

## HISTORY OF MATHEMATICS SECTION

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### The Pre-Pre-History of Mathematics

In earlier columns, we have looked at the mathematics of the past: sometimes the recent past, sometimes the rather more remote past. The earliest figure mentioned was Zeno of Elea, who lived about 2500 years ago. Zeno's own words have not come down to us but we do have accounts by Aristotle and Plato of what Zeno is supposed to have said. This then is history, but only just; history is what is vouched for in writing. The history of mathematics goes back a little beyond the Greeks of Zeno's day. We have some records of the mathematics of the Babylonians (about 2000 B.C.) and the Egyptians (at least as old and probably older).

To find out what happened before this we rely on the techniques of pre-history. This issue's column will be about the very dawn of mathematics and what we can know about it. The earliest mathematics we can imagine is the mathematics of counting, and all counting systems use a base, a special number that is used to express other numbers. Thus we say (e.g.) "forty-three" and mean "four times ten plus three". Similarly we write 43 to the same end. Our base is ten.

In fact all cultures of advanced numeracy today use a base of ten; only two such cultures have ever used bases other than ten. The Babylonians are usually said to have had a base of sixty, a loose and not entirely accurate account of the true situation. The Mayans, pre-Columbian inhabitants of Central America, used a base of twenty (see *Function*, Vol. 12, Part 4).

As the example of forty-three shows, the structure of our own language, English, reflects our adoption of a base ten system. English is one of a large family of languages, called the Indo-European family. All such languages descend ultimately from a single language called *Proto-Indo-European*, or PIE for short.

Linguists have been able to reconstruct PIE to a very impressive extent. Written records in Greek, Latin and Sanscrit take us back a long way and scholars then deduce what happened before that.

Take the example of the word "eight". The Greek (Ancient Greek) for "eight" is *oktō*; the Latin is *octō*, the Sanscrit *astau*. From these and other pieces of evidence, linguists have reconstructed the PIE *\*oktōu* (the asterisk is a device to show that the word is reconstructed and not directly attested). The *k* sound has turned into an *s* in the Sanscrit. This is in line with a general principle that consonants tend to move forward in the mouth as time goes by. Thus *k*, made at the back of the mouth, down in the throat, became an *s*, made at the front of the mouth, with the tip of the tongue.

Another line of evidence comes from languages still spoken today but which preserve many archaic features. The best example is Lithuanian. The Lithuanian word for eight is *astuoni*, and again we see the *k* → *s* shift that was observed in Sanscrit. The *k* is indeed preserved, but in an odd way, in English. True, we don't pronounce it, but we write eight with a *-gh-*. This corresponds to a *-ch-* in the German *acht*. We have dropped the guttural sound, as have the Italians, who have *otto*, from the Latin *octō*. The difference is that our spelling continues to reflect the older pronunciation.

Other Indo-European languages show variations on these themes. Thus we have in Old Irish *ocht*, Gothic *ahtau*, Old English *eahta*, Old Slav *osmi*, Armenian *ut'*, Hindi *āth*, Persian *hasht*, Tocharian *okāt* and so on. All these words derive from \**oktōu*.

This is just one example of how linguists can enter into a pre-historical world and deduce how things must have been before written records existed. By such means, very much of PIE has been reconstructed. Apart from a few details we know the numerals in PIE, and how its speakers counted. They very clearly used base ten.

There is some doubt as to who spoke PIE and when and where it was spoken. Probably about five to six thousand years ago and probably in Western Asia or the Middle East. This is about as much as we can say.

But now consider that PIE itself must have come from somewhere. And that the base ten system must have evolved from something less developed.

If we look at cultures today in which number notions are less important than in our own, we find a vast array of ways in which numbers are described. All languages have a word for "two", but by no means all have made what one authority (Menninger) calls "the step to three". There are languages (some Australian Aboriginal languages have this character) where the numeral concepts are *one*, *two* and *many*.

Where this happens, there are three forms of the word representing the objects being counted: singular, dual and plural. In English, by contrast, we have singular and plural (but no dual). We say, for instance, "one horse", "many horses", but we also say "two horses". It's horses as long as there are more than one of them.

Interestingly enough PIE had a singular-, dual-, plural system and this was preserved in (e.g.) Sanscrit. The Sanscrit word for "horse" is *asvas*, but for "horses" the word is *asvau* if there are two and *āsvās* if there are three or more. Similarly in Ancient Greek where the words are (respectively) *hippos*, *hippō*, *hippoi*. The dual survives up to a point in modern Lithuanian, but in large measure we have lost it from today's Indo-European languages.

But not entirely; about a year ago I set out to find vestiges of the dual in modern English usage. I ended up finding five such.

The first and most obvious one comprises the words *both* and *either*. These can only be used in the dual. For more than two items, they must be replaced by the words *all* and *each* which are not normally used if the situation is a dual one. Similarly with "the other" and "another".

A like situation exists with the prefixes *ambi-* and *amphi-*, as in "ambidextrous" and "amphibious". These prefixes have a clear dual implication, although in one case, it has partially extended to cover plurality as well: if a sentence were so unclear as to be capable of three or more interpretations, we would still speak of it as *ambiguous*. (This is to say there can be several *alternative* meanings. *This* word also was, until very recently, exclusively dual.) Thus that pressure which for thousands of years has been acting to exclude the dual is still operating today.

A somewhat more subtle case of a dual-plural distinction occurs with the difference between the comparative and the superlative forms of the adjective. The comparative form ("more", "larger", etc.) is used where there are two objects under discussion; the superlative ("most", "largest", etc.) when there are more than two.

Fourthly, there are special words signifying pairs. These differ from specialist words relating to other numbers ("quartet", for example) in that the word *pair*, the word

*yoke* and the word *brace*, etc. do not derive from the same root as the word *two*. "Quartet", by contrast, does derive from the PIE word \**kwetwores*, which is also the remote ancestor of our word *four*. (See *Function*, Vol. 8, Part 2, p.25.)

Finally, even when a word does derive from the same root as the word *two*, e.g. *twin*, our language allows it a wider use than it would (say) *triplet*. So we have *twin-tubs* and *twin-sets*, where we would say "3-stage", "3-piece", etc. almost universally, were the number *three* instead of *two*.

These then are the vestiges of the singular-dual-plural system that still stay with us today. Linguistic habits take a long time to die. Back in about 4000 B.C. when PIE was spoken, those linguistic habits were stronger and the dual was a more powerful influence. Where then did it come from?

Well, of course, we can't know, but we can put up the most plausible available hypotheses and see if these make sense. We are, if you like, asking questions about the pre-pre-history of our number-words.

We have seen that all languages of today allow a "one-two-many" distinction and some have no numerical concepts beyond this. This structure is mirrored in the singular-dual-plural system which was well-preserved in PIE. It is thus plausible to suppose that the grammatical structure of PIE was a relict of an earlier counting system: a "one-two-many" system pre-dating PIE's fully developed use of base ten.

Very recently the work of some influential Soviet linguists has started to become known in the west. Let us see what these linguists suggest.

The Indo-European languages fall into several sub-groupings, such as the Romance languages, the Slavic languages, the Germanic, the Celtic, and so on. These different sub-groupings together make up the Indo-European family. Other families exist: the Uralic (of which Finnish and Hungarian are examples), the Afro-Asian (Hebrew, Arabic, etc.), the Dravidian (Tamil and its relatives) and so on.

The Soviet linguists propose that some *families* are related to form "super-families", just as the sub-groupings together form families. In particular, they proposed that all the families mentioned above, together with some others (but also excluding yet others like Chinese) all derive from a very remote common ancestor called *Nostratic*.

This is regarded by many as rather speculative – but what little has been published in the West does lend some support to the historic base two notion in that proponents of Nostratic theory regard the expression of the numeral "two" as one of the strongest elements in their theory. Other numerals (with the possible exceptions of three and four, of which I will say more in a later column) do not seem to have been part of Nostratic (and the evidence for three and four is much more tenuous).

For further reading in this difficult but fascinating area, see *Scientific American*, October 1989 and March 1990.

To close, I quote an English translation of a poem written in Nostratic by the Soviet linguist Illič-Svitič. (In the original it rhymes and has a regular rhythm!)

"Language is a ford through the river of time,  
It leads us to the dwelling of ancestors.  
But he doesn't arrive there  
Who fears deep water."