HISTORY OF MATHEMATICS

Ethnomathematics

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A few months ago I came across an interesting story that I would like to share with my readers, but it also brought back into my mind other matters that have concerned me for some time and I want also to use this column to raise some of these more general issues.

1. A Somali Poem

I recently learned via the Internet of a Somali poem that has a mathematical flavour to it and which in particular can be related to the notion of a limit, so central to calculus. To reach the relevant site, first go to < http://www.dejanews.com/forms/dnq.html >. A dialogue box will then appear and under "SEARCH OPTIONS" click on "All" for the **Keywords Matched** and "Old" for the **Usenet Database**. Then, in the **Search For** box, type **somali fox calculus**. This will give you eight news items on the matter. The first part of this article is based on these postings, most especially the fourth, seventh and eighth.

They are concerned with a poem called **Qayb Libaax**, written in Somali, and associated with the Dervish movement.[1] The story line of the poem runs like this.

> The family of wild animals killed a camel and set about dividing the meat for their consumption. The lion (king of the beasts in Somali tradition as well as in our own) ordered the hyena to make a fair division. The hyena apportioned the flesh as follows: "One half for the king [lion] and the other half for the rest of us". This division displeased the lion, who punished the hyena, injuring its eye. The lion then asked the fox to take on the task – the fox being associated with cunning and opportunism in Somali tradition (again as also in our own). The fox produced a modified version of the hyena's apportionment: "One half of the camel meat for the

[1] The Dervishes are a branch of the Sufic (or mystic) stream of Islamic religion. They arose in the 12th and 13th centuries, and Dervish communities are still to be found today, despite the disapproval of mainstream Islamic thought. One of the postings describes the poem as "one of the last poems of the Dervish movement". I'm not quite sure what this means, but it may refer to the 13th century.

king [lion]; from the remainder, again one half for the king and so on". The lion then asked the fox "When did you learn this fairness and justice?", to which the fox replied "When I saw the injured eye of the hyena".

The mathematical point, of course, is that the lion gets everything. This can be discovered by summing a geometric series:

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots=1 \qquad (1)$$

and this is the route taken by some of those posting the various discussions.

What leaves me a little uneasy, however, is the somewhat exaggerated claims that some commentators seem to make for this fable. Among other things it is presented as an independent discovery of the paradoxes of Zeno.[2] This seems to me to claim far too much. In the first place, these latter constitute an elaborate and subtle argument as to the nature of Space and of Time. Are these to be thought of as continuous, and hence infinitely divisible, or else as composed of atom-like "places" and "instants"? There are four possibilities (each of Space and Time may or may not be infinitely divisible) and the four paradoxes are designed to show the impossibility of all of them. The conclusion we are invited to draw is that Space and Time are illusions.[3]

Thus the primary purpose of the Zeno analysis is metaphysical rather than mathematical; the mathematics is incidental, although important to the argument. Nonetheless, it is there, and at one point it comes very close to the point of the Somali poem.

The particular paradox in question is the first, known as the *Dichotomy*.[4] This takes Space to be infinitely divisible, in order to arrive at a contradiction. It may be presented in its starkest form by considering a journey. Before we can reach our destination, we must first reach the halfway point, and before we can reach *this*, we must reach the quarter-way point, etc. How could we ever get started? Now of course if we add up the half, the quarter and so on, then we get equation (1), and the sum of all these fractions (as an infinite series) is 1, the entire journey.

[2]The paradoxes of Zeno were discussed in this column of *Function*, *Vol* 14 *Part* 3 (June 1990).

[3]A similar point may be made in respect of a Buddhist version of one of these paradoxes; again see my earlier article.

[4]Meaning "division in two".

The important mathematical point, however, is that while we may *assign* the sum 1 to the series $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots$, in a very important sense its sum is *not* 1. I remember puzzling over exactly this point as a child, and not ever managing to resolve it. For *no matter how many terms of the series we take, we always fall short of* 1.

The actual mathematical resolution of the question of the sum is quite subtle. If we take (say) n terms of the series, we may show that the sum is $1-2^{-n}$. (I leave the proof to the reader; the actual arithmetic is quite simple.) We *never do sum infinitely many terms*. Such a task would be impossible. Rather, we note that we can, *in only finitely many steps*, get not to 1 itself, but *arbitrarily close to it*. So then we say that we *assign* the value 1 to the infinite case. No value short of 1 will serve (and of course no value greater than 1 would make sense.) But this is a completely new sort of sum, and we've only had such sums for something less than 300 years. Certainly Zeno never considered this subtle logic; nor does the Somali poem.

Rather we, from a more informed standpoint, find this *implicit* in Zeno as also in the fable of the shared camel. If we are to do this with the latter, it is perhaps more useful not to consider the geometric series at all, but instead to proceed from a much more elementary consideration. *There is no provision for any of the other animals to get any of the meat*; this is the simple reason why the lion gets the lot. (At one point in the poem, the other animals complain to the fox on this very account.) If we want to put a "mathematical spin" on this insight, we may do so. After a finite number of meals, the lion has eaten $1-2^{-n}$ of the camel and 2^{-n} remains. No other animal has yet eaten, nor may any do so now, for half of what remains is the lion's. This applies whatever the value of n. At no point may any animal other than the lion touch the carcass.

But notice that this mathematical analysis is *my* interpretation of the situation; there is no vouch for it in the text of the poem. If we are to understand what *that* is saying, then we need other background. Now I know next to nothing of Somali history and culture, so what I am about to say is offered only very tentatively.

But it seems to me that the poem is making not a mathematical point, nor a metaphysical one (as Zeno was when he queried the nature of Space and Time); rather the point of the poem is *moral*. It concerns "fairness and justice". I see it as a (somewhat rueful) recognition that "might" can take precedence over "right" in this imperfect real world we inhabit. (The other animals complain, surely with justice, about the unfairness of the fox's ruling.) Even if this interpretation is not strictly correct, I would hold that *some such point* is the main thrust of the poem; its purpose is not primarily mathematical at all. (In the original Somali, the "mathematics" occupies only some $7\frac{1}{2}\%$ of the total. The rest is concerned with the situation itself, the actual happenings, the dialogue on "fairness and justice" and then with applications of the story to other situations.)

I give this story in detail in part because it is interesting and the poem certainly not widely known, partly because it gives me the chance to explain once again some very central mathematics, and also because it is a convenient springboard to the discussion of a more general question: the validity of "ethnomathematics".

2. Ethnomathematics

Much, and to be quite frank most, of today's mathematics is a clear product of Western culture as that culture has developed since (say) the days of Newton and Leibniz (around about 1700). Of course the work of these two great mathematicians and their contemporaries and successors elaborated an already rich tradition: Euclid, Archimedes and the other mathematicians of ancient Greece. This is the clear pattern of the main lines of mathematical endeavour.

However, the tapestry of mathematics is richer than this simple description allows. There was early work in Babylon and in Egypt, in China and in India. Other cultures (Hebrew, Japanese, Javanese, Korean, Mayan[5], Persian and Tibetan) also reached high levels of numeracy, but without having major influence on the mathematics of today.

Possibly there are others we could add to this list.[6] And certainly special mention should be made of the Arab mathematicians[7], who not only preserved much of the ancient Greek heritage, but also added to it in many meaningful ways (and whose influence *has* been felt in the mathematics we learn today).

All these traditions are clearly mathematical in that lengthy, involved and precise arguments are advanced by means of symbolic techniques, either written or embodied in some type of hardware.

[5]See the cover story in *Function, Vol* 12 *Part* 4.

[6]For instance, many people might include as mathematics the wonderful navigational feats of the Polynesians.

[7]See my History of Mathematics column in *Function, Vol* 15 *Part* 2.

I will refer to these cultures in what follows as "cultures of high numeracy". But it then follows that there are other cultures that are *not* of high numeracy. Some people find this a disturbing conclusion. I don't, I'm afraid. Rather, some cultures had a need to develop (applied) mathematics; others even had the leisure to go on to develop pure mathematics. But these factors are not by any means universal.

Thus the mathematics involved in establishing calendars is very important once a society becomes reasonably complex. For example, of the groups I mentioned above, the Javanese and the Mayans showed their greatest mathematical prowess in this area. However, if the society has a less complex structure, then accurate calendars are less important to it and may well not be developed.

It is the same with other areas of mathematics. We define concepts as we need them. So when I read that the *Ormu*[8] word for "nine" prior to outside contact was *nen-rohi-fraja-nitje-ma*, I deduce that the *Ormu* made little use of the concept "nine". Evidently their traditional way of life had little call for it.

However, this has disturbed some people and they see such analyses as this as demeaning to groups such as the *Ormu*. They feel that (e.g.) the *Ormu* must have had a mathematical tradition and that if we think otherwise, the fault is ours for not recognising it. This is one origin of the rise of Ethnomathematics. It has become very fashionable in recent years.

What researchers in this area present for our consideration are various traditional customs and artifacts that have a strongly "mathematical" flavour. These may be games, intricate patters and designs, numeration devices, methods of keeping trade tallies, clan systems regulating who may or may not marry, and probably other such aspects of the cultures involved.

There is, for example, a game called *Mancala* (Arabic for "transferring") which may be found in one form or another in many African countries. It is deceptively simple to describe but extremely difficult to play well. Many even uneducated Africans excel at it.

Then again, many cultures have intricate geometric art. For example, the *Malekula* of Vanuatu have intricate sand-patterns which are to be drawn without lifting one's finger from the sand. We may analyse this endeavour in terms of *graph theory*[9] which deals with exactly such questions. See Figure 1

[8] A language from Irian Jaya.

[9] See *Function, Vol* 13 *Part* 1, pp. 20-27.

which shows a Malekula design produced by the four-fold replication of the element shown to the right of the full pattern.

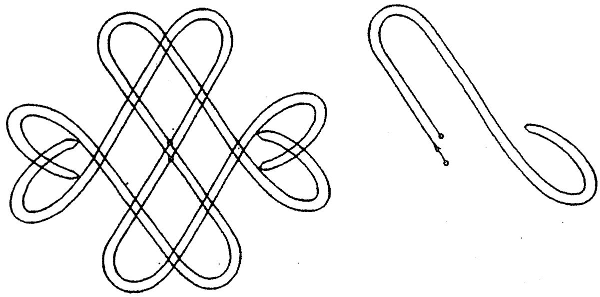

Figure 1. A Malekula sand-drawing (left), and (right) the basic unit which is replicated. It may be traced by following the direction of the arrow. (Adapted from a diagram in the article "Ethnomathematics" in the *Companion Encyclopedia of the History and Philosophy of the Mathematical Sciences.*)

What I find unconvincing about such cases is the way they are presented. *We* may analyse Malekula sand-art in terms of graph theory, but there is little evidence presented that the Malekula themselves do. There is a basic theorem of graph theory dealing exactly with the question of when a line drawing may be traced in one movement, and there is no evidence given at all that the Malekula are familiar with it. Certainly they must have found many intricate and appealing designs that may be drawn in this way, but this is not the same thing.

When the mathematician Euler was presented with a puzzle about whether the bridges of the town Königsberg could be traversed in a certain way (equivalent to drawing a pattern in one movement), he solved the matter by proving the theorem in question and then applying it to this particular case. He thus showed that the task was impossible. *That* is mathematics; to consider all the possible ways one might try to do the task and thus to eliminate all of them is not really mathematics, and Euler didn't do things that way.

The same point could be made in respect of (say) the clan structure of some societies, for example the Australian *Arunta*. An article in *Function* by

Hans Lausch[10] analysed this in terms of group theory, a branch of abstract algebra. But again notice that it was the author of the article who supplied the mathematics. *We* may find it implicit in the clan structure of the *Arunta*, but this is not the same thing as saying that the *Arunta* are engaging in group theoretical discourse.

One much studied and often cited case is that of the Inca *quipu*. A *quipu* is a form of physical representation of number made from knotted cotton cords. A *quipu* may be very intricate, with as many as 2000 separate cords and with a sophisticated hierarchy of knots and a pleasing design of different colours. Essentially each *quipu* records data as one or more numerals. Again, I don't really count this as mathematics, except in the sense that counting is mathematics. I find no evidence (or even claim) that any operation beyond simple addition was ever recorded by this means.

There are essentially two thrusts to the movement for ethnomathematics. The first is the wish to give dignity to cultures not normally regarded as cultures of advanced numeracy. I too regard this as an important and laudable aim, but I think its application misguided: all cultures have dignity and we do not enhance it by assigning to some of their aspects a significance they cannot really bear.

The second thrust is educational. With this I am entirely in sympathy. If one is teaching, say, graph theory in Vanuatu, then it makes eminent sense to use local examples and knowledge. Not only will it hold the students' attention the better, show that one *does* respect the local culture and provide good non-trivial examples, but it also makes use of the specialist knowledge the students already have.

Further Reading

There is a lot of material on Ethnomathematics. M Ascher's *ethnomathematics: A Multicultural View of Mathematical Ideas* is perhaps the fullest single account. M and R Ascher wrote the article on ethnomathematics in the *Companion Encyclopedia of the History and Philosophy of the Mathematical Sciences*. In this same collection is C Zaslavsky's article on mathematics in Africa: Explicit and Implicit. Both these sources give many further references. There is also a nice "Interchapter" on mathematics around the world in V Katz's *A History of Mathematics*.

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[10] *Function, Vol* 14 *Part* 3, pp 22-27.