Function

Volume 14 Part 4 August 1990

$$\pi = \frac{213440\sqrt{10005}}{27257006 \displaystyle\sum_{n=0}^{\infty} \left(\frac{\left(n + \frac{13591409}{545140134}\right)(-1)^n \, (6n)!}{640320^{3n} \, (n!)^3 \, (3n)!} \right)}$$

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FUNCTION is a mathematics magazine addressed principally to students in the upper forms of secondary schools.

It is a 'special interest' journal for those who are interested in mathematics. Windsurfers, chess-players and gardeners all have magazines that cater to their interests. FUNCTION is a counterpart of these.

Coverage is wide — pure mathematics, statistics, computer science and applications of mathematics are all included. There are articles on recent advances in mathematics, news items on mathematics and its applications, special interest matters, such as computer chess, problems and solutions, discussions, cover diagrams, even cartoons.

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Articles, correspondence, problems (with or without solutions) and other material for publication are invited. Address them to:

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FUNCTION

Volume 14 *Part 4*

At Monash University there is a rather unique sundial which not only tells the time but also the date. Function editor Dr Michael Deakin describes the problems involved in correcting it when it started to tell the wrong time.

We also investigate square-ish circles and look at an interesting number pattern. And for those who can't remember pi we give some mnemonics.

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THE FRONT COVER

Calculation of pi to a high degree of accuracy seems to have become fashionable, especially with manufacturers of high-speed computers. Within the last twelve months the record has gone up three times, from 500 million digits to 1000 million digits to 2000 million digits. The accompanying table gives the accuracy to which pi has been calculated. The formula on the front cover is the formula used to calculate pi to 1000 million digits.

Year	Number of digits of pi
1973	1 000 000
1981	2 000 000
1982	4 000 000
	8 000 000
1983	16 000 000
1985	17 000 000
1986	30 000 000
1987	138 000 000
1988	201 000 000
1989	500 000 000
	1 000 000 000
	2 000 000 000

Calculation to such an accuracy presents some unusual problems. The first problem is just finding somewhere to store the digits. They certainly would not be put on floppy disks – it would take approximately 1400 standard PC disks. Nor would they be put on the type of hard disk used by personal computers as it would take about 13 of them. It requires quite a high capacity hard disk.

The calculation has to be done on a high-speed computer with a huge amount of memory – enough to hold the value of pi in primary storage. Since the calculations were performed over a period of some eight months, a major problem was ensuring the validity of the calculations. It would have been necessary to ensure that the numbers were not affected by glitches in the circuitry or by errors reading or writing numbers. Ensuring the data integrity was of major importance.

The formula on the front cover is an infinite sum which converges quite rapidly. It is only necessary to take one term to get pi to the same accuracy as a calculator. The first term is

$$\pi \approx \frac{213440\sqrt{10005}}{272570067\left[\dfrac{13591409}{545140134}\right]}$$

which on my calculator gives 3·141592654, the same as the pi button. Using high precision arithmetic, it gives the value 3·14159265358973..., which is accurate to thirteen decimal places. Taking the first two terms gives the formula

$$\pi \approx \frac{213440\sqrt{10005}}{272570067\left[\dfrac{13591409}{545140134} - \dfrac{\left[1+\dfrac{13591409}{545140134}\right](6!)}{640320^3(3!)}\right]}$$

which gives the value $3{\cdot}141592653589793238462643383\ldots$, correct to 27 decimal places. (For other approximations to pi see *Function, Vol.* 13, *Part* 2.)

* * * * *

SHADES OF PYTHAGORAS VIA FIBONACCI

Garnet J. Greenbury, Brisbane

Let f be the Fibonacci sequence with first few terms

$$1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21,\ 34,\ 55,\ 89,\ 144,\ 233,\ 377.$$

f is defined by the rule

$$\begin{cases} f_{n+2} = f_{n+1} + f_n, & n \geq 1 \\ f_1 = f_2 = 1 \end{cases}$$

The Fibonacci sequence has many interesting properties, and we present a few examples.

Example 1.

$$(f_{n+1})^2 + (f_{n+2})^2 = f_{2n+3} \qquad (1)$$

To illustrate this, take the case $n = 4$. We verify that

$$5^2 + 8^2 = 89.$$

Notice that the sum of the subscripts of the terms on the left side of (1) equals the subscript of the term on the right side of (1).

Example 2.

$$(f_n\, f_{n+3})^2 + (2f_{n+1}\, f_{n+2})^2 = (f_{2n+3})^2 \qquad (2)$$

To illustrate this, take the case $n = 5$. We verify that

$$(5\times 21)^2 + (2\times 8\times 13)^2 = 11025 + 43264$$
$$= 54289$$
$$= 233^2.$$

Notice that the sum of the subscripts of the factors in each term on the left side of (2) equals the subscript of the term on the right side of (2).

Example 3.

$$f_{m+n} = f_{m-1}\, f_n + f_m\, f_{n+1}. \qquad (3)$$

To illustrate this, take $m = 4$, $n = 3$. We verify that

$$13 = 2\times 2 + 3\times 3,$$

that is,

$$13 = 4 + 9.$$

Example 4.

$$f_{2k}\, f_{2k+1} + f_{2k+1}\, f_{2k+2} = f_{4k+2} \qquad (4)$$

To illustrate this, take the case $k = 4$. We verify that

$$21\times 34 + 34\times 55 = 714 + 1870 = 2584.$$

* * * * *

Editor's note: The above article does not prove the relations (1) to (4). You may care to attempt to prove them.

Exercise:

a) Examine the relations carefully, and show that (1) and (4) can be deduced from (3).

b) Show that $(A^2+B^2)^2 = (A^2-B^2)^2 + (2AB)^2$, and hence deduce (2) from (1). (This relates to the note after the Letters to the Editor.)

c) Prove (3). For those who are familiar with matrices, the formula

$$\begin{bmatrix} f_{m+1} & f_m \\ f_m & f_{m-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^m$$

may be useful.

* * * * *

RUNNING ROUND IN SQUARES AND GETTING CROSS

Michael A.B. Deakin, Monash University

If we run around in a circle

$$x^2 + y^2 = 1, \qquad (1)$$

then our x- and y- coordinates are given by the equations

$$\left.\begin{aligned} x &= \cos t \\ y &= \sin t \end{aligned}\right\} \qquad (2)$$

where t is the distance we have travelled, starting from the point $(1,0)$.

The circle described by Equation (1) may be thought of as lying between two squares as shown in Figure 1. Suppose first that we run round the inner (tilted) square. Then

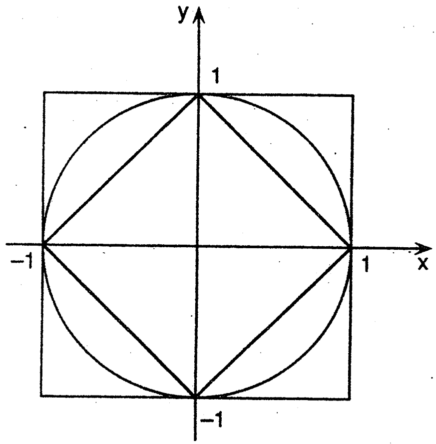

Figure 1

our x- and y-coordinates will be given by the equations

$$\left.\begin{aligned} x &= C_1(t) \\ y &= S_1(t) \end{aligned}\right\} \qquad (3)$$

where C_1, S_1 are the functions with graphs shown in Figure 2, and again t is the distance travelled from the point $(1,0)$. [Why I call these functions by these names will become apparent later.]

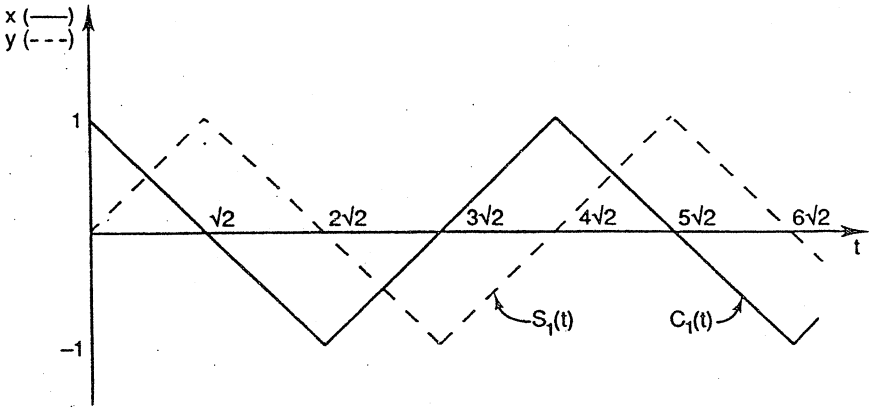

Figure 2

It will be noticed that $C_1(t)$ is a bit like $\cos t$ and $S_1(t)$ is rather similar to $\sin t$.

Now consider the outer square. This time the x- and y-coordinates are given by the equations

$$\left.\begin{aligned} x &= C_\infty(t) \\ y &= S_\infty(t) \end{aligned}\right\}, \tag{4}$$

where graphs of these functions (whose names will also be explained later) are given in Figure 3.

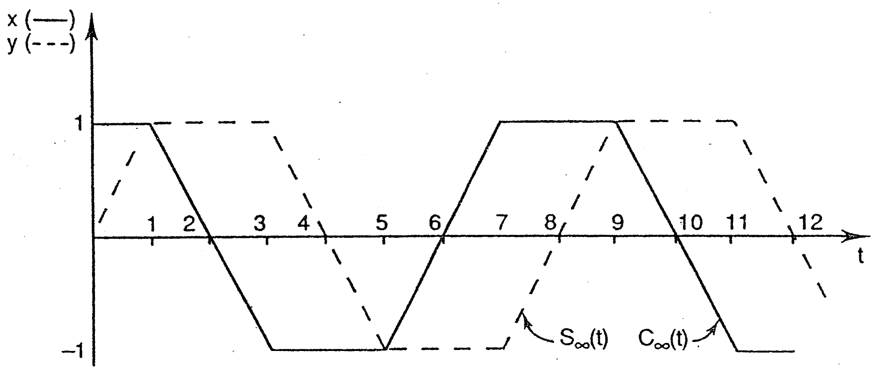

Figure 3

Again, these functions are rather reminiscent of our old friends cos and sin. Let us explore this matter further.

It will make life easier if we concentrate initially on the first quadrant ($x \geq 0$, $y \geq 0$). The functions C_1, S_1; C_∞, S_∞ may be built up completely from, in the case of the first pair, the values $0 \leq t \leq \sqrt{2}$, and, in the case of the second pair, the values $0 \leq t \leq 2$. These are the values corresponding to the first quadrant. (We could even extend both pairs of graphs into the region $t < 0$; this is quite easily done.)

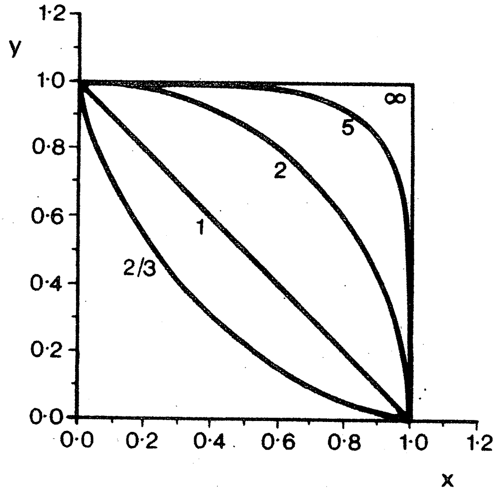

Figure 4

Figure 4 shows the first quadrant, and we see a number of curves. First look at the straight line joining (1,0) to (0,1). This has the equation

$$x + y = 1, \tag{5}$$

which, with a view to what follows, we will write

$$x^1 + y^1 = 1. \tag{6}$$

This curve is labelled with the numeral 1 in Figure 4.

Next out is the curve labelled "2", and this is the quadrant of the circle described by Equation (1), where the exponent is 2. This means that if C_1, S_1 correspond to an exponent 1 in Equation (6), we may similarly define C_2, S_2 in Equation (1). We thus have

$$\left.\begin{aligned} C_2(t) &= \cos t \\ S_2(t) &= \sin t \end{aligned}\right\}. \tag{7}$$

Further out from the origin in Figure 4 is the curve labelled "5", which is indeed the curve

$$x^5 + y^5 = 1. \tag{8}$$

Curves like that representing Equation (8) form a whole family of curves

$$x^n + y^n = 1. \tag{9}$$

These curves are called "supercircles" and you can read about them in Martin Gardner's column in the *Scientific American* of September 1965. Gardner applies the term only in the case $n > 2$, and there is some point to this. However, I will apply it whatever the value of n.

For each such case we will have a pair of functions (C_n, S_n) where

$$\left.\begin{aligned} x &= C_n(t) \\ y &= S_n(t) \end{aligned}\right\}. \tag{10}$$

Thus, our familiar cos and sin functions are (by Equations (7)) merely one pair from a whole infinite class of such function pairs.

As the exponent n in Equation (9) gets bigger and bigger, the supercircles bulge out more and more and come to approximate ever more closely the outer square, which may (loosely but conveniently) be thought of as the case $n = \infty$. Hence the notation of Equations (4) and the label in Figure 4.

When $n < 1$, the supercircles become concave figures rather than convex ones. The case shown in Figure 4 is that for which $n = 2/3$. I have chosen this deliberately as it is a very well-known curve termed the *astroid.* For more on the astroid and its properties, see *Function, Vol.* 2, *Part* 4 and *Vol.* 7, *Part* 4.

Again, we can envisage n tending to a limit, in this case 0. The path traversed then is a cross: from (1,0) in towards the origin, then up to (0,1), back down to the origin, then out to (–1,0), back to the origin, and so on. Such a curve would give rise to functions C_0 and S_0, where

$$\left.\begin{aligned} x &= C_0(t) \\ y &= S_0(t) \end{aligned}\right\}. \tag{11}$$

These functions are graphed in Figure 5. Once more there are many resemblances to the functions cos, sin.

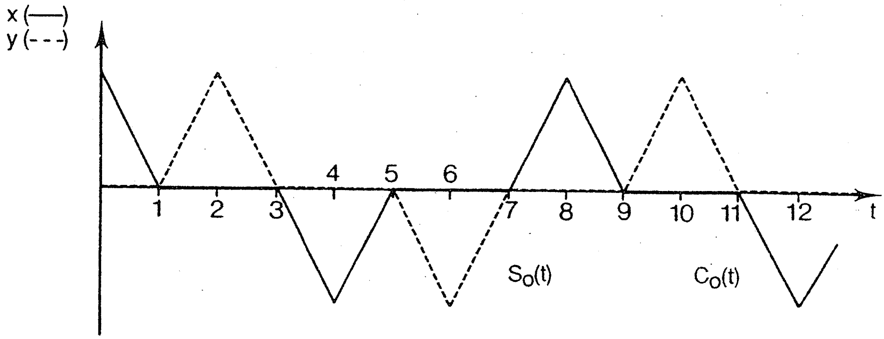

Figure 5

Figure 4 detailed only the first quadrant. Outside this, there are complications. To deal with these, we need the function $|t|$. $|t|$ may be defined as the distance (a positive quantity) between the point representing t on the number line and the point representing 0. In BASIC and other computer languages, $|t|$ is written as *ABS*(t), the absolute value of t. The equation for the full square corresponding to Equation (6) is actually

$$|x|^1 + |y|^1 = 1 \tag{12}$$

and in general we get, in place of Equation (9)

$$|x|^n + |y|^n = 1. \tag{13}$$

Sometimes the $|...|$ symbols are not needed. The familiar case $n = 2$ is a good example. For if x, y are squares any negativity is killed automatically anyhow. But this is not always so. Consider $n = 3$ and then think about $n = 1/2$. Equation (13) is the general case.

The function $|t|$, equivalently *ABS*(t), may be combined with another: $[t]$, equivalently *INT*(t), to produce explicit formulae for $C_1(t)$ and some of the other members of the family. $[t]$ means the greatest integer less than or equal to t. We then have:

$$C_1(t) = \left|\,\left|\frac{t}{\sqrt{2}} - 4\left[\frac{t}{4\sqrt{2}}\right]\right| - 2\right| - 1. \tag{14}$$

Can you show this to be true?

Equation (14) I derived from a formula given by Jack Thomas of La Sainte Union College of Higher Education (Southampton, U.K.) and published in the British Journal *Mathematical Gazette* (December 1988, pp. 307-309). Thomas introduces functions he calls clos and slin (the "l" stands for "linear") and these, in the terms of this article, may be defined as

$$\left.\begin{aligned} \text{clos } t &= C_1(t/\sqrt{2}) \\ \text{slin } t &= S_1(t/\sqrt{2}) \end{aligned}\right\}. \tag{15}$$

These functions relate to a square also, but one a little different from the inner square of Figure 1. Can you identify it and relate it, geometrically, to the circle in that figure?

What I've said so far only scratches the surface of the subject. There are a lot of questions left. It could make a nice project to investigate some of them. Here are five that come to mind.

(1) The functions C_n, S_n satisfy various properties of the trigonometric functions, e.g.

$$C_n(0) = 1,\ S_n(0) = 0$$
$$C_n(-t) = C_n(t),\ S_n(-t) = -S_n(t)$$
$$C_n^n(t) + S_n^n(t) = 1.$$

What about other properties?

(2) In discussion Equation (13), I have only considered the cases $n \geq 0$. What happens if $n < 0$?

(3) In what sense can we interpret the equations

$$|x|^0 + |y|^0 = 1$$
$$|x|^\infty + |y|^\infty = 1$$

as algebraic representations of the limiting curves described earlier? (In this connection, see the discussion of the meaning of 0^0, *Function, Vol. 5, Part* 4.)

(4) This is a rather harder question, and I don't think it's ever been answered, but a sophisticated calculator could do it fairly quickly. The period of C_1, S_1 is $4\sqrt{2}$, the perimeter of the corresponding supercircle. That is to say that the graphs of these functions merely replicate *ad infinitum* the pattern shown in an interval of $4\sqrt{2}$ in length. The graphs of C_2, S_2 have period 2π. Those of C_∞, S_∞ have period 8. What about other values of n? Figure 6 shows the general shape expected for the period T_n of the functions C_n, S_n expressed as a function of n. I greatly doubt that its fine detail has ever been explored.

(5) I have no idea as to the answer to this question, but, from Figure (6), we have that not only do we know that $T_2 = 2\pi$, but we can further see that for some number N, for which $0 < N < 1$, we have $T_N = 2\pi$. What is N ? And can we explain its value?

Well, there are some questions. Over to you!

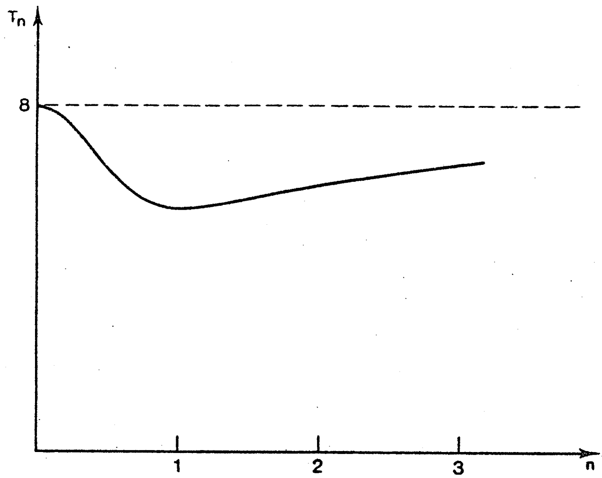

Figure 6.

* * * * *

HOW WE FIXED A SUNDIAL

Michael A.B. Deakin, Monash University

On the north wall of the Union Building at Monash University is a most remarkable sundial. We believe that it is the only sundial of its type in the world. Most people, looking at it, would say "Sundial! That's not a sundial; it looks nothing like one".

Most of us are conditioned by our experience to know only one of the (literally) hundreds of types of sundial that have from time to time been devised. That familiar type consists of a table, engraved with lines representing the hours, and having a pointer (technically called the "gnomon") whose shadow is read off the scale to give the time. Others are not horizontal, but vertical.

Many will have noticed with such sundials (a good example is to be found in Melbourne's Botanic Gardens) that the result is not especially accurate, but that the reading may be improved by adjusting it according to a tabulation (or graph) of corrections, often also engraved onto the sundial's surface. (See Figure 1.)

The need for these corrections arises because the sun, as well as traversing the sky each day, also varies its path with the seasons. (This description is, of course, from our point of view on earth!)

The late Carl Moppert, of Monash University's department of Mathematics, thought to use this fact to design a sundial that would tell both time and date; it would be, in other words, not only a clock, but a calendar as well. The need for correction would be dispensed with. Instead of using our prior knowledge of the date to correct the apparent time, we would *deduce* the date from the apparent discrepancy.

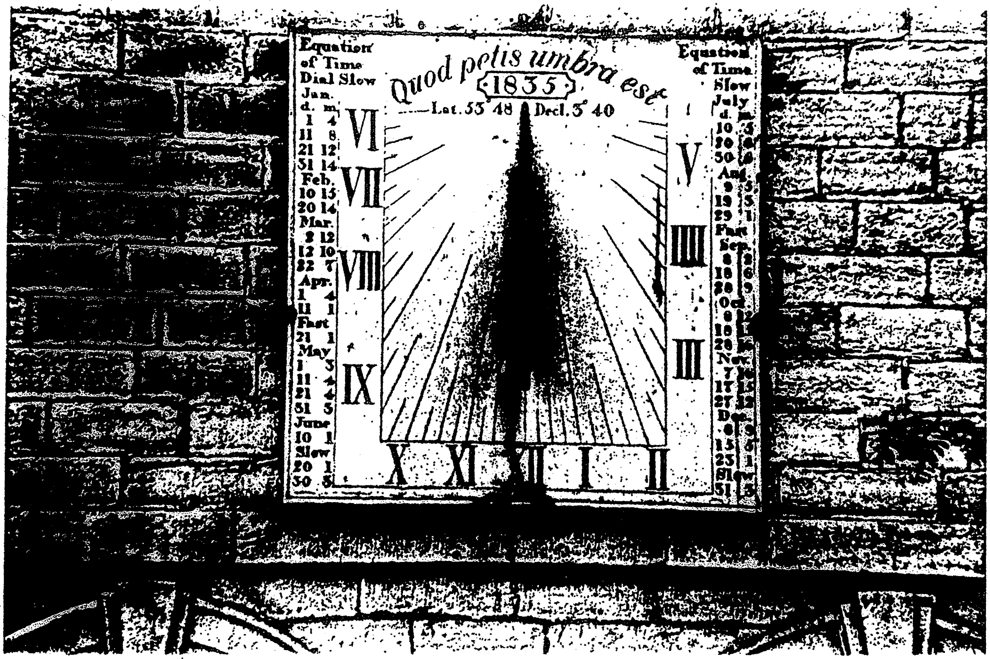

Figure 1 : *A vertically mounted conventional sundial. Note the marginal corrections to left and right. These must be applied to the apparent time to give the true time. The photograph is by Professor John Crossley of Monash University. The sundial itself is in Yorkshire.*

This idea was not entirely new. It is partially realised, but only partially, in a number of sundials at Britain's famous Greenwich Observatory. Furthermore, at almost exactly the same time as Carl was designing his sundial, C.K. Sloan in the U.S. produced similar designs. (These were written up in *Scientific American*, Dec. 1980, pp. 174-180.) Sloan's sundials, superficially similar to Carl's, are in fact simpler. They are meant to lie flat and, as at any given latitude and longitude there is only one horizontal plane, latitude and longitude are the only determining parameters in the underlying computations.

Carl's sundial, by contrast, is mounted on a vertical wall and this may have any orientation whatsoever. (In the case of the Monash Sundial, the wall faces 16·85° East of North. It is important to know this figure accurately.) The calculations for a vertical sundial are thus much more complex.

Sundials like Carl's and Sloan's are referred to as "analemmic sundials", because (see Figure 2) the loops that enable us to read off the hours* are technically known as "analemmas". It is believed that the Monash sundial is the only true vertically mounted analemmic sundial in the world.†

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* Notice that in December, when the sun is high in the sky, the shadow of the metal ring follows the bottom of the six curves, and in June the shadow follows the top curve. One can estimate the month from the position of the shadow between these curves, and the time of day from the position of the shadow in relation to the hourly analemmas

† A booklet by the late Carl Moppert describing this sundial is available from the department of Mathematics, Monash University, price $6.00. A very abridged account appeared in *Function*, *Vol.* 5, *Part* 5.

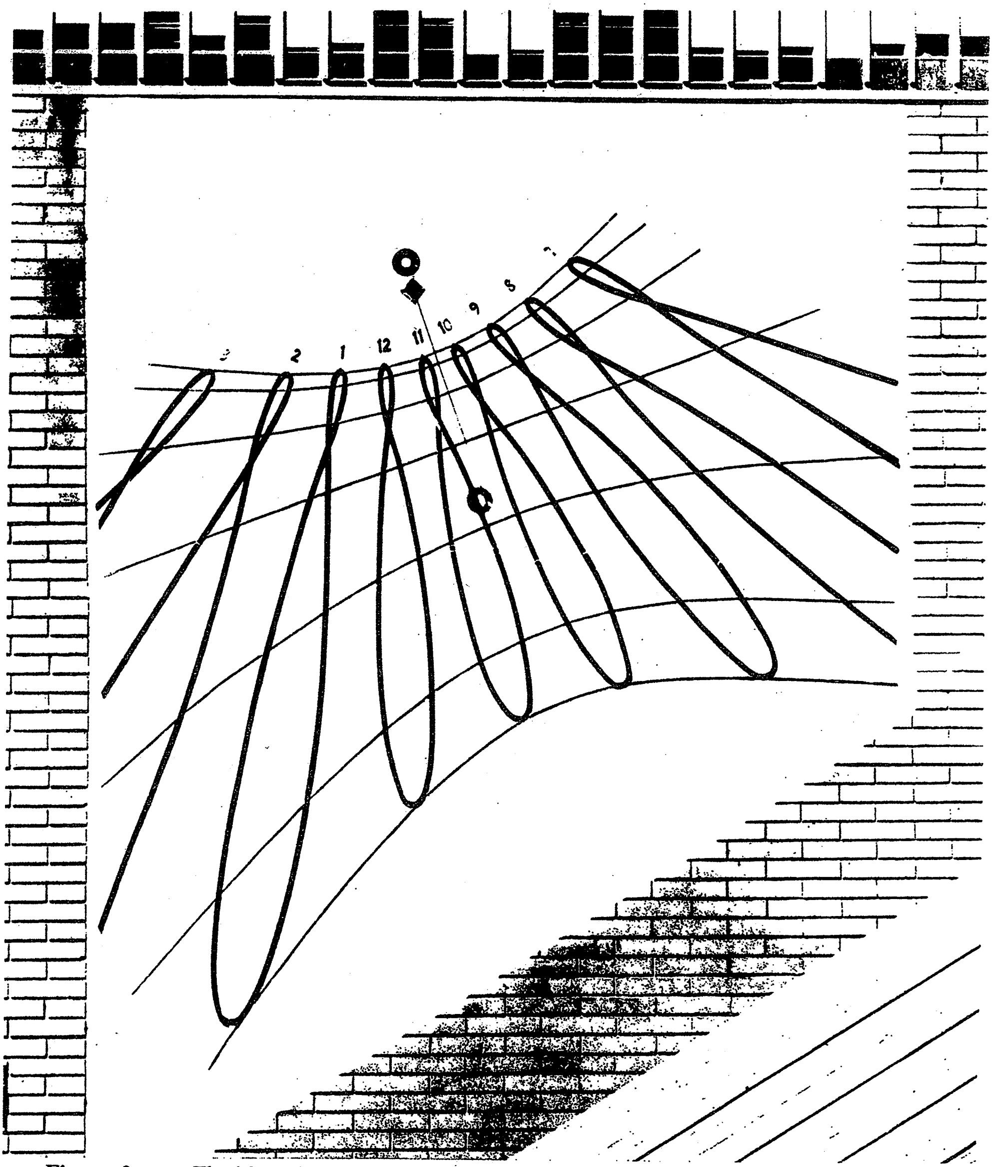

Figure 2 : *The Monash University sundial. The photograph was taken by Adrian Dyer of Monash University's department of Physics. The reading is 10:31 a.m., 13/10 when it should have been 10:26 a.m., 18/10. This was the photograph used to correct the sundial as described in the article.*

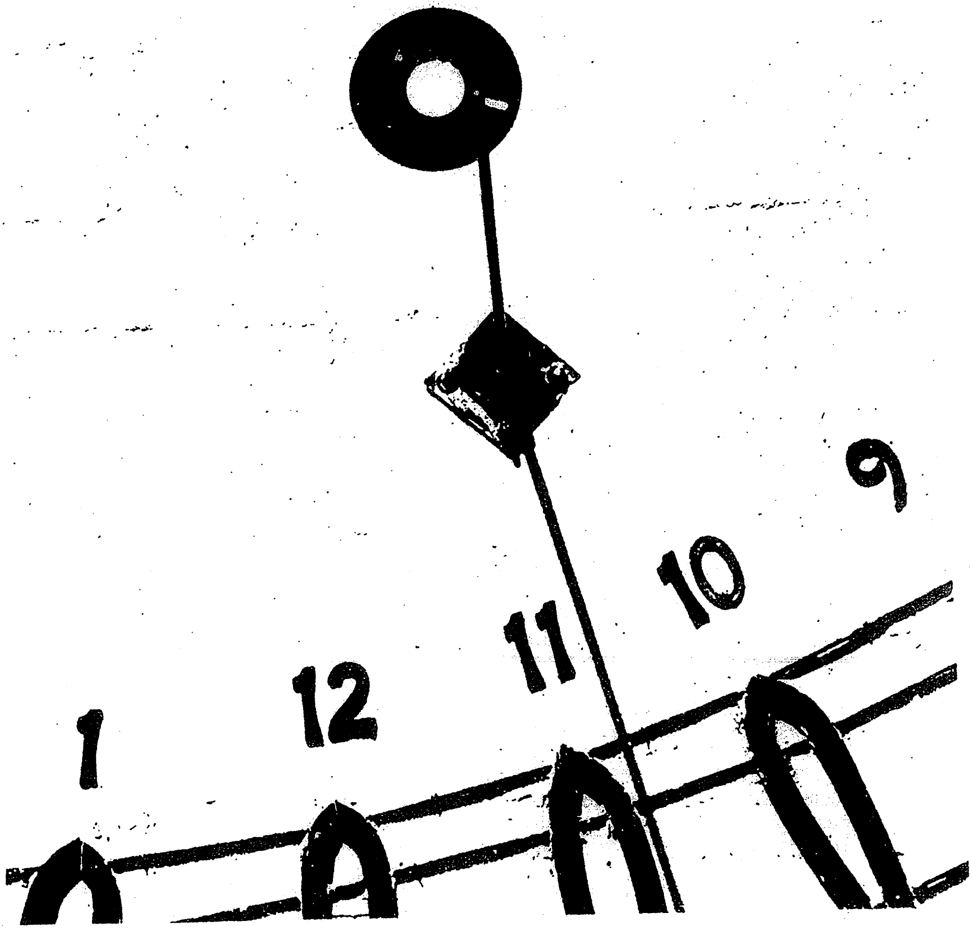

Figure 3 : *Close-up of the ring that casts the shadow. It is attached by a nut that screws onto the support. The ring may be rotated about the support and may also slide along a slit visible here to the right of the nut.*

With help from others, Carl built his sundial and it was officially "opened" by the then Chancellor of Monash, Sir Richard Eggleston, on March 24th, 1980. Carl himself died of leukemia not much later (September 16th, 1984). Had he lived, he would have been the first to notice when something went wrong with the sundial. It had once been accurate to well within five minutes, but suddenly it was out by up to fifteen. What had happened? How can a sundial be wrong? Especially when it has been right before?

Figure 3 provides the key to the explanation. The shadow is cast by a metal ring attached to the end of a horizontal metal rod protruding from the wall. This ring had worked loose and was hanging forlornly and out of position. With the ring now wrongly placed, its shadow too was awry.

In due course, a repair (or, more accurately, an attempted repair) was carried out. There seems to be no record of this earlier adjustment, nor of how it was done. Most probably the ring was positioned in such a way as to accord with a photograph taken for the paper *The Monash Reporter* back in the days when the sundial was under construction. Certainly this was the outcome, but the sundial was still inaccurate.

And so it remained for several years. Last year, however, I was asked to see if it could be fixed up properly.

The first thing I did was to satisfy myself that if it could be made to read true at some particular time and date, then it would read true for all times and dates. This is in fact the case, as long as the scale (i.e. the pattern drawn on the wall) is accurate. This is clearly so, because the sundial once used to be very accurate.

The sun's rays, falling on the wall, are (to an excellent approximation) parallel. (They meet, eventually, back at the sun, i.e. 150 million kilometres away!) Thus we can say that there is a ray falling on the right spot on the wall, but instead of encircling this ray, the ring had become mis-positioned and encircled the wrong ray.

There are two ways to go about the task of repositioning the ring correctly. One is experimental: on the 22nd day of some month (these are the days when the scale gives an accurate reading for date) and at a time which is an exact hour, move the ring till the shadow is in the correct position. This is the ideal way to do things, but it has major limitations. First, the sun must be shining; second, it must be a working day; third, we can't do it on Thursday or Friday because a market operates then and prevents access; fourth, December does not allow a very accurate fix nor do April, May, June, July or August. (In these months the change in position from one day to the next is small.)

We might nonetheless have satisfied all these constraints on March 22nd, 1989, but this was before we began the endeavour. The next feasible date was October 22nd, 1990. This seemed a little long to wait, and what if that day turned out, as it well might, to be cloudy?

So it seemed that other measures were called for and after waiting through what seemed like interminably many rainy weeks and drizzle-drenched days, we finally succeeded in getting a sequence of photographs taken on 18th October 1989. Figure 1 is actually the 5th of this sequence and the best. It was taken at 10:26 a.m.

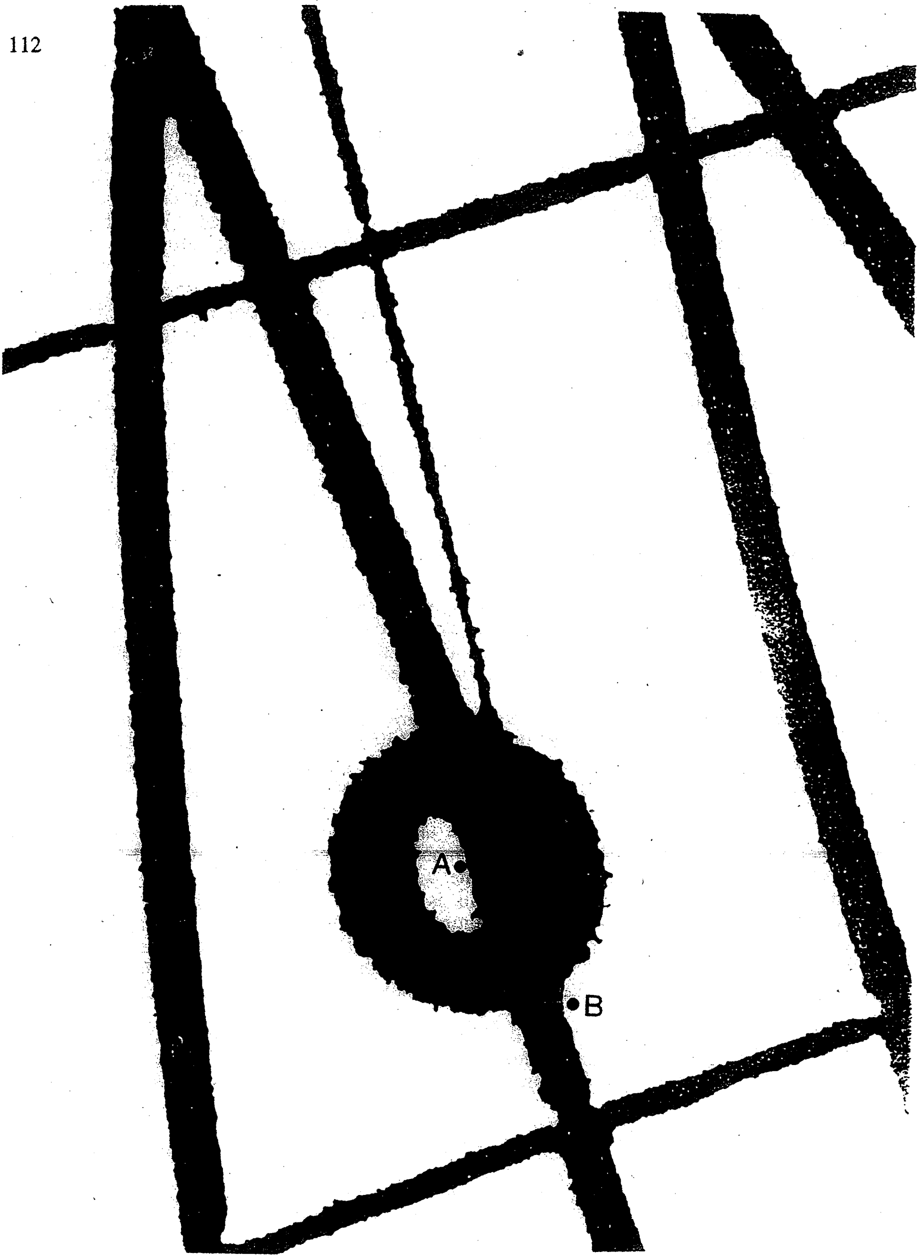

Figure 4 : *Close-up of the sundial reading from Figure 2, showing :* A. *where the shadow fell,* B. *where it should have fallen.*

Next, I photo-enlarged the relevant portion of the picture and taking careful measurements, worked out where the shadow should have fallen (Figure 4). [I also computed the time and date actually displayed – 10:31 a.m. on October 13th, but this is not needed for the calculation.]

Careful measurements enabled an estimate to be made of how far out the shadow was. I did this by using the bricks you can see in Figure 2 as a scale. (Each brick is 39 cm long and 8·5 cm wide.) As close as I could judge it, the shadow was 5·6 cm too far to the left (as we face the wall) and 5·9 cm too high.

As the sun's rays are parallel, this means that the ring itself is also 5·6 cm too far to the left and 5·9 cm too high.

Now look again at Figure 3. It shows that the ring is bolted onto its supporting rod and may slide in and out and may also rotate around it. Careful measurements based on the original of Figure 3 suggested that the centre of the ring was 4·9 cm to the left of the support and 1·6 cm above it.

It should have been (4·9–5·6) cm, i.e. –0·7 cm to the left, or 0·7 cm to the right and (1·6–5·9) cm, i.e. –4·3 cm above, or 4·3 cm below the support. The distance from the support to the centre of the ring was estimated to be 5·2 cm. On my calculations it should have been a distance of

$$\sqrt{0{\cdot}7^2+4{\cdot}3^2}\ \text{cm}$$

i.e. 4·4 cm. However, looking at Figure 3, we notice that the distance cannot be reduced any further. The centre of the ring is as close to the support as it will go.

This suggests that the 8 mm discrepancy between 4·4 cm and 5·2 cm is due to accumulated errors. The agreement is probably as close as we have a right to expect.

This leaves us with rotation to consider. Look at Figure 5. The points A, B show where the ring was and where it should be. We can thus read off that the ring needs to be rotated through $18^\circ + 90^\circ + 9^\circ = 117^\circ$ in the anti-clockwise direction.

Again the calculation is subject to error and what we can say is that we need an anti-clockwise rotation of about 120°.

The ideal day to adjust the sundial would have been March 22nd, the equinox, when we could have got a highly accurate fix for both time and date. Moreover, March 22nd, 1990 was a gloriously sunny day with not a cloud in the sky. The only trouble was: it was a Thursday Market Day. We were refused access.

The best possible compromise was the offer of a "cherry-picker" and two assistants on Wednesday March 28th. I booked them for 11:30 a.m. and, arriving at this time, found all in readiness. About 10 minutes before 12 noon, one of my offsiders climbed into the cherry-picker, while his assistant manned the controls to send him up to operation height. Just as this happened, an enormous cloud blotted out the sun.

This meant that all we had to rely on was the calculation of where the shadow should fall. The chap up in the cherry-picker carefully loosened the nut on the end of the iron rod and rotated the ring through 120° anti-clockwise.

And there matters might have ended but for the fact that while up in the cherry-picker he occupied himself in tightening the analemmas back onto the wall, as they had begun to work loose. It was then that we had a stroke of luck; with 50 seconds to go, the sky cleared and the sun shone through.

What emerged was that we had greatly improved the reading, but the result was still inaccurate. But now it was possible to rotate the ring (it took a further 35° anti-clockwise) so that it read 12 o'clock at precisely 12 noon. This was the best result we could achieve.

Two questions remain:

1. What was the source of the 35° error?
2. How accurate is the sundial now?

The first is relatively easily answered. The determination of where the shadow should fall (Figure 4) is not particularly accurate, owing to the large gaps between the analemmas and also the date marker-lines. The "vertical" scale in particular is stretched, especially at dates in late October. This has led to small errors in the determination of those distances 0·7 cm and 4·3 cm. Such small errors can lead to quite large errors in the calculation of the relevant angle. Hence the 35°.

Since the repair, the sundial has been checked many times for its correctness in giving the time of day, and it has done very well so far. On April 18th, 1990 the shadow fell below the line indicating April 22nd and on April 23rd it fell above it. April 22nd was a miserable overcast day and so no check was possible. But clearly it is now reasonably accurate. We may yet find that it needs some further fine adjustment, but we hope not.

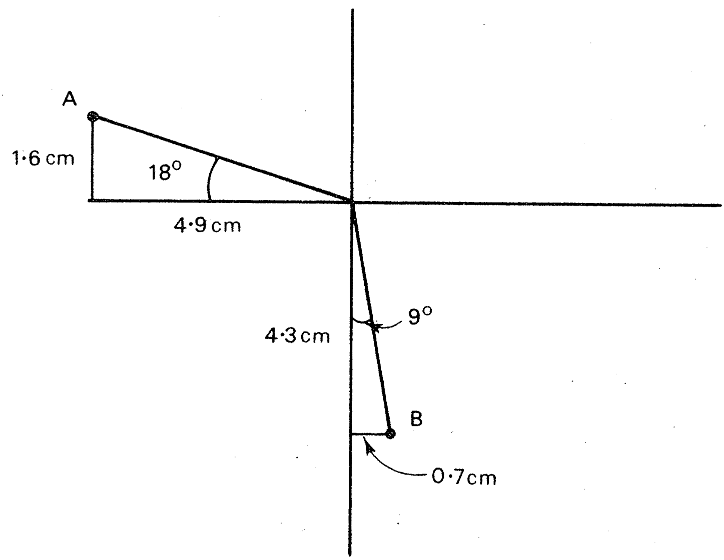

Figure 5

Calculating the angle of rotation.
***A* is the actual position, *B* is the correct position.**

LETTERS TO THE EDITOR

Even More on Cubics

Here is my response to D.F. Charles's article (*Function*, *Vol.* 13, *Part* 5). If (A,B,C) is an integral solution of $C^2 = A^2 + B^2 - AB$ then after rewriting the equation as $C^2 - B^2 = A^2 - AB$ and factoring to obtain $(C+B)(C-B) = A(A-B)$ we deduce

$$\frac{C+B}{A-B} = \frac{A}{C-B}, = \frac{m}{n}, \text{ say.}$$

Solving the equations

$$n(C+B) = m(A-B)$$

$$nA = m(C-B)$$

for B, C in terms of A, m, n gives

$$B = \frac{A(m^2-n^2)}{m^2+2mn}, \quad C = \frac{A(m^2+mn+n^2)}{m^2+2mn}.$$

In other words,

$$(A,B,C) = (m^2+2mn,\ m^2-n^2,\ m^2+mn+n^2) \times k \tag{1}$$

where k is some constant of proportionality. For the sake of simplicity, take $k = 1$, giving the solution

$$A = m^2 + 2mn,\ B = m^2 - n^2,\ C = m^2 + mn + n^2. \tag{2}$$

For example:

$m = 2$, $n = 1$ yields $(A,B,C) = (8,3,7)$,
$m = 3$, $n = 1$ yields $(A,B,C) = (15,8,13)$, and
$m = 3$, $n = 2$ yields $(A,B,C) = (21,5,19)$.

It may be observed that if A, B, C are the lengths of the sides of a triangle, then by the cosine rule

$$A^2 + B^2 - AB = C^2$$

when the angle opposite the side of length C is 60°.

K.R.S. Sastry,
Addis Ababa

Editor's notes:

If (A,B,C) is an integral solution to $A^2 + B^2 - AB = C^2$ then so also is (tA,tB,tC) for any integer t. We therefore concentrate on solutions for which A, B, C have no common divisor. Clearly A, B, C have a common factor t when m, n have common factor t. In addition, A, B, C can have a common factor of 3, even when m, n have no common factor, and this occurs when m, n have the same remainder on division by 3. Thus, for example, when $m = 4$, $n = 1$ we obtain from (2) the solution $(A,B,C) = (24,15,21)$. We can take $k = \frac{1}{3}$ in (1) and deduce the solution $(A,B,C) = (8,5,7)$.

Combining Sastry's solution, the above observation, and the remark of G.J. Greenbury in *Function, Vol.* 14, *Part* 2 we have the complete solution of the problem raised by D.F. Charles.

You may care to use the same technique to show that if A, B, C are the side lengths of a right-angled triangle, i.e., $A^2 + B^2 = C^2$, then $(A,B,C) = (2mn,\ m^2-n^2,\ m^2+n^2) \times k$. (In this case the solutions with A odd arise when m, n are both odd and $k = \frac{1}{2}$.)

* * * * *

ICME-7

The Canadian National Committee for ICME-7 is pleased to announce that the Seventh International Congress on Mathematical Education will be held at Université Laval in the city of Québec, Canada, from August 16 to 23, 1992. The program activities and informal meetings will offer many opportunities for establishing personal contacts and for disseminating ideas relevant to current problems and issues in mathematics education. A variety of social and cultural activities, as well as excursions, will be available for both participants and persons accompanying them.

English and French will be the official languages. However, it is anticipated that most sessions will be conducted in English. Simultaneous translation will be provided for certain sessions. Moreover, some services will be available in three languages: English, French and Spanish.

For further information, contact

The Local Organizing Committee of ICME-7
ICME-7 Congress
Université Laval
Québec, QC
Canada G1K 7P4
(Fax: (1) (418) 656-2000)

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PACKING, SUPERCIRCLES AND INTUITION

R.T. Worley, Monash University

We are all familiar with the standard way of packing circles in the hexagonal pattern

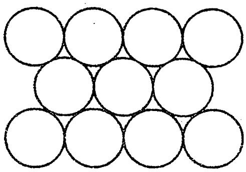


Figure 1

The points of contact lie on straight lines and if one looks closely at a circle and the

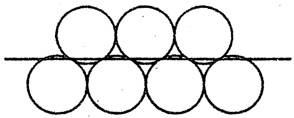


Figure 2

contact points on it one sees a regular hexagon inscribed in the circle. The sides of the

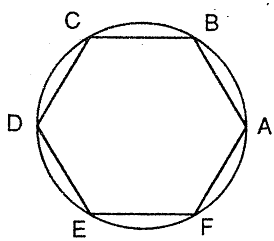


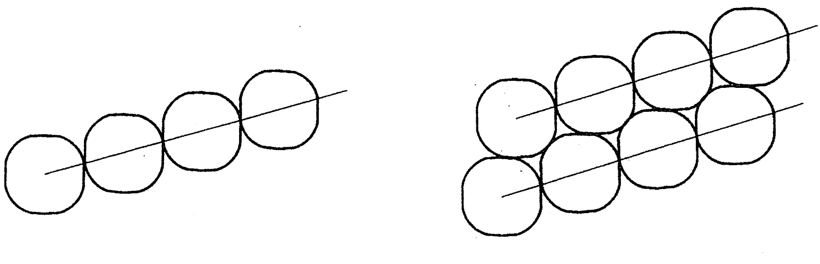
Figure 3

hexagon are equal to the radius of the circle.

Suppose now we try to pack supercircles

$$x^n + y^n = 1$$

where we will assume $n \geq 1$ so the supercircle is convex. One way of doing this is as follows. Choose a point A on the circumference of a supercircle and draw the line OA (where O is the centre of the supercircle). Form a line of supercircles centred on and touching on this line (Fig. 4a). Take identical lines of supercircles and fit them together (Fig. 4b). Fig. 5 illustrates the packings obtained with the point A on a) the x-axis and b) on the line $y = x$. These two were chosen because they are symmetry lines of the supercircle.



(a) (b)

Figure 4. Packing supercircles

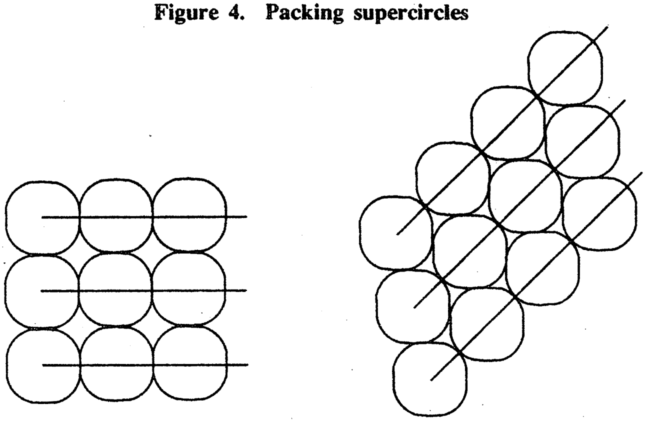


Figure 5. The symmetric packings of supercircles

As is the case with circles, if one looks at the contact points on a supercircle in such a packing one discovers a hexagon (Fig. 6). In this case, the hexagon is not normally a regular hexagon, but it still has parallel sides – in fact if the vertices are labelled *ABCDEF* in order, then the hexagon divides up into three equal area parallelograms *OABC*, *OCDE*, *OEFA*.

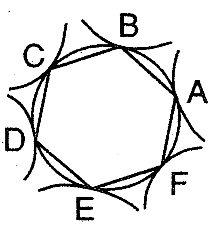


Figure 6

Which of these packings gets more supercircles in a given (large) area? This problem has an interesting history. H. Minkowski, a mathematician famous for his contributions to geometric number theory, made a couple of conjectures around the beginning of this century.

The number of supercircles in a given (large) area is the same as the number of hexagons. Since the hexagons cover 3/4 of the area being packed, the smaller the area of the parallelogram *OABC*, the more supercircles we will pack into the area. Minkowski's conjectures are phrased in terms of the area of *OABC*, which can be regarded as a function $f(\theta)$ of the angle θ (in degrees) that *OA* makes with the x-axis.

Because of the symmetries of the supercircle, $f(\theta)$ need only be investigated for $0 \le \theta \le 45$. Minkowski seems to have believed that either $f(\theta)$ increases as θ goes from 0 to 45, or it decreases, and consequently he conjectured that either the area is a minimum at $\theta = 0$ and a maximum at $\theta = 45$, or the area is a maximum at $\theta = 0$ and a minimum at $\theta = 45$. He went further and conjectured that for supercircles with $1 < n < 2$ the minimum is at $\theta = 0$ and that the minimum is at $\theta = 45$ when $n > 2$. (You may observe that when $n = 1$, 2 or ∞ the function $f(\theta)$ is constant.)

Minkowski could hardly have been more wrong. His intuition led him sadly astray. In the first place, for some values of n, $f(\theta)$ is neither increasing nor decreasing, having local minima at both $\theta = 0$ and $\theta = 45$. In addition to the values $n = 1, 2, \infty$ there is another value n_0 (approximately 2·5725) of n for which $f(0) = f(45)$. Furthermore, for $1 < n < 2$, $f(0) > f(45)$, so the minimum cannot be $f(0)$, and for $2 < n < n_0$, $f(0) < f(45)$ so the minimum cannot be $f(45)$.

These days, with the aid of computers, the smaller of $f(0)$, $f(45)$ can be determined for any given value of n. It is known to be $f(45)$ when $1{\cdot}01 \le n \le 1{\cdot}99$ and $2{\cdot}04 \le n \le n_0$, and $f(0)$ when $n \ge n_0$.

However, in the days of Minkowski the calculations required would have been daunting, so he can perhaps be excused for being so mistaken in his conjecture.

* * * * *

A NUMBER PATTERN

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Introduction

What is so special about the number 746 665 920?

If we choose eight of the nine positive digits 1, 2, ..., 9 and form all possible six digit numbers from them, take the sum of all these six digit numbers and divide that sum by the sum of the eight chosen digits, then the result is always 746 665 920 irrespective of the eight digits chosen.

In this paper we wish to explore this pattern further and investigate its general nature.

The Pattern

The example in the introduction is a special case of the following. Choose n of the positive digits ($n = 8$ in the example) and form all possible r-digit numbers ($r = 6$ in the example). What is the result of dividing the sum of the n-digit numbers by the sum of the chosen digits? We will denote this by $K_{r,n}$ (assuming that the result does not depend on the chosen digits).

Consider firstly the simple case, $n = 2$, $r = 2$. We select any two positive digits, say a and b. There are two 2-digit numbers that can be formed from a and b. We add them, then divide the result by the sum of a and b. For example, consider $a = 3$, $b = 2$. The possible two digit numbers are 32 and 23, and we calculate

$$\frac{32+23}{3+2} = \frac{55}{5} = 11.$$

We would like to decide if the value is 11 regardless of the values of a and b we chose. However, we must decide on some notation. If we were to write ab to denote the two digit number with tens digit a and units digit b, that is, the number $10 \times a + b$, we would probably get quite confused with the use of ab to denote the product of a and b. We will therefore use $\lfloor ab \rceil$ to denote $10 \times a + b$. Likewise we will use $\lfloor abc \rceil$ to denote the three digit number $100 \times a + 10 \times b + c$. (Of course, we don't confuse 32 with 3×2, so we will not use the $\lfloor\;\rceil$ notation if we are dealing with numbers). The two 2-digit numbers formed from a and b are $\lfloor ab \rceil$ and $\lfloor ba \rceil$, so we calculate

$$K_{2,2} = \frac{\lfloor ab \rceil + \lfloor ba \rceil}{a+b} = \frac{(10a+b)+(10b+a)}{a+b} = \frac{11(a+b)}{a+b} = 11$$

which, indeed, does not depend on the digits a and b.

In general we wish to consider the case of n chosen digits arranged r at a time, where $r \leq n$. Some or all of the n digits may be repeated, in which case we still consider them as distinct and obtain ${}^{n}P_{r}$ r-digit numbers.

For example, if we have chosen the digits 4, 5, 5 and consider 2-digit numbers, we obtain ${}^{3}P_{2} = 6$ 2-digit numbers by treating the 5's as distinct, and calculate

$$\frac{45+45+54+54+55+55}{4+5+5} = \frac{308}{14} = 22.$$

To verify that this is indeed independent of the digits chosen we form all six possible two-digit numbers $\lfloor ab \rceil$, $\lfloor ac \rceil$, $\lfloor bc \rceil$, $\lfloor ba \rceil$, $\lfloor ca \rceil$, $\lfloor cb \rceil$ of three digits a, b, c

and calculate

$$K_{2,3} = \frac{\lfloor ab \rceil + \lfloor ac \rceil + \lfloor bc \rceil + \lfloor ba \rceil + \lfloor ca \rceil + \lfloor cb \rceil}{a+b+c}$$

$$= \frac{10a+b\ +\ 10a+c\ +\ 10b+c\ +\ 10b+a\ +\ 10c+a\ +\ 10c+b}{a+b+c}$$

$$= \frac{22a\ +\ 22b\ +\ 22c}{a+b+c}$$

$$= 22.$$

Exercises

1. Comment on whether or not allowing 0 to be one of the digits affects the results. Can all the chosen digits be zero?
2. Show that $K_{r,n} = (111...1)(n-1)(n-2)...(n-r+1)$ where there are r 1's in the first number of the right side.
3. We have been using base-10 numbers. What happens if we use, for example, base-5 numbers?

* * * * *

PROBLEMS SECTION

EDITOR: H. LAUSCH

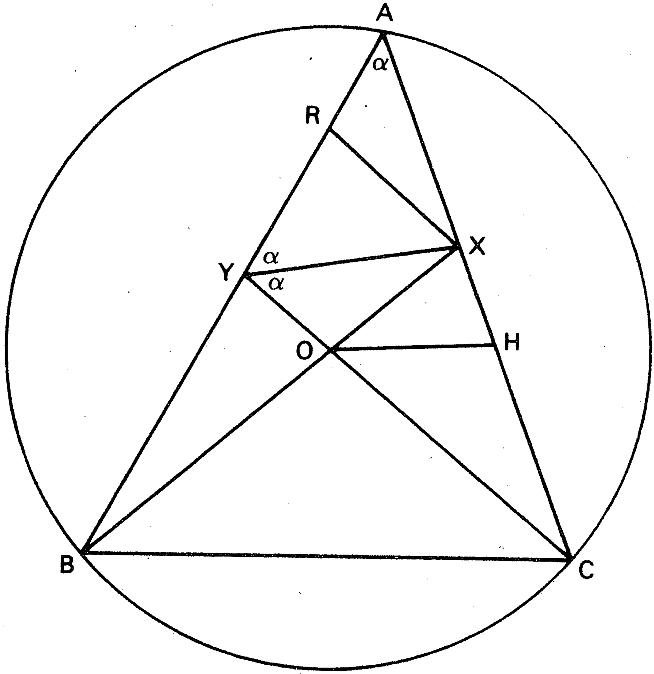
New problems and solutions to old problems have been received from our readers in Clayton, Australia and Ober-Döbling, Austria.

Solutions

Question 4, *Function, Vol.* 13, *Part 3* (June 1989), p.96. Let O be the circumcentre of the triangle ABC, and let X and Y be the points on AC and AB respectively such that BX intersects CY in O. Suppose $\angle BAC = \angle AYX = \angle XYC$; determine the size of this angle.

John Barton, North Carlton, who presented us with a solution to this problem (see Function, Vol. 13, Part 4, August 1989, p.127f.), commented: "There surely must be some relatively simple construction to allow a 'pure euclidean' solution (?). Note that the points B, Y, O, M are concyclic, but how can one show this without solving the problem?" Function editor J. Bruce Henry has an answer ready:

Solution. Let α be this angle. Draw XR parallel to CY, cutting AY at R. Draw OH to bisect angle XOC, cutting XC at H. Let $OB = OC = r$. Join OA. Since triangle AYX is isosceles, $AX = YX$. Then triangles ARX and XYO are congruent (ASA) and $OX = RX$. By the angle bisector theorem, in triangle OXC, $HC \cdot OX = OX \cdot XH$, and in triangle YCA, $XC \cdot AY = AX \cdot YC$. Then $HC \cdot OX = r(XC - HC)$. So $HC \cdot (OX + r) = rXC$, i.e. $HC \cdot BX = r \cdot XC = r \cdot YC \cdot AX / AY$. Now triangles AYC and YOX are similar, so $YC/AY = OX/OY$, and hence $HC \cdot BX = r \cdot OX \cdot AX / OY$. Further, triangles AYC and YOX are similar, so



$r/OY = BO/OY = BX/RX$ and hence $HC = OX \cdot AX / RX$. But $OX = RX$, so $HC = AX$. Now $OA = OC = r$ and angle OAX and OCH are equal (triangle OAC is isosceles), so triangles OAX and OHC are congruent. Thus angle AOX = angle COH. But

$$\text{angle } AOX = 2 \cdot \text{angle } ABO = 2 \cdot (180^\circ - 2(180^\circ - 2\alpha)) = 8\alpha - 360^\circ \quad \text{and}$$

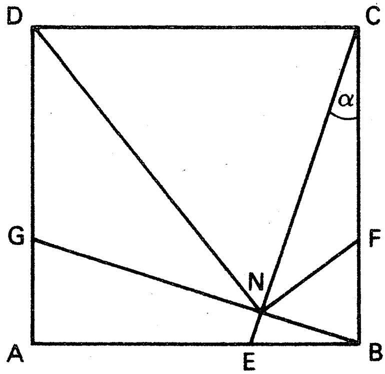
$$\text{angle } COH = (180^\circ - 2\alpha)/2 = 90^\circ - \alpha.$$

Hence
$$90^\circ - \alpha = 8\alpha - 360^\circ \quad \text{and so} \quad \alpha = 50^\circ.$$

And again, here is a new solution to an old problem. It has been sent by our reader Wenzel Julius Schöberl, Ober-Döbling, Austria. Here is Mr Schöberl's solution in its English translation:

Problem 14.1.8 Let $ABCD$ be a square. Choose any point E on AB and then let F be the point on BC which is determined by the condition $BE = BF$. Let N be the foot of the altitude of the right-angled triangle EBC. Show that $\angle DNF$ is a right angle.

Solution. In contrast to the solution given in the last issue, this one uses no trigonometry.



Let G be the intersection of BN with AD. Then $\Delta AGB \cong \Delta BEC$ (as EC and BG are perpendicular, $AB = BC$ and $\angle GAB = \angle EBC = 90°$). It follows that $AG = EB = BF$, whence $DG = CF$.

Next we observe that $\angle CDG + \angle GNC = 90° + 90° = 180°$, so that the quadrilateral $CDGN$ is cyclic. Because of $DG = CF$, the quadrilateral $CDGF$ is a rectangle and hence is also a cyclic quadrilateral. It follows that C, D, G, N and F lie on the same circle. In particular, the quadrilateral $CDNF$ is cyclic. Therefore $\angle DNF = 180° - \angle CDF = 180° - 90° = 90°$.

Problems

a. Some "psychological mathematics" for getting adjusted ...

Problem 14.4.1 (communicated by Louis Nottle, second-year student, Chemical Engineering, Monash University). Find an expression for the product $(x-a)(x-b)(x-c)...(x-z)$ that is simple and without brackets.

b. And a little bit of geometry – not too hard – ...

Problem 14.4.2 The bisectors of the angles C and D of a convex quadrilateral $ABCD$ meet at a point on AB such that $\angle CPD = \angle DAB$. Prove that P is the midpoint of AB.

Problem 14.4.3 ABC is a triangle, right-angled at C. Let CD be perpendicular to AB. The bisector of $\angle CDB$ meets CB in X, and the bisector of $\angle ADC$ meets AC in Y. Prove that $CX = CY$.

Problem 14.4.4 $\angle BAC$ is an obtuse angle. A circle through A cuts AB at P and AC at Q. The bisectors of angles $\angle QPB$ and $\angle PQC$ cut the circle at X and Y respectively. Prove that XY is perpendicular to the bisector of $\angle BAC$.

Problem 14.4.5 $ABCD$ is a square and P is a point on the circumcircle between A and B. The distances of P from A, B, C and D are denoted by a, b, c and d respectively. Show that $(\sqrt{2}+1)(a+b) = d + c$ and that $a - b = (\sqrt{2}+1)(d-c)$.

Problem 14.4.6 Let I be the incentre of triangle ABC, and let $A', B' C'$ be the circumcentres of triangles IBD, ICA, IAB respectively. Prove that the circumcircles of triangles ABC and $A'B'C'$ are concentric.

c. But life is not always meant to be easy ...

Function editor Marta Sved translated the **very hard** *problems of the 1989 Kürschák competition. Solutions are invited. This competition has quite a long history, and its name commemorates József Kürschák (1864-1933), who was born and educated in Hungary. He was professor of mathematics at the Polytechnic University in Budapest, member of the Hungarian Academy and permanent member of the Examination Board for prospective high school teachers of mathematics. The competition, originally known as the Eötvös Contest, has been open to Hungarian students in their last year of high school ever since 1894. Many of its winners turned into scientists of international fame, amongst them L. Fejér (see Function, Vol. 12, Part 1, The Front Page) and E. Tellér (who contributed substantially to the development of the hydrogen bomb).*

THE 1989 KÜRSCHAK COMPETITION

Problem 14.4.7 Two lines, e and f, do not interesect the circle C. Find a construction for the line g, parallel to f, and intersecting C and e at the points A, B, E in order to make the ratio $|AB|/|BE|$ as large as possible.

Problem 14.4.8 For any given positive integer n, denote by $S(n)$ the *sum of the digits* of n (in the decimal system). Determine all positive integers M for which $S(M) = S(kM)$ for all integers K for which $1 \leq k \leq M$.

Problem 14.4.9 A path in the coordinate plane consists of steps parallel to the axes and is subject to the following restrictions:

> From any point P we may step in the positive or negative direction, the *length of the step being equal to twice the* x-*coordinate of* P *if the step is parallel to the* y-*axis, and twice the* y-*coordinate of* P *for a step parallel to the* x-*axis.* No step is allowed to be followed by an immediate reversal.

Show that a path beginning at the point $P_0(1,\sqrt{2})$ cannot reach P_0 in a finite number of steps.

Function readers will appreciate the progress made by this competition with regard to the level of difficulty when they see Problem 1 of the 1894 contest:

Problem 14.4.10 Prove that the expressions $2x + 3y$ and $9x + 5y$ are divisible by 17 for the same set of integral values of x and y.

Marta Sved also draws attention to a (perhaps easier?) problem that turned up as Problem 4 in the 1990 Senior IBM Mathematics Competition:

Problem 14.4.11 It is known that all natural numbers can be written in the binary system, using only 0 and 1 as digits. But if (-2) is used as a *base* instead of 2, can *all* integers (negative or positive) be expressed as a sum of different powers of (-2)?

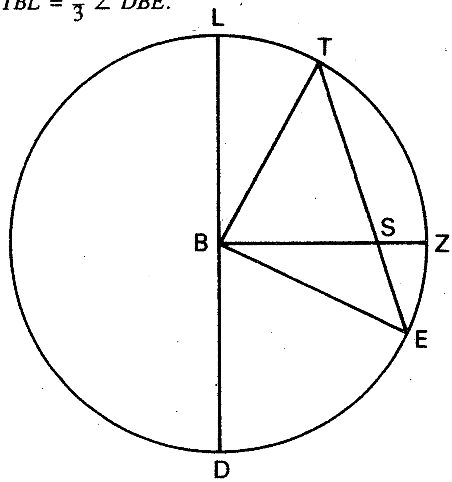
[Examples: $73 = (-2)^6 + (-2)^4 + (-2)^3 + (-2)^0$;
$-55 = (-2)^7 + (-2)^6 + (-2)^4 + (-2)^3 + (-2)^0$.]

Ed.: Keep this problem ready for the next issue, where more questions will be asked about (–2)-ary representations.

d. As history can tell

The following construction can be found in Theorem 20, Book 4 of De Triangulis (On Triangles) by the Dominican monk Jordanus Nemorarius (or Saxo), probably of Borgentreich near Paderborn, Westphalia, in North Western Germany. Jordanus joined the Dominican order in 1220, and he became its leader after its founder and first general Domingo de Guzman (1170-1221) had passed away. Jordanus used letter symbols to express general quantities to a hitherto unheard-of extent: numbers hardly ever occur in his work and if so, then never without being accompanied by letters. What made mathematical life still difficult in those days, despite the Dominican's abstract leap forward, was the almost total lack of symbols that would have enabled mathematicians to manipulate their letter combinations. Jordanus' sole exception is his notation abc *for the result of the addition of the magnitudes represented by these letters; incidentally, his contemporary Leonardo of Pisa, vulgo Fibonacci (for Fi'Bonacci = son of the good one, "good" having an ironic meaning) used the same notation for expressing the* **product** *of these magnitudes. Here is the Jordanian problem:*

Problem 14.4.12 Let C be a circle with centre B. On C choose two points D and E such that DBE is an acute angle. Let DB (prolonged) intersect C also in L. Let Z be the point on C such that DB and BZ are perpendicular, let S be on BZ and let ES (prolonged) intersect C also in T. Suppose that ST and BD have equal length. Prove that $\angle TBL = \frac{1}{3} \angle DBE$.



COMPUTER SECTION

EDITOR: R.T. WORLEY

Can we trust results obtained using a computer?

There are a number of different possible sources of error in results obtained using a computer. The most obvious one is programmer error. A programmer can make either a typing error or a programming error. Depending on the language the program is being written in, some typing errors may be picked up automatically, as the computer recognises the program is incorrect. For example, typing PRIMT A instead of PRINT A will be detected, whereas $A2 = A1 + 1$ in place of $A1 = A1 + 1$ may not be. Programming errors, resulting from the programmer not correctly understanding the problem, or making a mental error, or not understanding the computer language she/he is writing the program in, are not picked up automatically. Programming errors can be a problem in big programs where more than one programmer works on the program; the area of Software Engineering covers methods to reduce errors caused by misunderstandings between programmers. I would include under programmer's errors the case when a programmer does not understand the way numbers like $\sqrt{2}$ are normally stored as approximations accurate to only a certain number of decimal places, and the fact that such errors can accumulate or be magnified, and cause a program to give badly inaccurate answers.

The second error results from the fact that a computer program is translated from the language it is written in to a language understood by the computer. This can be done by an interpreter (as is often the case with BASIC) or a compiler (as is usually the case with PASCAL). An interpreter or compiler is, however, just a large program written by someone else and they could have made programming errors. These can be quite hard to detect, for compilers/interpreters usually go through a lot of testing, and it is normally only unusual code that is not thoroughly tested. My own experience is that there are more of these errors around than one would like, especially with compilers for microcomputers. This is probably because of the need to sell the compiler to generate income to cover the costs incurred while writing the compiler. The effect of these errors is usually obvious. For example, with one error I came across printing the powers of ten gave

```
10^0 = 1·000000
10^1 = 1·000000        (?!)
10^2 = 100·0000
10^3 = 1000·000.
```

Another case gave $4 + 4 = 4$, $2 + 6 = 6$, etc. (due to the compiler writer typing AND instead of ADD), and in another case adding 1 to a very large number (approximately 10^{38}) gave a very small number (approximately 10^{-38}). Other computer errors I have met involve i) a case where in accessing a variable, say A, it gave not the value of A but the location where the value of A was stored in the computer's memory, and ii) attempting to change the value of A changed some other value instead. In the case of this last error, despite my letters two revisions of the compiler have come out still with the same problem – I may be cynical but in some cases it seems more important to upgrade the compiler's "look and feel" to compete with the opposition than to fix genuine bugs. I have even come across a graphing package that seemed to dislike drawing graphs that cross and would change the points plotted to ensure the graphs did not cross.

A third source of bugs can be electrical glitches in the computer itself. This can cause problems for long-running programs, such as the one calculating the first thousand million digits of pi.

What impact does this have on the trust in answers produced by a computer? The third source of error is coped with by running the program a number of times – if the same answer is produced each time, then one can assume that no electrical glitches occurred. To cope with the first problem one can use two different programmers. If their programs produce the same results then one concludes that they made no errors. To cope with the second problem, one should compile the programs using different compilers – in fact it could be preferable to use different programming languages and different computers. There are instances in mathematics, such as the "four colour conjecture" which was proved using a computer program to exhaustively test a large number of possibilities. The possibilities were too many to check by hand, but the proof was not generally accepted until someone else had written another program to do the same work and run it on another machine.

In conclusion, I would encourage you to look very carefully at results obtained by computer to ensure they are sensible, and where possible check them by other means.

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The Factoring of F_9

A team effort has resulted in obtaining the factorisation of the ninth Fermat number $2^{512}-1$, using a factorisation method specialised for numbers of this form. This 154 digit number was known to have the factor 2424833; the remaining prime factors, with 49 and 99 digits respectively, have just been determined. Data for the computation was collected using hundreds of computer workstations around the world, and the final calculations from this data produced the factorisation.

* * * * *

Mnemonics

1. See, I have a rhyme assisting
   My feeble brain its tasks ofttimes resisting.

2. Sir, I send a rhyme excelling
   In sacred truth and rigid spelling
   Numerical sprites elucidate
   For me the lexicon's dull weight.

* * * * *

'PROOF' BY ILLUSTRATION

R.T. Worley

The following question appeared in the electronic news recently.

Q. Are there infinitely many integers n with the property that both n and n^2 have no zeros (in their decimal representation)? For example, $n = 11$ is one such n because 11 and $11^2 = 121$ contain no zeros.

A few days later the following appeared.

$$
\begin{aligned}
4^2 &= 16\\
34^2 &= 1156\\
334^2 &= 111556\\
3334^2 &= 11115556\\
33334^2 &= 1111155556\\
333334^2 &= 111111555556\\
3333334^2 &= 11111115555556\\
33333334^2 &= 1111111155555556\\
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333333333333333334^2 &= 111111111111111111555555555555555556\\
3333333333333333334^2 &= 11111111111111111115555555555555555556\\
33333333333333333334^2 &= 1111111111111111111155555555555555555556
\end{aligned}
$$

Two more days later there was a short note to the effect that this must be the "ultimate in 'proof' by illustration".

Do you believe that the "proof by illustration" is not a proof? If so, can you state and prove a result showing that the pattern illustrated above continues indefinitely?

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