MathSnacks

To Be or Not to Be, too
Four More Variations on Mathematical Existence

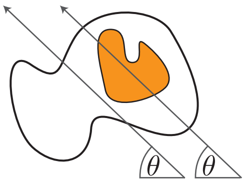
by Marty Ross, Burkard Polster, and QED (the cat)

Mathematical Monk

One day, a monk climbs up a mountain, starting at the foot of the mountain at 6 am and arriving at the top at 6 pm. On the next day he walks down, starting and ending at the same times. Is there some time on the two days at which he is at exactly the same spot?

Answer: Yes! Just imagine the two journeys occurring on the same day, with the monk walking up and his brother monk walking down. Obviously, they have to run into each other at some time.

Eggxactly Equal*



No matter how much of a mess you make breaking and frying an egg, it is always possible to cut your masterpiece along a straight line such that the resulting two halves contain the same amounts of egg yolk and egg white.

Proof: For every direction θ there is a line that cuts the yolk in half, and a parallel line that cuts the white in half. Suppose that, when θ=0, the yolk-line is to the *right* of the white-line. Then, when θ=180°, we have the same lines but with the opposite orientation, and so the yolk-line is now to the *left* of the white-line. Since the positions of these lines vary continuously, there must be an intermediate θ where the two lines coincide. Cutting along such a special coincident line then does the trick.

Similarly, it is possible to cut a ham sandwich along a plane to create fair shares of bread, ham, and cheese, though this is much harder to prove.

Mean Meow

If the Q.E.D. cat wanders 6 km from home in 2 hours, then at some time her speed is exactly 3 km per hour.

Why? If her speed was always below 3 kph, then she wouldn't have reached her target, and if her speed was always above 3 kph then she would have overshot it. So, since our cat's speed varies continuously, at some point her speed must be exactly 3 kph.

Note: The result is true, even if we don't assume the cat's speed varies continuously: this is the very important *Mean Value Theorem.*

Hairy Theorem

You cannot comb a hairy ball.

This is the famous *Hairy Ball Theorem*, but what exactly does it mean? Imagine a tennis ball that is evenly covered with hairs. Then the theorem states that, no matter how you comb the ball, there must somewhere be a part in the hair: the orientation of the hairs must be discontinuous.

We can look at this another way. Imagine the hairs represent the velocity of the wind blowing across the surface of the Earth. Then, if the wind velocity is continuous, there must be a point where the wind speed is 0.

Ripper Reference*

The Chickscope Project http://chickscope.beckman.uiuc.edu/explore/eggmath