FUNCTION AND FUNCTIONS

Michael A B Deakin, Monash University

I. THE HISTORY OF *FUNCTION* (SO FAR!)

Our journal *Function* is now twenty years old and so, in the nature of things, with five issues per year, we reach our 100th issue. Because we use base ten, this makes a hundred a "round" (and therefore "meaningful") number and celebrations are in order. (Of course, things would be different in a different base, but ... .) In consequence, I have been asked to tell our readers something of the history of our journal, a request that springs from (a) my having been there all along, (b) my having been the chief editor for much of *Function*'s history, and (c) my interest in history generally. So here goes.

In late 1976, the then chairman of Monash University's Department of Mathematics, Gordon Preston, called a meeting. Gordon had been appointed a professor of Pure Mathematics and taken up his appointment in 1963, very early on in Monash's history. He was an algebraist, with a special interest in semigroups, whose study constitutes a very difficult topic in abstract algebra, difficult because the subject is extremely general and begins from a very few assumptions. He had, when he arrived, just co-authored a very influential textbook, Clifford & Preston's *The Algebraic Theory of Semigroups.*

As part of his work, work undertaken with considerable energy, he interested himself in the content of secondary school syllabuses and the preparation that secondary students underwent for their later tertiary studies. By late 1976, he had come to the definite conclusion that secondary curricula (pressed as perhaps they were to do so many other things as well) were not challenging the more mathematically oriented students and were, furthermore, misrepresenting (indeed, badly misrepresenting) the nature of mathematics itself.

He thus called the meeting to address these issues. He envisaged a quality journal of real mathematics, mathematics as mathematicians themselves would recognise it, but addressed to secondary students. There were, he said, many journals addressed to teachers, but even here he found the focus of attention moving away from the actual mathematics. What he wanted was a journal addressed to the job of quality exposition of genuine mathematics. It would be published by Monash University and would, as well as filling a gap, also act as a showcase for the mathematics being taught at Monash.

The meeting endorsed these aims, and elected an editorial board comprising Gordon Preston himself as chairman, Neil Barnett, Neil Cameron, Ken Evans, Barrie Milne, Elizabeth (Liz) Sonenberg, Geoff Watterson and myself. Ken Evans and I have continued on the board for the entire period of *Function*'s life, and all of the others contributed very much for varying periods of time. An early addition to *Function*'s aims was the explicit policy of encouraging mathematics for all, most especially for girls who (then as now) were under-represented in the mathematical field.

This was very much in line with the official policy of the Department of Mathematics itself, which was very much in the lead in this matter in Australian tertiary education at that time.

(However, there was an amusing hiccup at one point somewhat later, when a radical, but obviously incompletely informed, feminist accused us of male chauvinism as we had *not a single female on the board*. It hadn't occurred to her that a *Dr* Sonenberg might be a woman!)

At that first meeting, there was some discussion on the name that might be given to the new journal. *Monash Mathematics Magazine* was suggested, but felt to be a bit of a mouthful and also to lack a little "oomph". There were (partially facetious) suggestions that we might instead use $3M$, M^3, or $M\hat{}3$. When the meeting broke up, we had no title, but each of us was enjoined to think of possibilities.

I left, somewhat euphoric that something I'd long thought about was likely to see fruition, and with the dynamism and the reputation of Gordon Preston behind it. I retired to the Monash University Club and joined my friends Gordon Troup (Physics) and Len Grant (Philosophy) for a beer, in fact as it turned out, quite a few beers. My excitement at the new venture must have been catching, and my two friends happily fell into a game of "brainstorming" as it is known in the advertising industry. You try to free the mind, or perhaps rather let it freewheel, which we did (probably aided by the various rounds of beer!). However when, in the course of the conversation, we came across a word that might serve as a possible title, this was carefully written down (on a rather soggy paper napkin, as I recollect). By the end of the evening, the napkin was covered in potential names, and next morning, I gave it to Gordon Preston.

My own favourite possible title was *Convergence*, perhaps in part because it was one of my own suggestions, and also because it seemed to have the connotation of a meeting between school and university mathematics. However, the name Gordon picked out was *Function*. His choice was based on his judgement that there is no more central concept in the whole of mathematics. This is explained in Part II of this article.

I personally never did remember which of the three of us had come up with this name, but Len always claimed it as his own and was apt at times to call *Function* his "godchild". (Len died of cancer in 1988, not long after he retired from Monash. He had a much better grasp of mathematics than do most philosophers, and saw it as being very important in his own field.)

The first issue of *Function* was produced in February 1977. Its leading article was an account by John Stillwell on the solution of the "Four Colour Problem". A proof of this conjecture had long been sought, and its ultimate resolution was attended by controversy and fanfare, much as was the later proof of Fermat's Last Theorem (later also to be written up by John, in *Volume* 18, *Part* 2, for the benefit of *Function*'s readers).

Also included in that first issue were two short articles from members of the editorial board, two articles from secondary school students, and a reprint of an account of the

Hungarian mathematical prodigy Louis Pòsa. The front cover was a computer graphic depicting the surface generated when the curve $y = \frac{\sin x}{x}$ is rotated about the y-axis. Many modern packages will today produce this quite routinely[1], but back then it was otherwise. We took it with permission from advertising material sent us by a company who were interested to sell software to the Department of Mathematics.

We celebrated the arrival of the first printed copies with several bottles of champagne in a semi-impromptu party in Gordon's office and by mailing sample copies to all Victorian secondary schools. We keep stocks of all back issues of *Function*, but the original printing of this historic issue has long been used up. (However, we can and do still supply photocopies.)

In the ensuing years, we have continued to publish what we hope you agree are high quality articles, and every issue has featured a front cover of mathematical interest. *Volume* 14 saw the inauguration of our new format with specialist columns on Computing and on History, together with a separate Problem Section. *Volume* 18 was the first with our new glossy cover. This year, we increased the size of a regular issue from 32 to 36 pages.

When I look back, perhaps the articles that most remain in my mind are those that deal with topics that lie outside the usual run of things, things that are hard or even impossible to find elsewhere. I think in particular of Peter Finch's "Mean, Mode, and Median" in *Volume*

[1]The graph on the front cover of this special issue was produced with *MAPLE*.

2, *Part* 1, John Barton's "The Emblem" (*Vol* 2 *Part* 3), Sir Richard Eggleston's "Bayes and the Island Problem" (*Vol* 4 *Part* 2), Carl Moppert's articles on the Monash Sundial (*Vol* 5 *Part* 5) and the Foucault Pendulum (*Vol* 6 *Part* 2), Tim Brown's discussion of the Poisson approximation to the binomial (*Vol* 8 *Part* 5), Cheryl Praeger on mathematics and weaving (*Vol* 9 *Parts* 4 and 5) Geoff Watterson's "AIDS and Bayes" (*Vol* 12 *Part* 3), Win Frost's "Pappus and the Pandrosion Puzzlement" (*Vol* 16 *Part* 3), John Stillwell's proof that there are in fact two distinct Möbius bands (*Vol* 16 *Part* 5) and Mike Englefield's law of logarithms (*Vol* 18 *Part* 2).

This list emphasises the more original articles, and thus may tend to show somewhat more difficult mathematics, mathematics that really challenges our target reader. More approachable for that reader will probably be the expository articles, but even here we do like to add a new twist to things where possible. See as recent examples Aidan Sudbury's account of the varying length of the day (*Vol* 19 *Part* 3) or Malcolm Clark's discussions of Tattslotto numbers (*Vol* 17 *Part* 3 and *Vol* 20 *Part* 4). The presence of some element of originality also distinguishes the most memorable front covers. Two I particularly like are Sean O'Connor's computer generated graph of the curve known as "Murphy's Eyeballs" (*Vol* 3 *Part* 2) (the first ever correct depiction)[2] and Jean Sheldon's careful reconstruction

[2]There are both a public joke and a private joke in the title of this curve. It was first drawn (incorrectly!) by the US mathematician H T Davis, who named it after the Murphy of "Murphy's Law": If anything can go wrong, it will! In his case, it did. However, it also became a pun on the name of John Murphy, the editor who usually produced the front cover in those days.

of an 18th-century engineering diagram (*Vol* 19 *Part* 1) (with particular relevance to Australia).

Student articles appear from time to time and have always been particularly welcome. From the first issue (which, as mentioned above, carried two) to this year, we have actively encouraged them. The most recent was Mark Eid's account of his discoveries with Pythagorean triples (*Vol* 20 *Part* 3). Another "occasional" feature is the April account of the work of Dai Fwls ap Rhyll; perhaps the best was the very first (*Vol* 4 *Part* 2) which actually fooled quite a number of people who should have known better.

Over the years, a good number of people have served on the editorial board, and there have been a number of chief editors. Cristina Varsavsky took over officially in 1995, but in fact had been very active in the previous year. I hope that readers will agree that under her guidance *Function* is going from strength to strength.

II. ABOUT FUNCTIONS

The modern notion of a function is that of a process that assigns to each member of one set a unique member either of the same set or else of another set. As an example, we might consider the set of real numbers and to each member x, say, assign from the same set the number $x + 1$. Or we might take the set of words in a dictionary and to each word assign a natural number corresponding to the number of letters in the word. Both of these correspondences are examples of functions.

On the other hand, suppose the first set was the set of positive real numbers and the second set the set of all reals. We seek to assign to each positive real number x a real number y such that $y^2 = x$. This does not define a function, because for each value of x, there are two such y. It is necessary to make the further stipulation that y is to be positive before we can have a proper function.

Much of this will doubtless strike the reader as somewhat pedantic and contrived. However, it was not a definition easily arrived at, nor of course was it done without a reason. Earlier definitions had made the idea of a function more akin to that of a *formula*, one that could be written down explicitly. But this led to difficulties. One was the need to use functions that had no simple formulaic expression. Another was the case of a formula given as a limit; we may get arbitrarily close to the true value of $\sqrt{2}$ but we never in fact reach it. In many cases, we may similarly define functions. What status was to be accorded to this?

Then there is the distinction to be made between the *value* taken by the process and the *procedure* itself. This becomes evident if we begin with (for example) a formula like $y = \sin x$ and seek to take a derivative. The derived function is $y' = \cos x$. If we evaluate y at, say, $x = 0$, we will find $y = 0$, but y' evaluated at $x = 0$ has the value 1, which is not the derivative of the function $y = 0$. When there are such possibilities for confusion, then mathematicians have to become very careful and resort to a very precise definition.

The evolution of the precise modern definition of a function was a long time coming. Just how involved that story is may be gauged from the fact that an entire book has (relatively) recently been devoted to it. This is Umberto Bottazzini's *The Higher Calculus: A History of Real and Complex Analysis from Euler to Weierstrass.*[3] Both Euler and Weierstrass were very famous mathematicians whose contributions are very much a part of today's mathematics; the modern definition of a function is essentially due to Weierstrass.

Here is an example I like to give that shows in a reasonably simple way how rather strange functions may be generated. There is a function that assigns to every real number another real number in a very simple way. It is called $\operatorname{sgn}(x)$, pronounced "signum of x". The word "signum" is Latin for "sign", not to be confused with "sine". The rule is simple; if x is positive, then $\operatorname{sgn}(x) = 1$, if x is negative, then $\operatorname{sgn}(x) = -1$, if $x = 0$, then $\operatorname{sgn}(x) = 0$. The function $\operatorname{sgn}(x)$ is actually quite useful and finds application in (for example) electrical engineering. But now consider a related function: $[\operatorname{sgn}(x)]^2$. This is a very simple function; it almost always equals 1, but exactly in the case $x = 0$, its value is zero.

To show that the story is not yet finished, let me close with a discussion of yet another strange function. This is the so-called "Dirichlet function" named after a German mathematician from the first half of last century. The Dirichlet function is written $D(x)$ and its value is 0 if x

[3]New York: Springer, 1986.

is a rational number, 1 if x is irrational. (Not exactly the sort of thing you can easily draw as a graph!) But now it has been asked whether this, apparently simple, definition is a proper one. We know, of course, that some numbers, like 2, are rational and that others, like $\sqrt{2}$, are irrational. We even know the status of more complicated numbers like e and π.[4] However, it is still not known if $e + \pi$ is rational or not. So until this question is decided we cannot say what value to assign to $D(e + \pi)$.

Furthermore, if we *do* manage to solve this problem, there are infinitely many other such numbers. It is now known that no systematic procedure can be devised that will enable us to decide if some given number is rational or irrational. This has led some mathematicians to query the status of the function $D(x)$. What do you think? Does it matter what we happen (at some or other particular moment of time) to *know*? Should mathematics be dependent on such questions?

[4]The irrationality of π, although known for some time, was only some 50 years ago the subject of a relatively simple proof. This was produced by the American mathematician Ivan Niven, some of whose work (but not this one, which is a bit above the target audience) we have been proud to reproduce in *Function Vol* 8 *Part* 1.