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Mathematics Goes to the Movies

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Chapter 3

Escalante Stands and Delivers

This is one of our favorites. *Stand and Deliver* (1988) is the story of Jaime Escalante, a mathematics teacher from Bolivia who comes to Garfield High, a poor and poorly run school in the Latino area of East Los Angeles. He is a humorous, charismatic and demanding teacher, who persuades his class to take the Advanced Placement exam in mathematics. The class performs so well that they are accused of cheating by the testing authorities.¹

The story is substantially true, with important qualifications which we detail at the end.² The real-life Escalante was made famous by the events at Garfield High, culminating in his winning the Presidential Medal for Excellence in Education in 1988, and being inducted into the National Teachers Hall of Fame in 1999. Sadly, Jaime Escalante died this year. But right up until his death, he was actively involved in promoting mathematics, in sharing his passion for mathematics. The movie succeeds primarily because Edward James Olmos, who plays Escalante, captures this passion.

Stand and Deliver is not a story about mathematics, nor even really a story about mathematics teaching; it is a story of the human spirit. Nonetheless, the movie contains lots of mathematics. And, unlike the vast majority of school-based movies, the mathematics is not only internally correct, it is contextually correct: the mathematics the students fight with is what they would have had to fight with. As the students progress, the mathematics progresses, from arithmetic and algebra, to trigonometry, to calculus. We shan't itemize it all, but certain scenes are notable, giving the flavour of Escalante's style and the role it plays in the drama.

¹ The Advanced Placement Program is a method for students in the US to gain university credit for their work in high school. The AP subjects are usually taught at school, but they are administered and independently assessed by the College Board, which is effectively the Bad Guys in *Stand and Deliver*.

² For a clear and thoughtful telling of the story see Matthews, Jay, *Escalante: The best teacher in the World*, Henry Holt, New York, 1988.

3.1 Welcome to the Finger Man

When Escalante first appears, he is confronted by a wild and totally disinterested class: *I don't need no math. I got a solar calculator with my dozen doughnuts.* The class is then ended by a premature bell, but the next class sets the tone. Escalante appears wearing a butcher's apron and wielding a huge cleaver. Chopping apples, he illustrates fractions to the students. Two cholos, Chuco and Angel, appear late, and Escalante enters into a whispered discussion with Chuco:

10:30

ESCALANTE: You know the times tables?

CHUCO (sticking up his thumb): I know the ones, [sticking up his second finger] the twos, [sticking up his middle finger, in an obscene gesture] the threes ...

ESCALANTE: Finger Man. I heard about you. Are you the Finger Man? I'm the Finger Man too. You know what I can do? [He holds his ten fingers splayed.] I know how to multiply by 9. 9 times 3: 1, 2, 3 [counting off with his fingers, to leave two fingers on one side of his crooked finger, and seven on the other side]. Whaddyou got? [wiggling his fingers] 27! 6 times 9: 1, 2, 3, 4, 5, 6. Whaddyou got? [wiggling his fingers] 54! Yeah. Want a hard one? How about 8 times 9? 1, 2, 3, 4, 5, 6, 7, 8. Whaddyou got? [wiggling his fingers] 72!



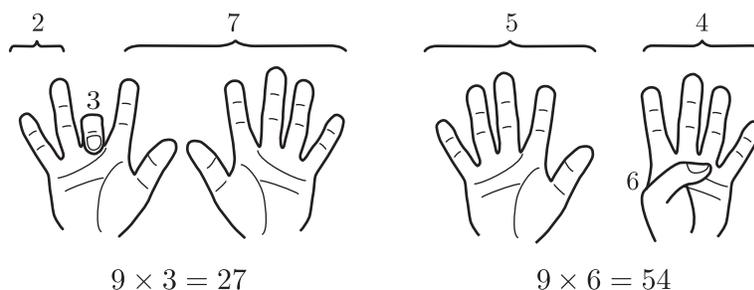


Fig. 3.1 Escalante demonstrates how to multiply by 9.

Escalante has demonstrated a clever trick for multiplying by 9 (which works because the sum of the digits in the answer is equal to 9); the trick is cute in itself, but it also contains a message relevant to the drama to come.³

3.2 Filling the Hole

In his next class Escalante explains zero, and negative numbers.

15:00

ESCALANTE: You ever dig a hole? The sand that comes out of the hole, that's a positive. The hole is a negative. That's it. Simple. Anybody can do it. Minus two plus two ... equals ... [He cajoles Angel into answering]. You going to let these burros laugh at you? ...

ANGEL (sullenly): Zero.

ESCALANTE: Zero. You're right. Simple. That's it! Minus two plus two equals zero. He just filled the hole. Did you know that neither the Greeks nor the Romans were capable of using the concept of zero? It was your ancestors, the Mayas, who first contemplated the zero, the absence of value. True story. You burros have math in your blood.

Escalante is substantially correct. The Mayans, unlike the Greeks and the Romans, had a certain conception of zero, of positional notation for numbers. For example, in the number 23 the 2 means “two tens”, and in the number 203, the 2 means “two hundreds”: it is the 0 which acts as a placeholder, keeping the distinction clear. However, that is not the same as understanding 0 to be a number in and of itself, as in the equation $2 - 2 = 0$; The Mayans seemingly did not have this more subtle understanding of 0. Around 800 AD

³ The Columbo-like detective Furuhata Ninzaburō performs a similar fingerman trick, proving $7 \times 8 = 56$ to introduce the episode *Murder of a Mathematician* (1995).

(and prior to the Mayans), a succession of Indian mathematicians struggled, ultimately successfully, with the true concept of 0 as a number.⁴

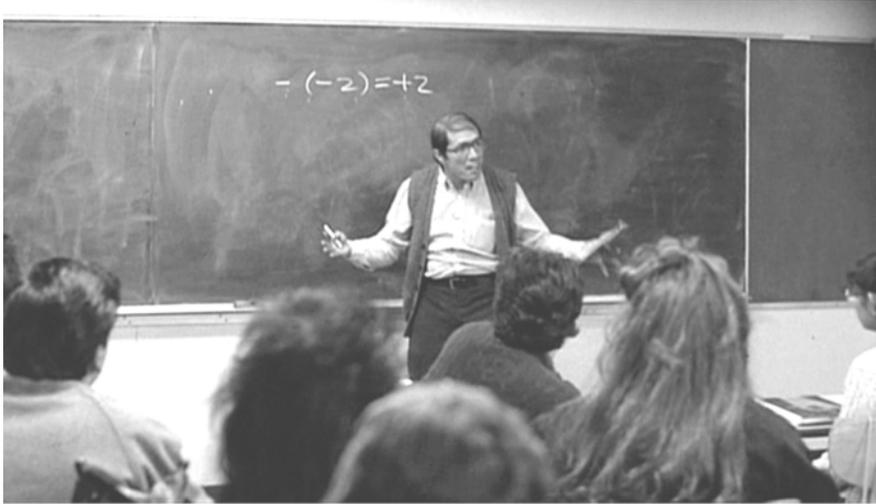


Fig. 3.2 Escalante hammers home the multiplication of negative numbers.

After his description of zero, Escalante goes on to discuss parentheses and negative numbers:

ESCALANTE: Oralé! Okay! Parentheses means multiply. Every time you see this, you multiply. A negative times a negative equals a positive. A negative times a negative equals a positive. Say it! A negative times a negative equals a positive. Say it!

CLASS AND ESCALANTE: A negative times a negative equals a positive.

ESCALANTE: Again!

CLASS AND ESCALANTE: A negative times a negative equals a positive. A negative times a negative equals a positive.

ESCALANTE: I can't hear you!

CLASS AND ESCALANTE: A negative times a negative equals a positive.

ESCALANTE: Louder!

CLASS AND ESCALANTE: A negative times a negative equals a positive.

ESCALANTE: Louder!

CLASS AND ESCALANTE: A negative times a negative equals a positive.

ESCALANTE (softly): Why?

This scene emphasises two aspects of the teaching of mathematics: first, the drilling of the fact, until it is accepted as true without question; and secondly

⁴ For a lovely exposition of the history of zero, see Kaplan, Robert, *The Nothing That Is*, Penguin, London, 1999.

the question, asking *why* it is true. (And the reader may well ask, why indeed is a negative times a negative equal to a positive?)

3.3 Let X Be the Number of Girlfriends ...

The movie continues with a number of scenes of Escalante teaching algebra. The mathematics mostly appears as background illustration, including a great image of Angel as sinner:

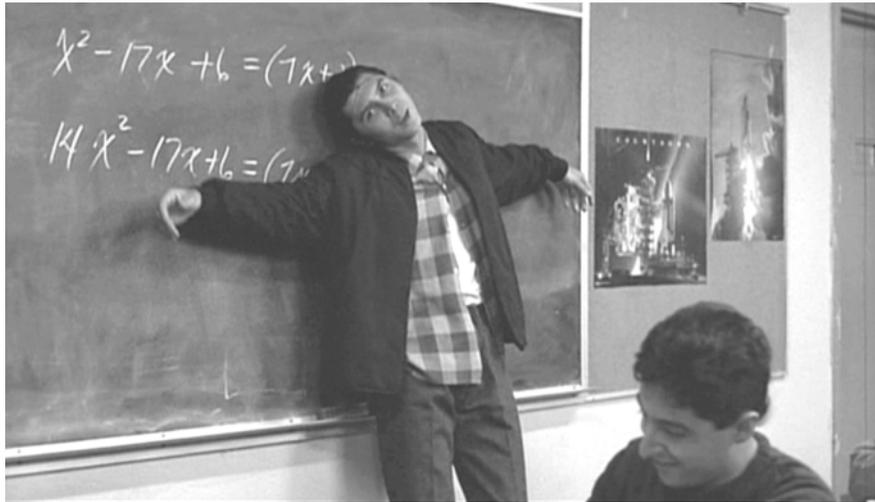


Fig. 3.3 Angel being sacrificed on the Altar of Polynomials.

After the algebraic introductions, we see a problem discussed in detail:

32:20

CLASS (reading from the blackboard): **Juan** has five times as many girlfriends as **Pedro**. **Carlos** has one girlfriend less than **Pedro**. The total number of girlfriends between them is twenty. How many does each gigolo have?

ESCALANTE: ... Anybody. [Tito raises his hand.] Think you got it, Einstein? You think you gonna do it?

TITO: Juan is X . Carlos is Y . Pedro is X plus Y . Is Pedro bisexual or what?

ESCALANTE: I have a terrible feeling about you. [Tito blows Escalante a kiss.]

CLAUDIA: Kemo. $5X$ equals Juan's girlfriends?

ESCALANTE: You're good now, but you're gonna end up barefoot, pregnant, and in the kitcheen. [Laughter from the class.]

RAFAELA: Can you get negative girlfriends?

ESCALANTE: No, just negative boyfriends. [He looks up to the heavens.] Please forgive them, for they know not what they dooo!

ANGEL: Carlos has $X-5$ girlfriends, que no?

ESCALANTE: Que no? is right. Que no. [Lupe raises her hand.] The answer to my prayers!

LUPE: May I go to the restroom please? [Laughter.]

ESCALANTE: In ten minutes. Hold it. [He walks up to Javier.] Señor Maya. Hit it.

JAVIER (Javier smugly wiggling his pencil): It's a trick problem, Mr. Kemo. You can't solve it unless you know how many girlfriends they have in common. Right?

ESCALANTE: It's not that they're stupid. It's just that they don't know anything.

JAVIER: I'm wrong?

ANA (who has just appeared at the door): X equals Pedro's girlfriends, $5X$ equals Juan's girlfriends, $X-1$ equals Carlos's girlfriends. X plus $5X$ plus $X-1$ equals 20, so $X=3$. [The class claps, as Ana sits down.]

Ana gives the expected answer, and the algebra is correct, but notice that Javier is also correct. Escalante is expecting the students to assume that no girl is the friend of more than one gigolo; as Javier has noticed, without this assumption, the problem is not solvable.

3.4 Newton was an Idiot

Escalante decides that he will prepare the class for the Advanced Placement mathematics exam (primarily calculus). The parents' permission is required attendance at the extra classes, and we witness one student cajoling her mother.

41:25

CLAUDIA: Mom, calculus is math that Sir Isaac Newton invented so he could figure out planet orbits, but he never bothered to tell anybody about his discovery until this other scientist guy went around claiming he had invented calculus. But the guy was so stupid that he got it all wrong, and so Newton had to go public and correct his mistakes. Don't you think that's neat?

CLAUDIA'S MOM: For a genius, Newton was an idiot.

Claudia is referring to Gottfried Wilhelm Leibniz, the co-inventor of calculus. Claudia's version of the history is correct, except for the implicit suggestion that Leibniz may not have come up with the ideas independently, and the explicit claim that Leibniz was stupid and got it all wrong.

We next see the students, who have come to class very early in the morning. They are being taught a standard calculus technique for calculating volumes

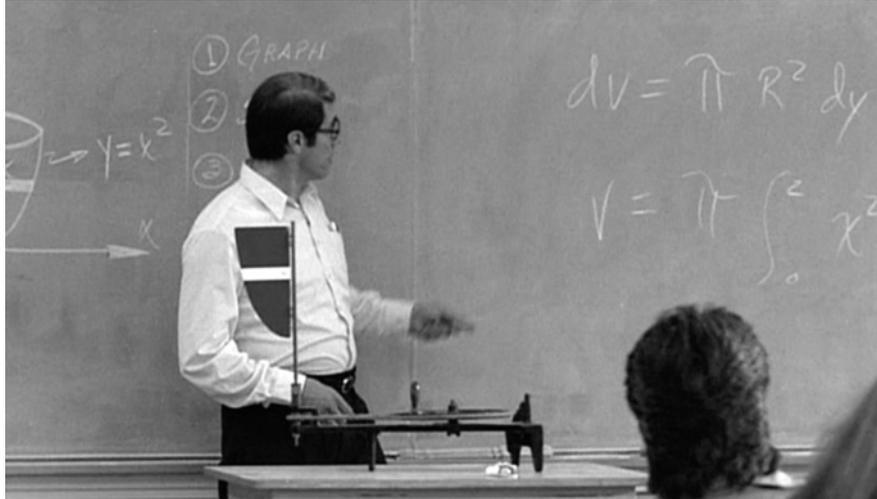


Fig. 3.4 Escalante teaches volume by disks.

of revolution, “by disks”. So the graph $y = x^2$ from $x = 0$ to $x = 2$ is rotated around the y -axis, creating a bowl. The ‘radius’ of the bowl at any height y is then x , which equals x^2 , giving the area πx^2 of a circular slice through the bowl. This leads to Escalante’s formula on the board,

$$V = \pi \int_0^2 x^2 dy.$$

However, the students are struggling merely to stay awake:

44:15

ESCALANTE: One, you got the graph, right here. Two, the strip, the most important part is right here. It’s the radius of rotation. That’s it. Anybody got any questions? [Escalante sees Tito, asleep, and Escalante starts slapping him gently with a towel.] ... Wake up this morning! How are you? ...

TITO (groggily): I was swimming with dolphins whispering imaginary numbers, looking for the fourth dimension.

ESCALANTE (pushing Tito’s head down onto the towel): Good! Go back to sleep. That’s very good.

In the next math scene, we see Pancho at the blackboard struggling, and failing, with an integration by parts problem. He is attempting to compute the integral

$$\int x^2 \sin x dx = -x^2 \cos x + 2x \sin x + 2 \cos x + C.$$

Pancho's calculations are correct but unhelpful, since he has made the wrong choices for u and dv .

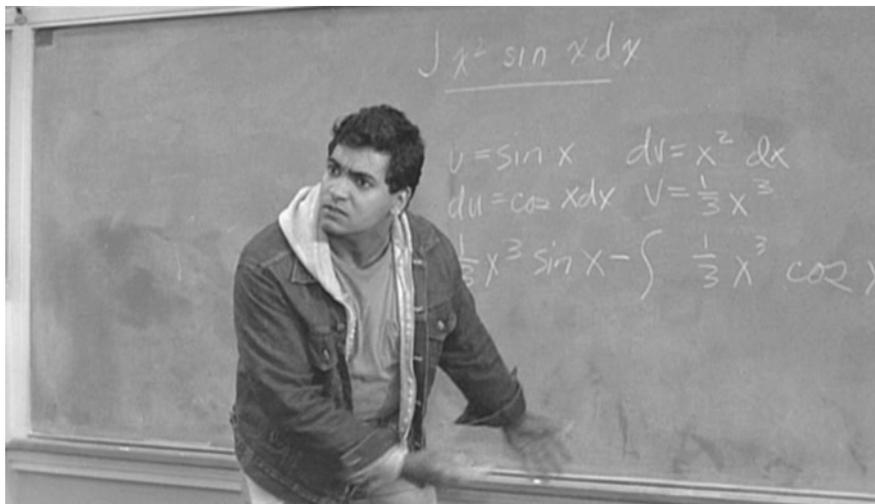


Fig. 3.5 Tito struggles with integration by parts.

48:10

ESCALANTE: Try the shortcut. This is easy. Baby stuff for boy scouts.

PANCHO: Kemo, my mind don't work this way!

ESCALANTE: Tic-tac-toe! It's a piece of cake upside down. Watch for the green light. [Pancho stares at the board, and then thumps it with his fist.]

PANCHO: I've been with you guys two years! Everybody knows I'm the dumbest. I can't handle calculus! ...

ESCALANTE: Do you have the ganas? Do you have the desire?

PANCHO: Yes! I have the ganas!

ESCALANTE: You want me to do it for you!

PANCHO: Yes!

ESCALANTE: You're supposed to say "no"! [Escalante continues with the integration problem, and talks to Pancho as he writes.] *Tic ... tac ... toe. Simple.*

Escalante actually gives a very nice method for performing certain integration by parts problems, where the u term, a power of x , will be differentiated out: the first column contains u and its derivatives, until we arrive at 0; the second column contains $\frac{dv}{dx} = \sin x$ and its antiderivatives; and the third column contains alternating positives and negatives. Then, scoring off as indicated, he obtains all the terms produced by the integration.

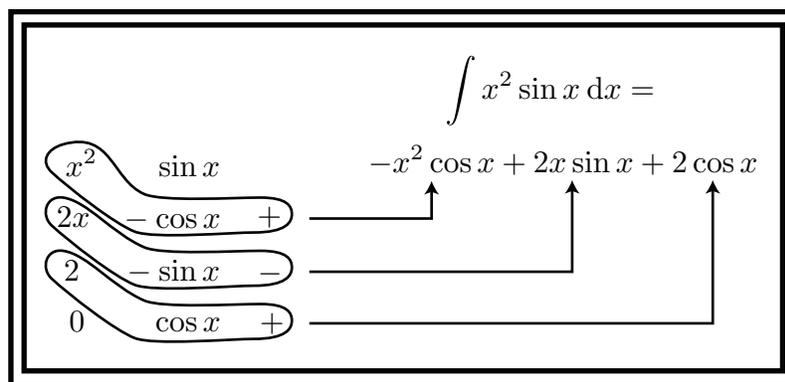
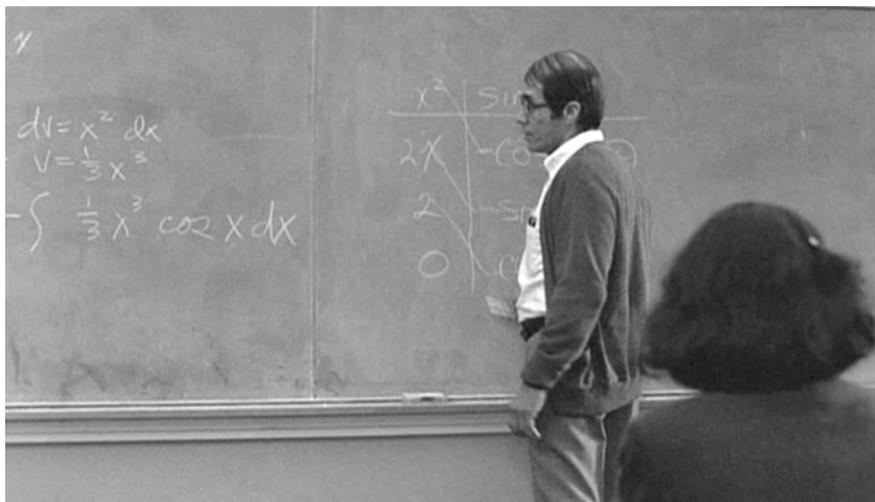


Fig. 3.6 Escalante's shortcut.

Notice that by emphasising his shortcut, Escalante also ignores Pancho's misunderstanding. As with the Finger Man trick for multiplication by 9, this is indicative of Escalante's style (in real life as well as in the movie): he places much more emphasis on drill and techniques than on fundamental understanding. The scene also uses Escalante's magic word: *ganas*. For Escalante, all that matters is *desire*. That is the message of the movie.

Escalante works himself into the ground, to the point of having a minor heart attack. We see this coming when Escalante asks the class about the function $f(x) = 3\sin(2x + \pi)$, but is unable to see what's going wrong.

52:00



Fig. 3.7 Escalante misleads the students.

ESCALANTE: We're looking for the area in the first quadrant bounded by the curve. What are the limits? Anybody.

TITO: 0 to $\frac{\pi}{2}$, sir.

ESCALANTE: Wrong. Lupe.

LUPE: 0 to $\frac{\pi}{2}$?

ESCALANTE: What's wrong with you? This is review.

LUPE: Kemo, I checked my work twice.

ESCALANTE: I'm giving you the graph. Check it again.

ANGEL: No, Kemo, I'm getting the same answer as the gordita.

LUPE: Don't call me gordita, pendejo.

ANA: It's 0 to $\frac{\pi}{2}$, sir.

JAVIER: Yeah. I got the same thing.

ESCALANTE: You should know this. No, no way. You should know this. What's wrong with you? This is review! You're acting like a blind man in a dark room looking for a black cat that isn't there. What's wrong with you guys? I don't believe it. You're giving me a shot from the back! No way! No way!

The problem is that Escalante has misdrawn the graph: the function $f(x) = 3\sin(2x + \pi)$ equals 0 at $x = 0$ and $x = \frac{\pi}{2}$, but the graph is *below* the x -axis

(i.e. in the fourth quadrant) between these limits. Thus, in contradiction to Escalante's graph, the correct limits are $\frac{\pi}{2}$ to π .

Escalante's description of the students is actually a famous quotation by Charles Darwin, describing *all* mathematicians and the very nature of their pursuit. Escalante's use of it as a description of the students (supposedly) making simple mistakes is quite different.

3.5 The Students Stand and Deliver, and Deliver Again.

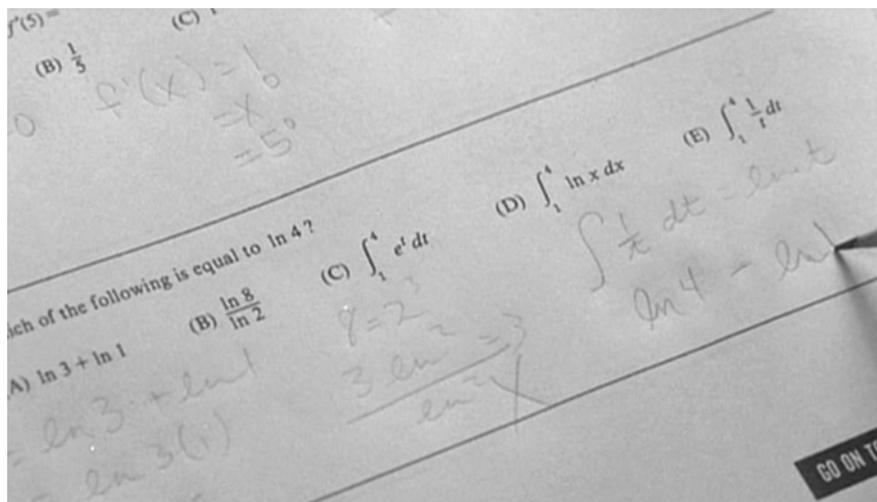


Fig. 3.8 Working on the Advanced Placement exam.

Finally, the students take the Advanced Placement exam. We see them struggle with the questions, and the relief afterwards. But then comes the central drama of the movie, when the students are accused of cheating by the external authorities. Reluctantly, the students retake the exam. In the climactic scene, we hear the students' scores being read out over the phone: they've all passed, and the movie ends with Escalante walking away, pumping his fist in victory.

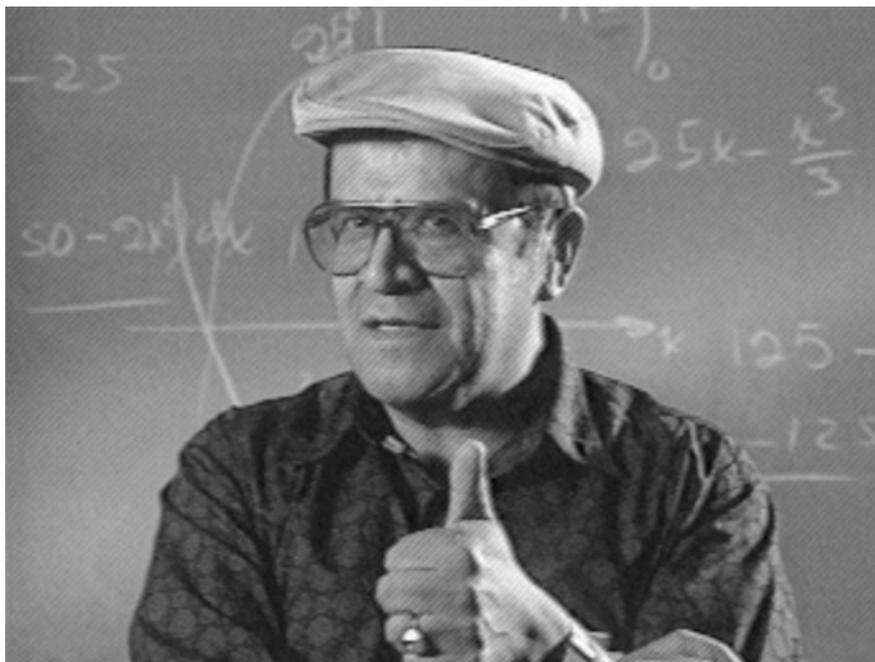


Fig. 3.9 The real Jaime Escalante at Garfield High, 1991.

3.6 Will the Real Jaime Escalante Please Stand Up?

As we remarked above, the story of *Stand and Deliver* is by and large true. The major fictional aspect of the movie was to compress many years of work, with many classes of students, into two years with one class.

Jaime Escalante started at Garfield High in 1974. The school already had an Advanced Placement math program, but it was in disrepair. In 1978 Escalante went to work on it. The first such class started with 14 students; 5 students lasted to take the exam in 1979, of which two students passed.

Each year, the size and the success of Escalante's program grew. *Stand and Deliver* tells the story of the class of 1981-1982, when 18 students took the exam. 14 of the students were (implicitly) accused of cheating; all those who were not accused passed the exam. The accused students were offered an alternate exam: two students declined, having already received university offers. The other 12 accused students sat the alternate exam, and all 12 students passed. As the movie indicates at the very end, Escalante's program then continued to grow, becoming arguably the most successful such program in the country.

Though the compression of time leads to a dramatically powerful movie, it is also importantly misleading. The movie overstates how quickly Escalante

could turn around the mathematics program, and how quickly he could take a student from mathematical ignorance to formal success. This also goes some way to explaining why Escalante's classes continued to do better, year after year.

Another element of confusion in the story concerns Escalante's goal. Through teaching mathematics, Escalante attempted to make his students appreciate their own abilities and possibilities. The goal was not primarily to have the students understand and appreciate the mathematics, the goal was to pass the exam, which was the meaningful and practical achievement for the students. This part of the story is under-emphasised in the movie.

The quick-trick approaches Escalante instilled would also tend to lead to the students making similar and peculiar mistakes, and goes part of the way to explain why the students' performance triggered suspicion. However, there is another reason why the students' performance triggered suspicion: their performance was in fact suspicious. In reality it is not clear whether some of the students cheated or not (though by passing the test they proved that they didn't need to cheat). This is discussed in some detail in Jay Matthews' excellent book; see the footnote on page 43.

Finally, there is one last manner in which the movie is misleading. In *Stand and Deliver*, Escalante is presented as a strong-willed and somewhat acerbic character, but the movie gives no real sense of the extent to which Escalante was willing and able to really piss people off. In real life, Escalante did not suffer fools, and at Garfield, Escalante's pushy, critical arrogance made him plenty of enemies. Effectively forced out, Escalante left Garfield High in 1991. The Advanced Placement program at Garfield then quickly crumbled.

But these are the most minor of quibbles. *Stand and Deliver* is a wonderful movie about an incredible story, and of a truly great teacher.