HISTORY OF MATHEMATICS SECTION

EDITOR: M.A.B. DEAKIN

Hypatia of Alexandria

The first† woman in the history of mathematics is usually taken to be Hypatia†† of Alexandria who lived from about 370 A.D. to (probably) 415. Ever since this column began, I have had requests to write up her story. It is certainly a fascinating and a colourful one, but much more difficult of writing than I had imagined it to be; this is because so much of the good historical material is hard to come by (and not in English), while so much of what is readily to hand is unreliable, rhetorical or plain fiction.

I will come back to these points but before I do let me fill in the background to our story.

Alexander the Great conquered northern Egypt a little before 330 B.C. and installed one of his generals, Ptolemy I Soter, as governor. In the course of his conquest, he founded a city in the Nile delta and modestly named it Alexandria. It was here that Ptolemy I Soter founded the famous Alexandrian Museum, seen by many as an ancient counterpart to today's universities. (Euclid seems to have been its first "professor" of mathematics; certainly he was attached to the Museum in its early days.) The Museum was for centuries a centre of scholarship and learning.

Alexandria fell into the hands of the Romans in 30 B.C. with the suicide of Cleopatra. Nevertheless, the influence of Greek culture and learning continued. Two very great mathematicians are associated with this second period. *Diophantus* (who lived around 250 A.D.) wrote a number of books but most particularly an algebra text that will come into the story later. A little less than a hundred years after Diophantus came the great geometer *Pappus*. Shortly after Pappus, however, the Museum fell into a decline. Alexandria became a prey to sectarian violence between various factions of Christians, different groups of Greek "pagans" (including a number of so-called Neoplatonic groups), Jews and others.

Riots occurred and did much to damage the Museum, in particular destroying its great libraries – the last going up in smoke in 392 when the temple of Serapis was put to the torch by a riotous throng.

The last known member of the Museum (very likely its last president) was *Theon of Alexandria*, a minor mathematician and astronomer. He made few if any original contributions to mathematics but his work as an editor has been very useful to later generations. His daughter, *Hypatia*, is the heroine of our story. She was not associated with the Museum, but headed the Neoplatonic school, another institution. She thus belonged to one of the "pagan" groups, and met her death on this account, brutally hacked to pieces by a Christian lynch-mob in a year that is usually put at 415.

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† This assessment may, however, need to be revised. Winifred Frost of the University of Newcastle believes she has found an earlier claimant. We hope to bring this development to you in a future issue.

†† The strictly correct pronunciation probably approximates *heew–pah–TEE–ah*, but it is usual and acceptable to pronounce the name as *high–PAY–sha*. (Much as we say "Paris" in the usual way and don't attempt the French pronunciation, which is more like *par–HEE*.)

After this, and perhaps in part because of it, the focus of Neoplatonist thought moved to Athens. In the years that followed, *Proclus* (to whom we owe the preservation of much of Euclid's *Elements*) and other mathematicians frequented the Neoplatonic school in Athens. The last two names associated with the school are those of philosophers rather than mathematicians: *Isidorus* and his pupil *Damascius*[†] . In 529, the emperor Justinian, enforcing Christianity, closed the school and Damascius went into exile in Persia.

Let us now look more closely at those turbulent times around the year 400. A good place to begin finding out about a mathematician from the past is a 16-volume work called the *Dictionary of Scientific Biography* (DSB). The article on Theon in that work is by G.J. Toomer; it is authoritative and well-researched. It tells us what happened and also *how we know* that it happened. There are notes with clear and detailed references to where Toomer got the information. All this is how a scholarly article should be.

[These articles in *Function*, by contrast, are popular articles and are not intended as scholarly. So in most cases I don't give all my sources; usually they are readily accessible, and in any case readers seeking further information can always write to me, as some have. In the present case, I will give rather more detail than is my usual custom, but not to the point of excess. I hope to prepare a scholarly article on the subject for publication elsewhere.]

Toomer, to get back to *Theon*, tells us that Theon was the author of a number of "Commentaries". These were editions, with extra notes, of the works of famous authors. In many cases, the original works get lost and modern editors have to work from such Commentaries. Theon wrote Commentaries on Euclid's *Elements* (and in places these provide the basis for the modern text), two other books by Euclid, the *Data* and the *Optics*, and two works by the astronomer *Ptolemy* (about 100-170 A.D.)[††] , the *Almagest* and the *Handy Tables*. Over and above these he wrote a book on the astrolabe, an astronomical instrument with navigational applications. This book may or may not have been a Commentary on an earlier (now lost) book by Ptolemy. It too is lost, but perhaps not entirely.

So Toomer tells us a lot about Theon. Regrettably the DSB article on Hypatia is not up to that work's usually high standard. The sources are only perfunctorily indicated, and in many cases not given, credence is given to a work of avowed fiction, and some statements are plain wrong. So I had rather more work to do than I anticipated when I set out to write this article; however, the extra work has led me to some very interesting reading.

Historians distinguish between *primary* sources (the original documents on which all subsequent work depends) and *secondary* sources (those which retell, explain, comment on and judge the material in the primary sources). Unless one is *oneself* expert in the period, the language (in this case patristic Greek) and the questions of textual authenticity and interpretation, secondary sources are vital. In this instance, the sources are rather hard to come by. I have succeeded in laying my hands on all the primary sources and most but not all of the best secondary ones.

The primary sources on Hypatia come under two headings: (a) the *Suda Lexicon*, (b) the *Patrologiae Graecae*. The *Suda Lexicon* is a 5-volume work from the 10th Century A.D. It is an alphabetical compilation for all the world like an encyclopedia of today. Until recently it was called the *Suidae Lexicon* and its supposed author was called *Suidas*

[†] Damascius may have some minor claim on mathematical history as an editor of Euclid, but the case is disputed.

[††] Note that this is not Ptolemy I Soter, but a different chap.

(rather as if some 30th Century writer were to talk of Britannicus and his wonderful Encyclopedia!). Anyhow, the *Suda Lexicon* (the name is now thought to be related to the Greek for "fortress" – the stronghold of knowledge) is a compilation from earlier sources.

There is quite a long entry on Hypatia in the *Suda* and this derives from an earlier such encyclopedia (the *Onomatologus* of *Hesychius Milesius*) and also from Damascius' *Life of Isidorus*.

Hesychius Milesius was also known as Hesychius the Illustrious. The name Hesychius was quite common and so one needed to say which Hesychius was being discussed. (In particular, and confusingly, Hesychius of Alexandria has nothing to do with the story.) Hesychius Milesius' *Onomatologus* now survives only through one very imperfect copy and what has found its way into later works like the *Suda*. *Damascius* we met briefly earlier. His life of Isidorus is now lost, but fragments survive as quotations in other writings.

The *Patrologiae Graecae* are by and large in better shape. They form a work of over 150 volumes collecting the writings (in Greek) of persons important in the early Christian Church. The most scholarly edition comes with a parallel translation into Latin. Of the texts in this collection that concern Hypatia, most are letters from *Synesius*, one of her pupils, but who either was or became a Christian, indeed a bishop. There are also letters from Synesius not to Hypatia but making mention of her. The other major source in the *Patrologiae Graecae* is a passage in the *Ecclesiastical History* by *Socrates Scholasticus*, who lived only shortly after Hypatia. (This is not, of course, *the* Socrates; *he* was some 850 years dead by this time.) The remaining references are meagre. There is a sentence in the 6th Century *Chronicle of John Malalas* and a short paragraph in an early 5th Century chronicle by the ecclesiastical historian *Philostorgius*. It is to Philostorgius that we owe the opinion that Hypatia was a better mathematician than her father Theon, and it's possible that he had many more interesting things to say – but we don't know. The version of Philostorgius that has come down to us is an abridgement by the 9th Century scribe *Photius*.

Photius himself wrote a sentence on Hypatia. It will endear him neither to women nor to mathematicians. It went:

> "Isidorus greatly outshone Hypatia, not just because he was a man and she a woman, but in the way a genuine philosopher will over a mere geometer."

It is believed that this sentence is in fact copied from Damascius' *Life*, the lost work that in part informed the *Suda*.

The last and least of the Christian fathers with anything to say is *Nicephorus Callistus* who lived in the 14th Century and whose account merely paraphrases Socrates Scholasticus.

So – there are our sources. What do they tell us?

As always, much less than we'd like to know. But a good deal is agreed. Hypatia was a public figure who taught philosophy and mathematics. She attracted a large following and probably held some kind of official post. She was unmarried – in fact determinedly celibate. She was a Neoplatonist, born probably sometime around 370 and murdered in (almost certainly) 415 by a mob of Christian fanatics.

There are arguments over details: which of the many brands of Neoplatonism did she profess? Which Christian faction killed her and why? Was Cyril, the bishop of

Alexandria, implicated in her death? And so on.†

I will not dwell on these matters, but turn rather to what we can learn of Hypatia's mathematics and what the sources tell us of *that.*

Precious little really, I'm afraid. *That* she was a mathematician is widely agreed. She is variously described as a philosopher, a mathematician, a geometer and an astronomer. What we would like to know is what as a mathematician, geometer or astronomer it was that she did.

The most explicit statement is a 12-word passage in the *Suda.* Yes, just 12 words (and almost half of these subject to disputed readings or various interpretations). However, there is a general consensus as to what they say:

> "She wrote a Commentary on Diophantus, [one on] the astronomical Canon, and a Commentary on Apollonius's Conics."

Take these in reverse order. *Apollonius*, who lived around 200 B.C., was a very great geometer. He codified much of what we know about the conic sections (ellipse, parabola, hyperbola). Regrettably, Hypatia's Commentary on this work is totally lost.

When it comes to the "astronomical canon", we are on slightly firmer ground. Most scholars agree that what she wrote was a Commentary on one of Ptolemy's works: either the *Almagest* or the *Handy Tables.* It will be remembered that Theon, Hypatia's father, wrote Commentaries on both these works. Various authors have suggested that Hypatia collaborated with him in one or other or both of these enterprises.

Theon's Commentary on the *Almagest* has twice been edited in modern times: once last century and once this. The 20th Century edition is a work of great scholarship. Its editor, a Professor *Rome*, suggests that what Hypatia did was to revise her father's Commentary on Book 3 of the *Almagest.* An inscription by Theon is preserved in the best manuscripts saying that he is using the text as revised by 'my philosopher-daughter, Hypatia'.

Neugebauer (a historian of Mathematics whom we met in *Function, Vol.* 15, *Part* 3), however, thinks that this is not the work referred to in the *Suda*, which he thinks is a now lost Commentary on Ptolemy's *Handy Tables.*

The remaining work attributed to Hypatia is her Commentary on Diophantus. Most writers assume that Hypatia's Commentary was an edition of his major work, the *Arithmetic.*

By "arithmetic" we should understand "number theory", which used to be called "higher arithmetic". Diophantus has given his name to several branches of modern mathematics. A diophantine equation, for example, is one to be solved in integers. Thus, for example, the diophantine equation

$$x^2 + y^2 = 25$$

has solutions $(0, \pm5)$, $(\pm3, \pm4)$, $(\pm4, \pm3)$, $(\pm5, 0)$.

But I digress. Diophantus's *Arithmetic*, like Euclid's *Elements*, was a collection of 13 "books". We know this from the introduction. Of these 13, however, we have only six (presumed to be the first six).

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† We may dismiss a further ground of dispute. She was *not* Isidorus's wife, although the *Suda* says at one point that she was. The passage is almost certainly spurious (some blame Photius). Besides, Isidorus was either unborn or in nappies when Hypatia was killed.

Diophantus's writings were collected and edited by the 19th Century French scholar, *Paul Tannery*. Tannery put forward the suggestion that it was these six books on which Hypatia "commented". This could have been because Books 7-13 were already lost by Hypatia's time, but Tannery preferred the alternative view that Books 1-6 were preserved *because* Hypatia commented on them. (Recent research based on Arab translations has complicated this theory however; it is the subject of great controversy.) Tannery and Heath, however, suggest that what has come down to us is in fact not Diophantus's original but Hypatia's Commentary. If this is so then we are much indebted to Hypatia, for without her we would miss most of the surviving work of Diophantus.

But now, if this is right, then what survives of Diophantus's work would incorporate whatever comment Hypatia wrote, and so we would have a small legacy of her mathematics hidden in the work of Diophantus. In 1885, Sir Thomas Heath brought out the first English edition of Diophantus. This suggests that the most obvious place to look for such interpolated material is at the start of Book 2. Problems 1-5 of Book 2 are mere repetitions of problems that already appeared in Book 1. Problems 6, 7 look "different" from Problems 8, 9, etc.

It seems very much as if what we see is an edition designed as a student text. Problems 1-5 could be seen as "revision". Then Problems 6, 7 are "exercises" before we move on to the "new theory" of Problems 8, 9, etc. Thus, if we do see Hypatia's hand in Diophantus's *Arithmetic*, she poses (in essence) the problem of solving for x, y the simultaneous equations

$$x - y = a, \quad (x^2 - y^2) - (x - y) = b,$$

where a, b are known. This is Problem 6. Problem 7 is essentially the same.

Hardly, I'm afraid, stuff to raise one's voice about.

The other source of specific information on Hypatia's mathematics is the writings of Synesius. Of Synesius' letters to Hypatia, six and a little bit survive. She is mentioned in a number of others – the precise number depending on which editor one believes. Two of these letters are relevant to an evaluation of Hypatia's mathematics.

One is Letter 15. It is puzzling. He writes that he is "in such a bad way" that he has to have a "hydroscope". He asks her to make him one and sends quite detailed instructions and specifications. Clearly he greatly respects her abilities – indeed relies on them.

But what is he talking about? What is a "hydroscope"? And why should he be in such urgent need of one? The question has attracted attention for over 300 years at least. Normally a "hydroscope" implies a water-clock, but why should he be so desperate for a water-clock? *Fermat*, the 17th Century mathematician, suggested that what Synesius needed (being very ill) was a *hydrometer* to measure the density of drinking water or medicine of some sort. Now, certainly, the letter has a text which is compatible with this story. But does one really judge drinking water or measure medicine by finding its *density*? Was he perhaps *making* his own medicine? The whole matter leaves me perplexed.

Finally we return to the astrolabe. The term "astrolabe" is applied to a wide variety of astronomical or navigational instruments. (For an accessible article on the astrolabe see *Scientific American*, Jan. 1974.) Essentially all the various instruments that went by the name were models of the heavens. Some were "armillary spheres" – large, and necessarily clumsy, 3-D structures. Other, later, varieties were portable 2-D instruments in which geometric projections made for what were handy dedicated analogue computers.

This account comes from Neugebauer who suggests that Theon's "lost" work on the astrolabe is alive and well – the common core to various suspiciously similar works which he sees as Commentaries on an earlier work: Theon's. It may well be that it was Ptolemy who showed how to construct the handier 2-D instrument and that Theon's book in its turn derived from Ptolemy. This seems to Neugebauer the most likely course of events.

In any case, Synesius wrote a covering letter (it isn't listed as a letter and is to be found elsewhere in his writings – however, it is a letter) to one *Paionos* to accompany the gift of an astrolabe. In it, he states that he designed the astrolabe himself but with help from Hypatia and had it crafted by the very best of silversmiths. The implication is that the knowledge derived by (probably) Ptolemy was passed on through Theon to Hypatia and thus to her pupil Synesius.

This then exhausts all we know of Hypatia's mathematics. It is commonly said that Theon was a transmitter of mathematics rather than a creator of it. He edited the works of others rather than developing theories of his own. The same would seem to be true of his daughter. She was widely respected as a teacher – eminent, influential, even charismatic in her day. But we have no evidence that she was anything more than this.

There has been an often stated view that she was a better mathematician than Theon. This derives from the passage in Philostorgius, which may however mean merely that she was the more widely acclaimed in her day. Indeed, we could argue that Theon was in fact the greater. In 640 or 642, the Arabs conquered Alexandria. What texts we have of Greek mathematics often come to us through Arab translations and Commentaries. This is true of Diophantus's *Arithmetic* and also of some of Theon's work. It is not unreasonable that there is a principle of selection here – the best work is what has survived; the Arabs saved what they thought worth saving. One would not like to push this notion too far, nonetheless a good proportion of Theon's work survives and almost none of Hypatia's.

Whatever judgement we make of her contribution to mathematics, she was certainly a remarkable woman. She certainly was a mathematician, a philosopher and a charismatic teacher. It would be nice to know more of her.

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A True Equality

The German university town of Göttingen is famous for its mathematicians and theoretical physicists, among them the very great mathematician David Hilbert (1862-1943). Overlooking the town are two hills known as *die Gleichen* (the equals). Hilbert was fond of saying that this was not because they were the same height, nor because they presented the same aspect to the viewer.

"Why the name then?", people would ask.

Hilbert is said to have attributed the name to the incontrovertible fact that they were the same distance from one another!

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