k \in R$ and $k > 0$.

The inverse function of h is defined by $h^{-1}: R \to R$, $h^{-1}(x) = \dfrac{e^{kx}-1}{2(e^{kx}+1)}$.

The area of the regions bound by the functions h and h^{-1} can be expressed as a function, $A(k)$.

The graph below shows the relevant area shaded.



You are not required to find or define $A(k)$.

i. Determine the range of values of k such that $A(k) > 0$. 1 mark

ii. Explain why the domain of $A(k)$ does not include all values of k. 1 mark

Question 4eii.

Marks	0	1	Average
%	91	9	0.1

If $h \geq h^{-1}$ for $x > 0$ or ($h \leq h^{-1}$ for $x < 0$) there will be no bounded area. There will be no bounded area for $0 < k \leq 4$.

This question was not answered well. Some students discussed $k \leq 0$ or referred to the asymptotes.