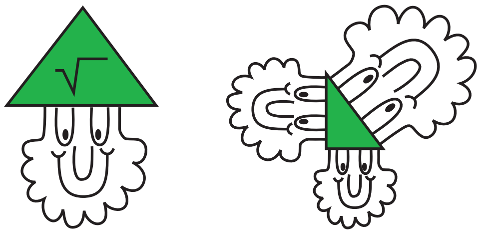
MathSnacks
Pythagoras & Co.

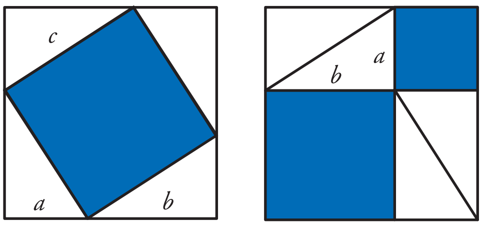
by Marty Ross, Burkard Polster, and QED (the cat)

Scary Scarecrow



In *The Wizard of Oz*, the Scarecrow is granted his Doctor of Thinkology and promptly recites: *The sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side.* What are the three mistakes? Are there sides of *any* isosceles triangle which satisfy the Scarecrow's equation? Danny Kaye sings a correct version of the Pythagorean Theorem in *Merry Andrew*.

Perfect Proof



Of course, the Pythaogorean Theorem says that for a right-angled triangle the area of the large square is the sum of the area of the two smaller squares. In algebraic terms, this is the famous equation

$$a^2 + b^2 = c^2.$$

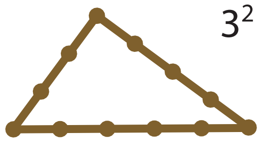
Proof: Fit four copies of the triangle in the corners of a square of size $a+b$ *(above, left)*. This leaves a tilted square of sidelength c in the middle. Summing the areas, we have

$$c^2 + 4 \cdot 1/2 \cdot a \cdot b = (a+b)^2,$$

and thus $c^2 = a^2 + b^2$.

We can dispense with the algebra by reshuffling the triangles *(above, right)*. We then simply observe that the blue areas in both diagrams are equal: c^2 on the left and $a^2 + b^2$ on the right. There are many other lovely proofs of the Pythagorean Theorem.*

Tantalising Triples



$$3^2 + 4^2 = 5^2$$

A *Pythagorean triple* consists of three natural numbers that satisfy the Pythagorean equation. This leads to an ancient method of constructing a right angle: taking a loop of rope with 3+4+5=12 equally spaced knots, then the *converse* of the Pythagorean Theorem guarantees that the triangle is right-angled.

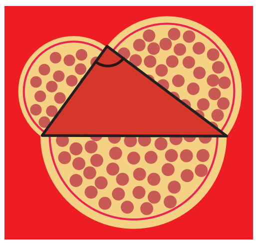
A simple way to construct some (but not all) Pythagorean triples is based on the equality

$$(n+1)^2 = n^2 + (2n+1).$$

Just choose $2n+1$ to be an odd square! For example, $2n+1=25$ corresponds to the triple 5:12:13. There is also an elegant formula which gives rise to *all* Pythagorean triples.*

The presence of Pythagorean triples on the Babylonian clay tablet Plimpton 322 demonstrates that people knew of the Pythagorean Theorem long before Pythagoras.

Pizza Puzzle



A large pizza costs the same as a medium and a small pizza together. Which should we buy? To answer this, cut the pizzas in half, and make a triangle out of the diameters. If the angle opposite the large pizza is a right angle, then the area of the large pizza is exactly the sum of the areas of the two smaller pizzas: that's Pythagoras in action! If the angle is greater than a right angle, then it's better to buy the large pizza, and otherwise we should buy the two smaller pizzas.

Brilliant Books*

R. Nelson, Proofs Without Words, I & II, MAA, 1993, 2000.
J. Stillwell, Numbers and Geometry, Springer, 1997.