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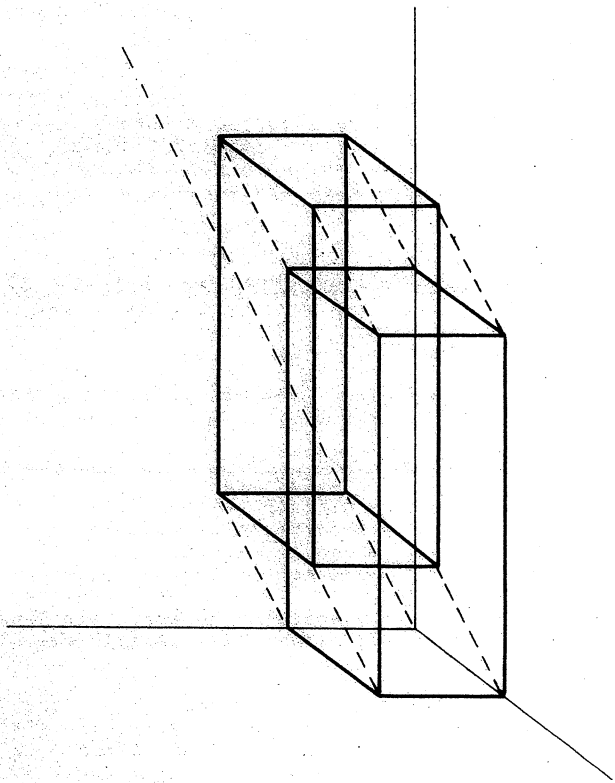

A SCHOOL MATHEMATICS MAGAZINE

Published by Monash University

Function is a mathematics magazine addressed principally to students in the upper forms of schools. Today mathematics is used in most of the sciences, physical, biological and social, in business management, in engineering. There are few human endeavours, from weather prediction to siting of traffic lights, that do not involve mathematics. *Function* contains articles describing some of these uses of mathematics. It also has articles, for entertainment and instruction, about mathematics and its history. Each issue contains problems and solutions are invited.

It is hoped that the student readers of *Function* will contribute material for publication. Articles, ideas, cartoons, comments, criticisms, advice are earnestly sought. Please send to the editors your views about what can be done to make *Function* more interesting for you.

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Articles, correspondence, problems (with or without solutions) and other material for publication are invited. Address them to:

The Editors,
Function,
Department of Mathematics,
Monash University,
Clayton, Victoria, 3168.

Alternatively correspondence may be addressed individually to any of the editors at the mathematics departments of the institutions shown above.

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Some things more impossible than others? Yes, indeed, and to see how this can be, read John Stillwell's account of the tribar, previously discussed in *Function*, *Vol*.7, *Part* 2. That such "impossible" things can become possible is a consequence of the *abstraction* process involved in mathematical thought - mathematics has its roots in the world of experience, but at the same time is able to transcend that world. One spinoff from this is that when we come to apply mathematics we may find ourselves doing this in what at first seem to be strange ways. Professor Sharfuddin gives a very elegant example of exactly this phenomenon.

Function is a journal for school students and, as such, does not normally address itself directly to teachers. We make a minor exception in this issue by printing Professor Tergan's article. This paper first appeared in a German periodical addressed to mathematics teachers. Its beauty and importance induced us to break our usual rule and publish it here.

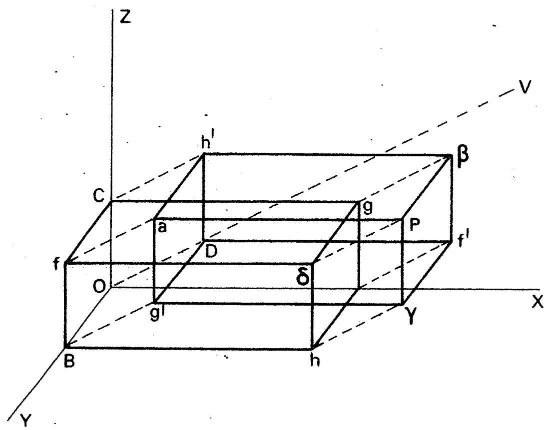
And, of course, there is much more besides.

CONTENTS

THE FRONT COVER
M.A.B. Deakin, Monash University

At first sight, the object drawn looks rather straightforward, but look again. Four axes meet at O, and all the angles are perspective renderings of right angles. The object represented is the four-dimensional analogue of a cube - known technically as a *tesseract*. (See p.25.) Perhaps you will feel as you look at it that you can almost glimpse the fourth dimension for brief instants.

Let the lines drawn all have length a. The sixteen vertices are then connected by 32 linear edges of length a. Each adjacent pair of edges defines a plane face and there are 24 of these in all. Each adjacent triad of edges defines a three-dimensional equivalent of a plane (known as a flat) and there are eight flat regions surrounding the entire tesseract. These eight regions are called cells.



The lines

$$Bh,\ f\delta,\ Cg,\ g'\gamma,\ Df',\ h'\beta,\ \alpha P,$$

are parallel to the axis OX; the lines

$$Cf,\ g\delta,\ Ah,\ h'\alpha,\ Dg',\ f'\gamma,\ \beta P,$$

are parallel to the axis OY; the lines

$$Ag,\ h\delta,\ Bf,\ f'\beta,\ Dh',\ g'\alpha,\ \gamma P,$$

are parallel to the axis OZ; the lines

$$Af', \; Bg', \; Ch', \; f\alpha, \; g\beta, \; h\gamma, \; \delta P,$$

are parallel to the axis OV.

The coördinates of P are (a,a,a,a) and the coördinates of the various angular points in the diagram, other than P, are as follows:

$$\left.\begin{array}{ll} A & \text{is the point } a,0,0,0, \\ B & \text{is the point } 0,a,0,0, \\ C & \text{is the point } 0,0,a,0, \\ D & \text{is the point } 0,0,0,a, \end{array}\right\} ; \quad \left.\begin{array}{ll} \alpha & \text{is the point } 0,a,a,a, \\ \beta & \text{is the point } a,0,a,a, \\ \gamma & \text{is the point } a,a,0,a, \\ \delta & \text{is the point } a,a,a,0, \end{array}\right\} ;$$

and

$$\left.\begin{array}{ll} f & \text{is the point } 0,a,a,0, \\ g & \text{is the point } a,0,a,0, \\ h & \text{is the point } a,a,0,0, \\ f' & \text{is the point } a,0,0,a, \\ g' & \text{is the point } 0,a,0,a, \\ h' & \text{is the point } 0,0,a,a, \end{array}\right\} .$$

Further, the lines

$$\begin{array}{ll}
\left.\begin{array}{l} Of,\; A\delta,\; D\alpha,\; f'P, \text{ are parallel to one another} \\ BC,\; gh,\; g'h',\; \gamma\beta, \ldots\ldots\ldots\ldots \end{array}\right\} & \begin{array}{l}\text{and of length} \\ a\sqrt{2}\;; \end{array} \\
\left.\begin{array}{l} Og,\; B\delta,\; D\beta,\; g'P, \ldots\ldots\ldots\ldots \\ CA,\; gh,\; h'f',\; \alpha\gamma, \ldots\ldots\ldots\ldots \end{array}\right\} & \begin{array}{l}\text{and of length} \\ a\sqrt{2}\;; \end{array} \\
\left.\begin{array}{l} Oh,\; C\delta,\; D\gamma,\; h'P, \ldots\ldots\ldots\ldots \\ AB,\; gf,\; f'g',\; \beta\alpha, \ldots\ldots\ldots\ldots \end{array}\right\} & \begin{array}{l}\text{and of length} \\ a\sqrt{2}\;; \end{array} \\
\left.\begin{array}{l} Of',\; B\gamma,\; C\beta,\; fP, \ldots\ldots\ldots\ldots \\ DA,\; g'h,\; h'g, \alpha\delta, \ldots\ldots\ldots\ldots \end{array}\right\} & \begin{array}{l}\text{and of length} \\ a\sqrt{2}\;; \end{array} \\
\left.\begin{array}{l} Og',\; C\alpha,\; A\gamma,\; gP, \ldots\ldots\ldots\ldots \\ DB,\; h'f,\; f'h,\; \beta\delta, \ldots\ldots\ldots\ldots \end{array}\right\} & \begin{array}{l}\text{and of length} \\ a\sqrt{2}\;; \end{array} \\
\left.\begin{array}{l} Oh',\; A\beta,\; B\alpha,\; hP, \ldots\ldots\ldots\ldots \\ DC,\; f'g,\; g'f,\; \gamma\delta, \ldots\ldots\ldots\ldots \end{array}\right\} & \begin{array}{l}\text{and of length} \\ a\sqrt{2}\;. \end{array}
\end{array}$$

Again, the lines

$$\left.\begin{array}{l} O\alpha \\ AP \end{array}\right\}, \quad \left.\begin{array}{l} Bh' \\ h\beta \end{array}\right\}, \quad \left.\begin{array}{l} Cg' \\ g\gamma \end{array}\right\}, \quad \left.\begin{array}{l} Df \\ f'\delta \end{array}\right\},$$

$$\left.\begin{array}{l} O\beta \\ BP \end{array}\right\}, \quad \left.\begin{array}{l} Cf' \\ f\gamma \end{array}\right\}, \quad \left.\begin{array}{l} Dg \\ g'\delta \end{array}\right\}, \quad \left.\begin{array}{l} Ah' \\ h\alpha \end{array}\right\},$$

$$\left.\begin{array}{l} O\gamma \\ CP \end{array}\right\}, \quad \left.\begin{array}{l} Dh \\ h'\delta \end{array}\right\}, \quad \left.\begin{array}{l} Ag' \\ g\alpha \end{array}\right\}, \quad \left.\begin{array}{l} Bf' \\ f\beta \end{array}\right\},$$

$$\left.\begin{array}{l} O\delta \\ DP \end{array}\right\}, \quad \left.\begin{array}{l} Af \\ f'\alpha \end{array}\right\}, \quad \left.\begin{array}{l} Bg \\ g'\beta \end{array}\right\}, \quad \left.\begin{array}{l} Ch \\ h'\gamma \end{array}\right\},$$

are of length $a\sqrt{3}$, each bracketed pair of lines being parallel.

Finally, the eight lines

$$OP;\; A\alpha,\; B\beta,\; C\gamma,\; D\delta;\; ff',\; gg',\; hh';$$

are of length $2a$; and each of them has $(\frac{1}{2}a,\; \frac{1}{2}a,\; \frac{1}{2}a,\; \frac{1}{2}a)$ for its middle point.

Each plane face has area a^2 and each cell has volume a^3.

The entire tesseract has a hypervolume of a^4.

This account and the diagram are based on those found in A.R. Forsyth's *Geometry of Four Dimensions* (C.U.P., 1930).

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MATHEMATICS ACCEPTS THE IMPOSSIBLE

John Stillwell, Monash University

We are often inclined to think of mathematics as a very factual discipline, with no room for fantasy or paradox. History shows, however, that the fantastic and "impossible" are sometimes useful, and eventually mathematicians find a way to accept them. This was the case, for example, with negative and complex numbers. We still call complex numbers "imaginary", even though we now view them quite concretely as points of the plane, but they were called "impossible" once. In geometry, it was once thought impossible for the angle sum of a triangle to differ from 180°, but such triangles are now accepted in noneuclidean geometry, and even in the real world of astronomy (see for example my article "What is noneuclidean geometry?" in *Function, Vol*.3, *Part* 2). It may be that the only truly impossible objects are ones which are self-contradictory such as triangles with four edges.

The White Queen in *Through the Looking Glass* was able to believe in six impossible things before breakfast. This takes practice, but by the end of this article I hope you will at least be able to believe in the Penrose tribar, shown opposite, which featured in *Function, Vol*.7, *Part* 2.

As a warmup to believing in this difficult three-dimensional object, let us believe in an impossible two-dimensional object first. The object I have chosen is a polygon with two (!) straight edges, and angles of 90° and 270°. Impossible? It becomes believable if you know where to look for it, namely on the surface of a cylinder:

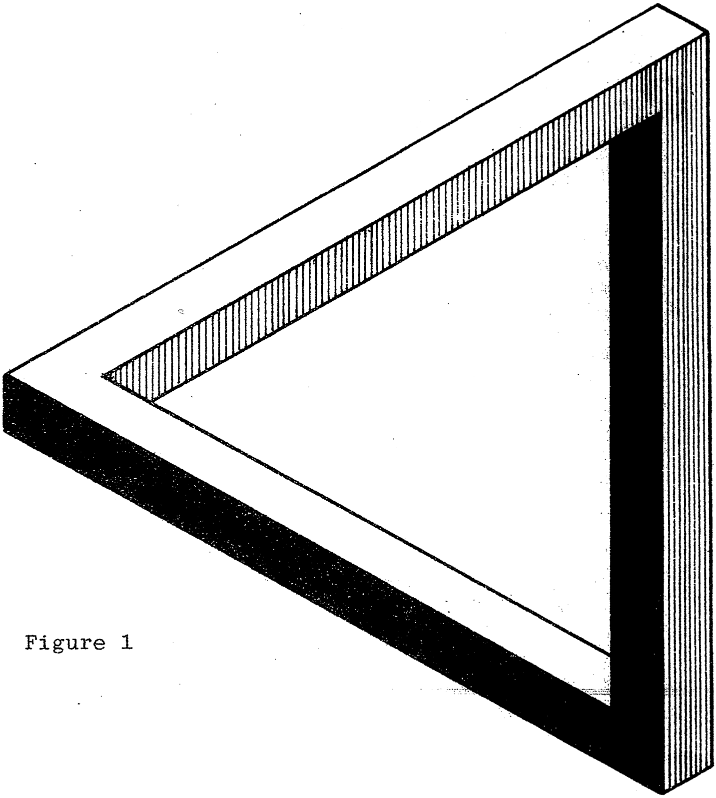

Figure 1

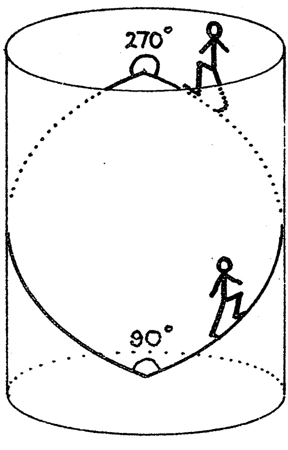

Figure 2

Admittedly, one cannot say which is the "inside" and "outside" of this polygon, but at any rate the two angles are on the *same* side (on which the man is walking). And the edges are as straight as lines can be on a cylinder, since they become ordinary straight lines when the cylinder is rolled flat (they are shortest paths, or *geodesics*, on the cylinder):

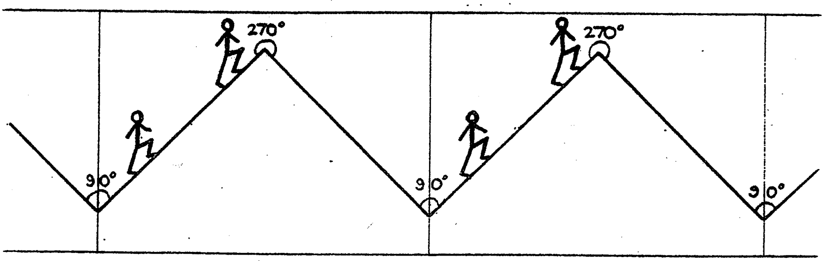

Figure 3

If the cylinder were a self-contained world in which light rays travelled along these "straight" lines then the strip picture above would be what the inhabitants of this world actually see. People would see infinitely many images of themselves, resulting from light rays travelling round the cylinder again and again in "horizontal" circles (i.e. those which are horizontal in Figure 2). It might even be easier for them to view their world as an infinite strip in which everything repeats periodically, rather than attempt to imagine it bending back on itself in a higher dimensional space.

It is certainly true that we cannot easily imagine our three-dimensional space bending back on itself, enabling us to see the backs of our own heads if we look far enough ahead. But we can easily imagine this happening in a periodic universe. Consider the following picture.

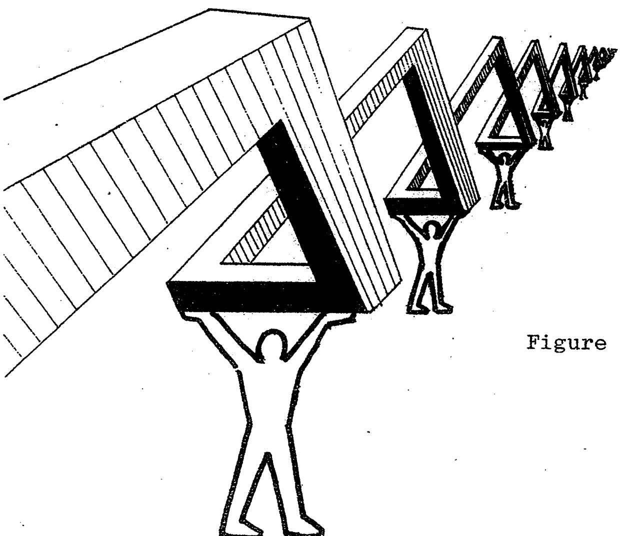

Figure 4

If we accept that the infinitely many men in this picture are really multiple images of the same man, then the pipe he is carrying is in fact finite; and consists of three sections joined together at right angles - precisely a tribar!

Mathematically speaking, the space in which the man lives is called $\mathbb{R}^2 \times \mathbb{S}^1$ or "a plane cross a circle". Points in this space have ordinary (x,y,z) coordinates, with the only difference being that each point has infinitely many different coordinates. If we choose the y axis along the line of images of the man (as shown), and choose units so that the distance from each image to the next is 1, then (x,y,z), $(x,y,z\pm1)$, $(x,y,z\pm2)$,... will all be coordinates of the same point. Conversely, if (x_1,y_1,z_1) and (x_2,y_2,z_2) are coordinates of the same point, then $x_1 = x_2$, $y_1 = y_2$ and $z_1 - z_2$ is an integer. The (x,y) coordinates describe a plane $\mathbb{R}^2$, and the z coordinates describe a circle S^1 (of circumference 1), which is why the space is called $\mathbb{R}^2 \times \mathbb{S}^1$.

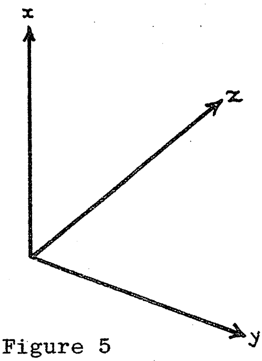

Figure 5

Portions of this space which are "small" (i.e. in which z coordinates differ by less than 1) are exactly like ordinary space. Hence it is possible in principle that *we* live in an $\mathbb{R}^2 \times \mathbb{S}^1$, with an $\mathbb{S}^1$ of very large circumference, and that (very large) tribars are physically possible.

Actually, physicists have proposed something much stronger than this. A theory of Kaluza and Klein in the 1920's has been revamped recently to suggest that space is in fact not ordinary three-dimensional $\mathbb{R}^3$, but $\mathbb{R}^3 \times \mathbb{S}^7$, where $\mathbb{S}^7$ is a 7-dimensional sphere of very tiny radius. This means that what we presently take to be points are really very tiny (below atomic size) 7-dimensional spheres! If this is so, then tiny tribars are also physically possible, because $\mathbb{R}^3$ contains $\mathbb{R}^2$, $\mathbb{S}^7$ contains $\mathbb{S}^1$ and hence $\mathbb{R}^3 \times \mathbb{S}^7$ contains $\mathbb{R}^2 \times \mathbb{S}^1$. To explain what else might exist in this extraordinary world would unfortunately require a lot more practice in believing the impossible than I can give in this short article.

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GEOMETRY AND DIPLOMACY[†]

S.M. Sharfuddin, I.A.S.T.T., Bangladesh[††]

A statesman at an international congress was faced with the delicate task of forming committees to perform various functions and decided that it would be tactful to form these committees in accordance with the following conditions:

(a) Any pair of nations should appear on at least one committee;

(b) Any pair of nations should not appear on more than one committee;

(c) Any two committees should have at least one nation in common;

(d) Every committee should have at least three nations on it.

To the statesman it appeared to be a very complicated task so he went to a mathematician. The mathematician at once pointed out that these are equivalent to:

(1) Any particular combination of two nations will appear in one and only one committee;

(2) Any pair of committees will have one and only one nation in common;

(3) Every committee should have at least three nations represented on it. (This could still leave many combinations of three nations that do not appear on any committee.)

The mathematician recognised that the conditions on nations and committees were precisely like the following statements about

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[†] An excerpt from a talk to a conference on Mathematics and Language (Calcutta, January 1984). We thank Professor Sharfuddin and the conference organisers for their permission to reproduce this material here.

[††] Mailing Address: G.P.O. Box 809, Dhaka, Bangladesh.

points and lines[§]:

(a') Any pair of points appear on exactly one line.

(b') Any pair of lines meet in exactly one point.

(c') Every line contains at least three points.

The only difference between the two sets of statements is that words "point" and "line" take the place of "nation" and "committee", respectively. Thus the absence of well-defined meanings for the undefined terms "point" and "line" proved to be a great advantage.

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MORE ON YAN-A-BUMFIT

Professor Clive Probyn has sent us another version of the Anglo-Cymric score. It comes from H.L. Gee's *Folktales of Yorkshire* (Edinburgh, 1952) and runs

Yain, tain, eddero, peddero, pitts,
Tayter, later, overro, coverro, dix,
Yain-dix, tain-dix, eddero-dix, peddero-dix, bumfit,
Yain-o-bumfit, tain-o-bumfit, eddero-bumfit,
peddero-bumfit, jiggit.

From Tobias Dantzig's *Number - The Language of Science* comes the origin of those mysterious English words *eleven* and *twelve*. They too are base ten, but they go back to a very early Germanic base *lif* which means "ten", but has been replaced in modern times by *zehn*, a close relative of English *ten* and *-teen*. The older words for "eleven" and "twelve" were *ein-lif* and *zwö-lif* and it is clear, particularly in the latter case, how these have given rise to modern usage.

The older root survives also in modern German - the words there are *elf* and *zwölf* (but "thirteen" is *dreizehn*, and the pattern is as in the English - so it was already established at the time of the Saxon invasion of England: C5AD).

Lif derives from the root **lekwos* meaning "left over" and we see it in words like *reliquary* and the word *left* itself, so *ein-lif* originally meant "one left (beyond ten)", etc.

See also p.25.

[§] There are geometries (known as *projective geometries*) in which these axioms hold true. For one example, see Professor Preston's article on "The Seven Point Geometry" in *Function, Vol.4, Part* 4. Axioms (a'), (b') are the basic axioms of *plane projective geometry*. Eds.

GUIDO GRANDI — THE ROSARIAN'S MATHEMATICIAN[†]

Malcolm J. Cameron, Burwood, Victoria

To Guido Grandi, priest and mathematics professor at Pisa in 1723, his newly created 'rosaces' curves had a mathematical beauty equal to that of nature's rose.

This is the story of this man and his graceful curves. It is written for rosarians - but one can hardly divorce a mathematician from his mathematics, can one? A story of a hybridist such as Francis Meilland[††] must include some technical information. A story of a painter such as Pierre-Joseph Redouté[§] must include his painting. And so for Guido Grandi, the rosarian's mathematician, some high mathematical terms must be written in passing. However, for the rosarian one hopes that this does not spoil the unusual history of this man.

The 'Rosaces' Curve.

The 'rosaces' are a family of mathematical curves resembling flowers - the multipetalled 'Roses of Grandi'. Our diagram shows the rosaces with five and eight petals but varying the number n in the formula will give curves with almost any number of petals.

[†] This article first appeared in *The Australian Rose Annual*, 1982 and is reproduced here with their kind permission and that of Dr Cameron. The topic is also discussed in Dr Cameron's book *Heritage Mathematics* (Hargreen Publ. 1983) and in *Function*, *Vol.1, Part 2*.

[††] 'For Love of a Rose' by Antonia Ridge.

[§] 'The Man Who Painted Roses' by Antonia Ridge.

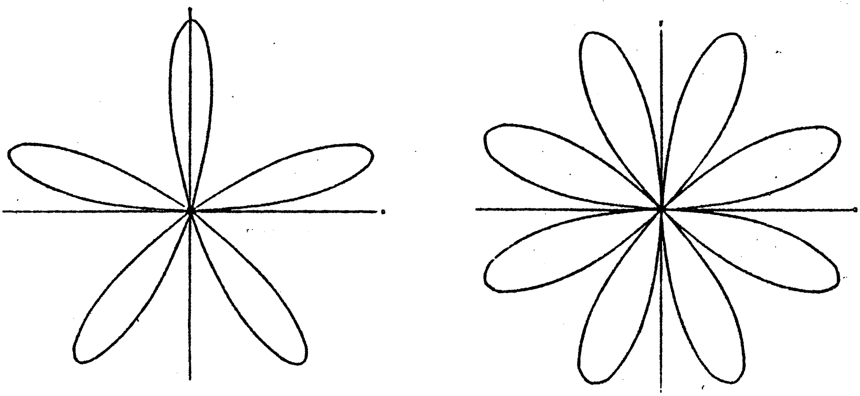

Figure 1. The rosaces curves created by Guido Grandi with 5 and 8 petals. The curves have the form: $r = R\sin n\theta$ or $x = R\sin n\theta\cos\theta$, $y = R\sin n\theta\sin\theta$. The curve has $2n$ petals when n is an even number and n petals when n is an odd number. As θ takes values from 0° to 360° the point (x,y) inscribes the curve.

To the modern rosarian the resemblance may be more to a daisy or a buttercup. This is not Guido's fault but due to the enthusiasm of modern rose hybridists in giving us tight multi-petal roses.

Obviously Professor Grandi had early eighteenth century roses in mind, perhaps the common Field Rose (Rosa Arvensis - Figure 2). This is an old Gallica rose similar to the five petal vigorous Dog-Rose often used as a stock for modern hybrids.

That is not to say that some old European roses did not have plenty of petals, outnumbering our Olympic Torch with 40-45 petals or Folklore with 35-40. The white Rose of York had 50-60 petals while the Old Cabbage Rose was not called the 'Hundred Petalled Rose' without reason. Its mutation the Moss Rose has 100-120 petals, but did not appear until 1750.

Compared with the eighteenth century roses, the modern rose is one of composite beauty - brilliant colour, perfect spiral form, longer petals, more pointed formation and perfume.

But to get back to Professor Grandi.

Figure 2. The common Field Rose (Rosa Arvensis) a five petal old Gallica rose with myrrh-scented flowers.

Guido Grandi - Mathematician and Philosopher

Guido Grandi was born in humble circumstances at Cremona, Italy in 1671. He died at Pisa in 1742 where he is commemorated by a bust and plaque at the Church of St Michael at Borgo.

As a youth he studied at the Jesuit School in his own city and entered the Camaldolite Order where he distinguished himself in philosophy, theology, and the study of the history of his own order. In 1693 he went to Rome to continue his studies at the Camaldolite Monastery of St Gregory.

In Rome he realised that a knowledge of mathematics was necessary to teach philosophy well. After reading the classical works he continued by studying his contemporaries such as Torricelli, Huygens, the Bernouillis, Leibniz and Newton.

A few years later he returned to Pisa where, in 1700, he was appointed Professor of Philosophy at the University of Pisa. Concurrently he held more practical appointments as Mathematician and Theologian of the Grand-Duke, Director of the Waterboard for the Grand-Dukes of Tuscany, and Pontifical Mathematician for the questions of the waters of the Romagne. What these duties involved is by now anyone's guess.

The Times of Guido Grandi

Today, as rosarians we live in the golden age in the development of the rose. Likewise Guido lived in a golden age in the development of analytic geometry, calculus and astronomy.

This was after Newton's classical work in physics, astronomy and calculus, at the time when Newton was engaged in a bitter debate with Leibniz, of Germany, concerning the originality and superiority of their differing forms of calculus. In the calm of Italy, Grandi was able to teach the merits of both systems, in much the same form as is done today.

Closer to home was the great debate, sparked by Galileo, that the sun was the centre of the solar system. By defending Galileo, Grandi himself was enveloped in bitter controversy, to the extent of being accused of inventing documents. Be assured, however, the only problem was Grandi's love for the truth.

This was an age when an individual could cover the whole field of knowledge. Grandi wrote, for example on astronomical problems, geometry, gravity, light, hydraulics and the nature of infinity. He experimented with a vapour machine, wrote prose and poetry for the 'Journal of Literary Man', published in Venice, and was an active member of the Italian and foreign academies.

Analytic Geometry

For all this, it is only for the rosaces curves - in the field of analytic geometry - that Grandi will be remembered. A few words of explanation and history.

A fundamental mathematical advance was achieved when René Descartes (1596-1650) combined the geometry of antiquity with the newly developed algebra. He saw that a point could be determined by x and y, its distances from two lines drawn at right angles. The equation of a curve could then be written as a function of x and y. And, from this, all its properties could be calculated.

This set the scene for the application of algebra to geometry as the equations of all sorts of curves were formulated - straight lines, circles, parabolas, hyperbolas, etc.

It was toward the end of this period that Guido created his curve. He was not alone. Many mathematicians of the time achieved some fame with curves of their creation.

Blaise Pascal (1623-1662) challenged the mathematicians of the world to reproduce the results he calculated for his cycloid - the curve traced out by a point on the circumference of a rolling hoop. Such public challenges were a feature of this age of science.

James Bernouilli (1654-1705) was so pleased with the properties of his 'spiral' curve that he begged that an equiangular spiral should be engraved on his tombstone.

Maria Agnesi (1718-1799), an associate of Grandi, created a curve the shape of a witch's hat, now called the 'Witch of Agnesi' or the 'Versiera', as she preferred. Her story is worth telling. Firstly her abilities allowed her to fill her father's professorial chair at the University of Bologna during her father's illness. Secondly Maria Agnesi was a somnambulist. Several times it happened to her that she went to her study while in the somnambulist state, made a light, and solved some problem she had left incomplete when awake. In the morning she was surprised to find the solution carefully worked out on paper. Oh, that mathematics would come so easy to us!

Again I have drifted from the story of Guido Grandi!

Amongst all these curves, the 'rosace' is the most graceful. It was published in 1723 in the Philosophical Transactions of the Royal Society of London under the title of 'Florum Geometricorum'. In 1728 Grandi extended the idea to his 'Clelia' curve - an analogous three-dimensional curve drawn on a sphere!

The writer Montucla records that our priest Grandi was so pleased with the result that he ceremoniously offered 'the noble woman Countess Clelia Borromeo of Grillo an elegant bunch of "flores geometrici" - geometrical flowers'.

Truth and Beauty

What manner of man was Guido Grandi, this mathematician of the rosaces? Did naming the curve after the rose reflect his own love of roses and beauty?

The old manuscripts give us no clue. On the other hand, since he chose the rose rather than the daisy or the buttercup, we may safely include him in the rosarian ranks.

A love of beauty he certainly had. Grandi saw beauty and mysticism in a mathematical function guiding a point along a graceful curve. There is a beauty in the world when a hybridist creates a new rose. As with a poet, a novelist, or a musician linking notes together. Equally this applies in the creation of a new mathematical curve.

A rosarian's temperament he also had. He never avoided discussion or controversy, yet he still won wide respect. As president of the Royal Society of London, Newton invited him to become a member. A clue to his wellbeing in the church after his defence of Galileo is obtained in the comment that 'his critics did now know how to compete with him, because he expressed himself so powerfully and knew how to choose the opportune moment to expose fallacious arguments...'.

All the pontiffs of his time held him in the highest esteem. Saint Benedetto XIV, in a handwritten letter dated 1 July 1741 wrote: "He reflects the affection we have always had for him, the high esteem we have always had for his value ... the appreciation his name has received ... so great is our delight in receiving his letter and evidence of his good health, so important is this literary man ...".

Guido himself wrote in May 1736 'God preserve us no more than is necessary to preserve the honour of our country...' and prayed that God would keep him busy in his journeys and research of documents of interest to him.

'He Lives on the Mountain Tops'

Guido Grandi is one among mathematicians, hybridists, poets, musicians and writers whose endless time and patience in the search for beauty was rewarded by his curves.

Such curves have a permanent place in mathematics, usually in the early university years in the study of functions expressed in this form.

Yet time is against us in all things. When we plant a new rose, do we know or remember the hybridist who created the variety? Likewise today's students are unlikely to hear of Guido Grandi. Even the old history books spare only a few lines.

Nevertheless, Newton once explained that 'if he had seen further than other men, it was only because he stood on the shoulders of giants'. Among those giants was Professor Guido Grandi - the rosarians' mathematician.

Acknowledgements

The bulk of information for this article originated from the paper 'Guido Grandi' by Luigi Tenca of Florence (*Physics Vol.* 2, 1960, pp.84-89) obtained from the University of Adelaide Library and translated by Mr Raphaël Papandrea of Dee Why, N.S.W. Additional comment was obtained from mathematical histories published in 1893, 1919 and 1966.

Figure 2 is reproduced from 'The Rose Garden' pamphlet of the Royal Botanic Gardens, Melbourne.

THE GENERAL TRIANGLE†

Bernhard Tergan, Wandin Valley
Institute of Educational Innovation 1

Summary: The concept of the general triangle is introduced, the general triangle and its most important properties are described. This concept is a valuable aid for the teaching of geometry.

According to Lockhead's Principle of Cognitive Specialisation[2], the student tends to attain abstract concepts from concrete examples and to construct them from special cases as required. Charlton[3] indicates that the educational aim, namely the confidence with which the student can reproduce the general rule, depends essentially on the kind of cognified example. Investigations involving primary school teachers have shown that formal rules are often better retained than their underlying assumptions; Pythagoras' Theorem[4] and the theorem on the sum of the angles in a triangle[5] are more readily reproduced than the fact that the first theorem holds only for right-angled triangles, the latter for all.

In view of this, a study of good examples appears to be urgently needed, and this paper is intended as a first step in this direction. It deals with the situation in which good examples are especially hard to find, but nonetheless most important: the theory of triangles in elementary geometry (c.f. the example given above!). Everyone who has ever learned elementary geometry will know of the following dilemma - to demonstrate a fact using an acute-angled triangle, we draw one on the board only to discover that the diagram illustrates either a right-angled or an isosceles triangle. But this is just what we wanted to avoid so that the student does not decide on too narrow a Specialisation in the sense of Lockhead's Principle.

---

†This article is a translation of one which first appeared in the German periodical *Journal für Mathematik Didaktik, Vol*.1 (1980), pp.102-107 and which has been widely anthologised and has given rise also to several sequels. We thank Dr H. Lausch for the translation and the adaptation of this article to Australian conditions and Professor Tergan for permission to publish this material in *Function*. The numbered notes are to be found at the end of the article, on p.20.

The question as to whether it is at all possible to draw a triangle which is neither right-angled nor isosceles seems not to have been studied - probably because it seems absurd to the mathematician. Of course it is easy to speak of such triangles. However, classroom experience shows that this answer is absolutely Laputan. A triangle with angles 89°, 45°, 46° is no better for classroom demonstration than an isosceles right-angled triangle. It is irrelevant to the Principle of Cognitive Specialisation whether the triangle on the board is actually right-angled or isosceles or not. Rather, what is essential to the Principle is whether or not it is *perceived* as being right-angled or isosceles. It is here that we discover the missing key to the construction of good examples. Hence, we will call an acute-angled triangle a *general triangle* if it is seen as neither right-angled nor isosceles by a sufficiently large percentage of the students.

First it must be determined when two angles are distinguished by the student. We thank Dr Wimsey, headmaster of Black Stump High School, Wandin Valley, who has taken up this suggestion. (His results are very shortly to be published.) One of his SDA teachers, in a field experiment with Year 11, stream d, students at the aforementioned high school, has studied the ability to distinguish plane angles.

To this end, the students were presented with pairs of angles and were asked to decide immediately whether they were equal or different. The results are summarised in the following table.

Difference of angles in degrees	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
No. of students answering "equal"	18	18	17	16	14	12	10	9	7	5	4	3	2	2	1

If we graph these results (Figure 1) we see that the values are normally distributed, and we can calculate the standard deviation[6], which is found to be

$$\sigma = 5\cdot 77 .$$

Statistical wisdom has it that fewer than 1% of the population lie further than $2\cdot 6\sigma$ from the mean. We may thus proceed on the assumption that at least 99% of all students (and this we may very well take to be a sufficiently large proportion!) distinguish two angles if they differ by 15° or more. The figure of 99% may appear to be chosen arbitrarily, but we shall see later that for completely different reasons that a higher level of significance cannot be attained.

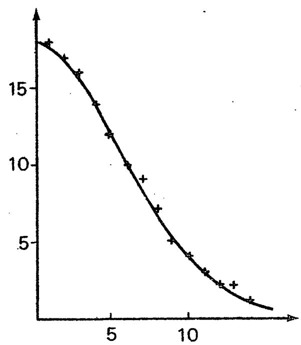

Figure 1

We thus know more precisely now how we have to define the general triangle: an acute-angled triangle is general if its angles all differ by at least 15° from a right-angle and if also each pair of its angles differ by at least 15°.

At this point the author experienced a (he cannot conceal it, pleasant) surprise. For if we consider the possible forms of the general triangle in more detail, we get:

> The largest angle in the general triangle is at least 15° smaller than a right angle, and so let its value be
>
> $$\alpha = 75° - \delta \quad \text{for some} \quad \delta \geqslant 0 .$$
>
> The second-largest angle, β say, is at least 15° smaller still and thus we obtain
>
> $$\beta = 60° - \delta - \zeta \quad \text{for some} \quad \zeta \geqslant 0 .$$
>
> Finally, by the same argument, the smallest angle has the value
>
> $$\gamma = 45° - \delta - \zeta - \xi \quad \text{for some} \quad \xi \geqslant 0 .$$
>
> Calculating the sum of the angles, we find

$$\begin{aligned} 180° &= \alpha + \beta + \gamma = 75° - \delta + 60° - \delta - \zeta + 45° - \delta - \zeta - \xi \\ &= 180° - 3\delta - 2\zeta - \xi , \end{aligned}$$

from which it follows immediately that† $\delta = \zeta = \xi = 0°$.

We can thus state the

MAIN THEOREM: There exists (up to similarity) one and only one general triangle (Figure 2), whose angles are 45°, 60°, 75°.

Indeed a beautiful and a happy result! Now it is clear what we must do when we want to draw a general triangle on the board. Furthermore the general triangle has several very nice properties. The reader may try to find some for himself. One is that the altitude measured from the longest side of the general triangle cuts it into a right-angled isosceles triangle and a triangle with the convenient angles 30°, 60° and 90°. We use this property below.

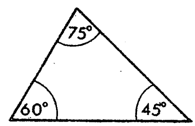

Figure 2

The reader may well now ask how he should actually go about drawing the general triangle on the board. Of course the general triangle is, after all, special in its own way, but according to the Lockhead Principle the student must never take the general

---

†Which is why no greater level of discrimination is possible. Eds.

triangle for a special triangle. A mechanical construction of the general triangle, for example with a protractor, must therefore be avoided, as it would turn the intended effect into its exact opposite. We can state as a simple rule:

SIMPLE RULE: The general triangle is general only if it is drawn surreptitiously.

The distinguished staff of Study Group IV in Wandin Valley were kind enough to trial the classroom use of the general triangle. Only this much of their results (to be published in the near future) can be revealed here:

(1) Freehand drawing of the general triangle is hazardous, even after practice (only a few staff members were able to draw the general triangle without practice).

(2) Surreptitious marking of the vertices on the board leads to good results. In the case of wooden boards it is enough to hammer in, and then remove, a thin nail. With glass boards, the same effect can be achieved with a good electric drill (due to the risk of breakage, it is best to get the janitor to do this). This method has several disadvantages: first, there is the time and trouble involved; and secondly, in some cases the marks were noticed by the students, which led them to respond by inserting further marks.

It has even been suggested that blackboards be equipped by the manufacturer with surreptitious traces of the general triangle.

(3) In most cases such preparations are superfluous, however, if a board with a square grid is available. In this case, it is easy to draw a sufficiently good approximation to the general triangle. We utilise the fact that $4^2 + 7^2 = 65$, which is approximately 8^2. So it is easily seen that a good approximation to the general triangle can be obtained as shown in Figure 3.

Begin by drawing a baseline 11 units long and then erect a perpendicular from a point dividing the baseline in the ratio 4:7. The perpendicular is required to be 7 units long, so that the triangle has the vertices (0,0), (11,0) and (4,7). We can see immediately (and quickly calculate) that this triangle is almost identical with the general triangle.

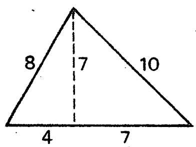

Figure 3

Experiments have shown that it is possible without much practice to draw the general triangle by following this recipe, and without attracting the attention of the students. This approximation to the general triangle has the further advantages that the remaining sides have lengths which are very nearly integral. Besides 11, we have the values of 10 and 8 units. The area of about 38·5 square units, however, gives the student a credibly general impression.

The author believes that the general triangle should find a place in the repertoire of all geometers. He would appreciate feedback from classroom trialling and also suggestions on further properties of this mathematical object.

Envoi:

I am grateful to Professor Profitt for his remark that in this paper the general triangle has merely been rediscovered. Previously that great teacher G. Pólya had pointed out the G.T. and its educational significance. The author feels honoured to have rethought Pólya's thoughts. The importance of this topic has already been revealed in the very lively discussion which evolved from the first rumors of this present paper. Rarely have I received so many responses to a publication as I have to this; some were complementary and some were complimentary, others were nugatory and superficial. Indeed, since the first publication, the topic has been addressed in several sequels and has been considerably developed.

Notes

1. The author is the director of the Wandin Valley Institute of Educational Innovation (a branch of the Brammer Institut für pädagogische Innovation, affiliated with the Technische Hochschule, Darmstadt) and is the author of *The Eternal Triangle: A Primer of Modern School Mathematics*.

2. B. Lockhead: *Education Education*, Mills & Boon, 1970.

3. E. Charlton: *Foul Stroke, Four Away - a Practical Introduction to Plane Geometry*.

4. Pythagoras: On harmonic triangles and triangular harmonies when two sides are perpendicular, In *Ancient Contributions to Mathematics* ed. Julius Wintner Miller. Parrot Books, Melbourne.

5. Euclid: On the sum of the angles in a triangle. *Ibid*.

6. Our special thanks to Professor Uncertanian of the Statistics cadre of the Mathematics co-operative, Erivan, for his friendly assistance in statistical matters.

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By and by comes Mr Cooper ... of whom I entend to learn Mathematiques; and so begin with him today ... . After an hour's being with him at Arithmetique, my first attempt being to learn the Multiplicacion table, then we parted till tomorrow.

- Samuel Pepys, Diary entry for 4 July 1662.

Remark: Pepys was at that time something like a modern Secretary of the Navy. From *American Mathematical Monthly*, January, 1984.

SELF NUMBERS AND JUNCTION NUMBERS†

D.R. Kaprekar, 311 Devlali Camp, India

If a number N has the sum of all its digits equal to d then $N + d = K$ will be called a generated number from N. K is the generated number and N is the generator. Thus take the number 75. It has $7 + 5 = 12$. $75 + 12 = 87$. 87 is the generated number and 75 is the generator. Now after 87 we can go further in the same way. 87 has $8 + 7 = 15$. $87 + 15 = 102$. 102 is the generated number from 87; further, 102 has $1 + 0 + 2$ and $102 + 3 = 105$. 105 is generated number; its generator is 102. Now 75, 87, 102, 105 and so on form a series†† and it is called a digitadition series. The series will go on increasing for ever. After 105, the next is $105 + 6 = 111$. Then $111 + 3 = 114$, $114 + 6 = 120$, and so on. 75, 87, 102, 105, 111, 114, 120 ... is a digitadition series. Now go in reverse order. What is the generator of 105? It is 102, and for 102 the generator is 87, and for 87 the generator is 75, but what is the generator of 75? Can it be 71? No, because $71 + 1 + 7 = 79$. Can it be 69? No, because $69 + 6 + 9 = 84$. Can it be 65? No, because $65 + 5 + 6 = 76$. Thus, you will find that 75 is such that it has no generator. 75 is called a self number or self born number, similarly you will find that 53 is also a self number. From 53 we can go further, $53 + 5 + 3 = 61$ and so on, then, similarly 42 is also a self number; it has no generator.

1, 3, 5, 7, 9, 20, 31, 42, 53, 64, 75, 86 and 97 are the self numbers between 1 and 100.

Every number either has a generator or, if it has no generator, it is a self number.

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†An edited excerpt from the author's "Meaning of Self Number" supplied by him to *Function*. For more on this topic see Martin Gardner's column in *Scientific American*, March 1975.

††We would call this a *sequence*.

E.g. What is the generator of 35? It is 31, but 31 has no generator; it is a self number. 1 000 000 is a self number.

See further: Some numbers have two generators. From 100 we get $100 + 0 + 1 = 101$. 101 is the generated number but also from 91, we get $91 + 9 + 1 = 101$. 101 is the generated number. Thus 101 has two generators. So 101 is called a junction number and is represented as

$$\begin{tikzcd}[row sep=small] & 91 \\ 101 \arrow[ur, dash] \arrow[dr, dash] & \\ & 100 \end{tikzcd}\ .$$

Similarly 713 is a junction number; the two generators are 694 and 703 and this is represented as

$$\begin{tikzcd}[row sep=small] & 694 \\ 713 \arrow[ur, dash] \arrow[dr, dash] & \\ & 703 \end{tikzcd}\ .$$

1015 is also a junction number; the two generators are 989 and 1007, represented as

$$\begin{tikzcd}[row sep=small] & 989 \\ 1015 \arrow[ur, dash] \arrow[dr, dash] & \\ & 1007 \end{tikzcd}\ .$$

The first few junction numbers are 101, 103, 105, 107, 109, 111, ... I have found many more junction numbers.

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AN ELEGANT ARGUMENT

Problem: Can it be that there are two *irrational* numbers a, b, such that a^b is *rational*?

Solution: Consider $\sqrt{2}^{\sqrt{2}}$. Either

(a) this is rational

or (b) it is irrational.

(a) If it is rational, the answer is *yes*. (b) If not, choose $a = \sqrt{2}^{\sqrt{2}}$ as this number will be irrational. In this case, put $b = \sqrt{2}$. Then

$$a^b = (\sqrt{2}^{\sqrt{2}})^{\sqrt{2}} = \sqrt{2}^{\sqrt{2}\times\sqrt{2}} = \sqrt{2}^2 = 2\ ,$$

which is rational and again the answer is *yes*.

Either way we have our answer, but we did not need to know whether $\sqrt{2}^{\sqrt{2}}$ was irrational.

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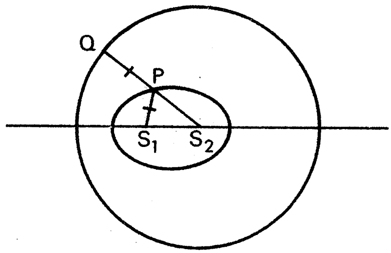
A PROPERTY OF CONIC SECTIONS

Garnet J. Greenbury,
Taringa, Queensland

Besides the usual "focus-directrix" definition of the conic sections (see John Mack's articles in *Function, Vol.7, Parts* 3 and 4), many others are possible. I give one here as an example.

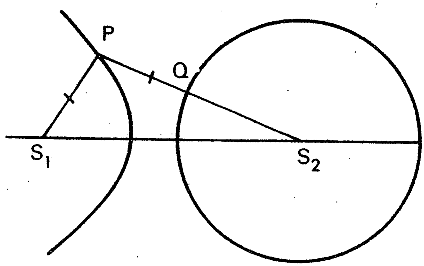


We may define a conic section as the locus of a point which remains equidistant from a fixed point and a fixed circle.

(a) If the fixed point S_1 is within the circle centre S_2,

$$S_1P = PQ ,$$
$$S_2P + PQ = \text{constant radius},$$
$$PS_1 + PS_2 = \text{constant}.$$

Therefore the locus of P is an ellipse.



(b) If the fixed point S_1 is outside the circle centre S_2,

$$S_1P = PQ = PS_2 - S_2Q ,$$

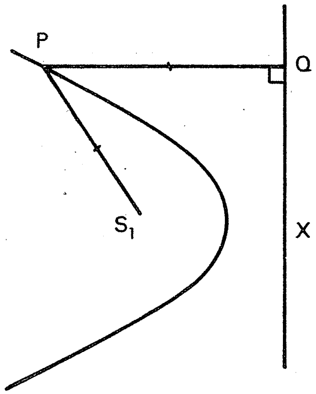
so that $PS_2 - PS_1 = S_2Q =$ constant radius.

Therefore the locus of P is a hyperbola.

(c) If the circle is of infinite radius it becomes a straight line QX .

S_1 the fixed point is on one side of this line, and $S_1P = PQ$ (perpendicular to QX). Therefore the locus of P is a parabola.

You may care to explore for yourself the effect on the shape of the conic of the distance between the point S_1 and the circle.



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THE REASON FOR *FUNCTION*

Go to a mall. Go to a bookstore —— a Waldenbooks, say. Go to the science section. What will you find? Four three-foot shelves: astronomy gets one shelf; weather, physics, and general science gets one shelf; biology gets one shelf; miscellaneous science gets one shelf. John McPhee is there, with *The Curve of Binding Energy*. Lewis Thomas is there. Isaac Asimov is there. *The Soul of a New Machine* is there. Is there any mathematics? One book and one only: *Hofstadter's Gödel, Escher, Bach ...* .

Go to a public library. Go to the science section and see that the books on mathematics take up about one-twentieth of the total space for science. And what books they are! Three works on the slide rule, *Arithmetic for the Practical Man, Introduction to Plane Trigonometry*. Old books, undisturbed for years, the dust thick on them. I suspect that most of them were gifts to the library from people discarding books no longer of any use. Why would anyone want to keep a mathematics book?

Go to *Books in Print*, the 1982-83 edition. How many of the tens of thousands of books listed are classified under "Mathematics - Popular Works"? One hundred, would you think? Fifty? The number is eight. Eight only, and 37.5% of those were published in the Union of Soviet Socialist Republics. One of the remaining five is *Technical Shop Mechanics*, popular only in a restricted group. You can maintain, probably rightly, that the classifiers of the J.J. Bowker Company are not expert in classifying mathematics, but the fact that there are so few titles classified under Popular Mathematics shows that such works are rare and strange.

From a recent review in *American Mathematical Monthly*.

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LETTERS TO THE EDITOR

A NUMBER'S FAMILY TREE

The suggestion of a 1-shift in the meaning of the word *sethera* (*Function, Vol*.8, *Part* 1, p.18) is intriguing but inconclusive. Generally speaking, such shifts do not occur. Consider the following case.

The primitive Indo-European root for *four* is believed to have been **kwetwōres*. (The asterisk denotes a reconstructed form.) This appears in Sanskrit as *catvāras*, in Old Slav as *četyre* and in Latin as *quattuor* (whence the Spanish *cuatro*, the French *quatre*, the Italian *quattro*, etc.). In Irish the word is *ceathair*, and (as noted in the article), the initial sound turns to a *P* in the Welsh, to give *pedwar*.

Similar *Q* - *P* shifts took place elsewhere. In Boeotian, the word is *pettares* and in Gothic *fidwor* (the *P* being softened to an *F*). This last has given rise to the German *vier* and the English *four* (via, in the latter case, the Old English *fēower*).

In Greek, the initial sound was modified, not to *P*, but to *T*, to give the Ancient Greek *tèttares* (from which we get our words "tetrahedron", "tetralogy" and "tetrarch") and Modern Greek *tèssereìs* (from which we get the word "tesseract", meaning the four-dimensional analogue of the cube).

This might lead us to think of the West Cumbrian *tethera* as meaning "four", whereas it means "three" and is quite likely linguistically related to the word "three", so that no 1-shift need be invoked.

Finally, it should be noted that whereas all the Welsh, Cornish and Breton words given in the article relate to other Indo-European number terms, the West Cumbrian words *lethera* and *dovera* are difficult to relate to any other words for "seven" and "nine".

[*The above "letter" was put together from various sources - most of them notes from Dr H. Lausch of Monash University. Eds.*]

SQUARE ROOTS AND RIGHT TRIANGLES

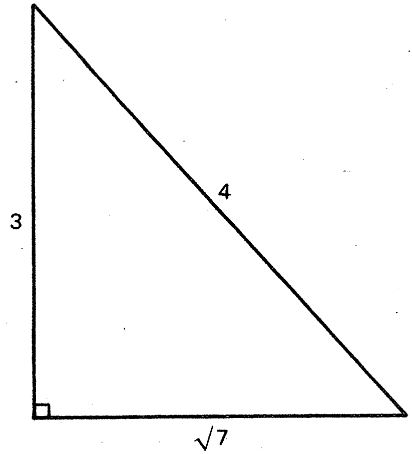
Take a guess at the square root of n. Let the guess be $\sqrt{n} - e$, and let $(\sqrt{n} - e)^2 = n - D$, where D, the discrepancy, can be obtained by squaring the guess and subtracting the square of the guess from n.

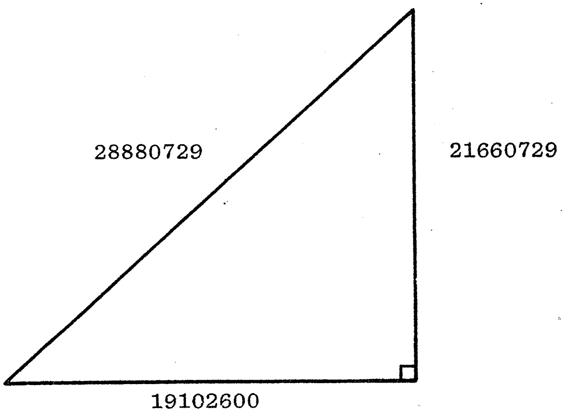
$$\begin{aligned} \text{Now} \quad (\sqrt{n} - e)^2 &= n - 2e\sqrt{n} + e^2 \\ &\cong n - 2e\sqrt{n} \\ &\cong n - D\,. \end{aligned}$$

$\therefore\ 2e\sqrt{n} \cong D$ and $e \cong \dfrac{D}{2\sqrt{n}} \cong \dfrac{D\sqrt{n}}{2n}$. Let $n = 2$. From the diagram $3\sqrt{2} > 4$ so $\sqrt{2} > \dfrac{4}{3}$.
Let $\sqrt{2} - \quad = 1\cdot4$. Then $D = 0\cdot04$ being $(2 - 1\cdot96)$.

$\therefore\ e \cong \dfrac{0\cdot04 \times 1\cdot4}{2 \times 2} = 0\cdot01 \times 1\cdot4 = 0\cdot014$.

$\therefore\ \sqrt{2} \cong 1\cdot414$.





In the above right-angled triangle every side is a whole number, and the quotient obtained by dividing three times the shortest side (the base) by the second shortest side (the perpendicular height) after being rounded off to six decimal places gives a result of 2·645751.

In case any readers think that this triangle was obtained by trial and error, the next triangle in this remarkable series has sides

2919 2746 1461 1441
2189 4559 6109 1441
1930 9186 5959 0800 .

Quotient = 2·645751311, accurate to nine decimal places.

The problem is this. Can you find another right-angled triangle in which every side is a whole number, but such that this time the quotient obtained by dividing three times the shortest side by the second shortest side after being rounded off to just four decimal places gives a result of 2·6457 (and not 2·6458)?

THE SOLUTION

The solution uses Newton's First Theorem[†].

When it is realised that 2·645751 and 2·645751311 are $\sqrt{7}$ accurate to six decimal places and nine decimal places respectively, then it becomes obvious that these triangles are approximations to the $(\sqrt{7},3,4)$ triangle. That is so obvious it hardly needs saying. The real problem is that $7 \neq n^2 \pm 1$, and so Newton's first theorem cannot strictly apply.

G.K. Chesterton would have said that $\sqrt{7}$ is the exception that proves the rule, and in this spirit let us proceed boldly.

Let $x = \sqrt{7}$, then $x^2 = 7$ and subtracting 4 from both sides yields $x^2 - 4 = 3$.

Now $x^2 - 4$ is the difference of two squares.
$(x - 2)(x + 2) = 3$,

$$\therefore \quad x - 2 = \frac{3}{x+2} = \frac{3}{4 + (x + 2 - 4)} = \frac{3}{4 + (x - 2)}\,.$$

Substituting for $(x - 2)$ in the R.H.S. from $(x - 2)$ on L.H.S.,

$$x - 2 = \cfrac{3}{4 + \cfrac{3}{4 + (x-2)}} = \cfrac{3}{4 + \cfrac{3}{4 + \cfrac{3}{4 + \cfrac{3}{4 + \dots}}}} \quad \text{ad infinitum.}$$

Let the rth rational be $\frac{n}{d}$. Then the $(r + 1)$th rational approx. is $\dfrac{3}{4 + \frac{r}{d}} = \dfrac{3}{4d + \frac{n}{d}} = \dfrac{3}{4d + n}$. But the first approximation is $\frac{3}{4}$.

$$\therefore \frac{n_2}{d_2} = \frac{12}{16 + 3} = \frac{12}{19}\,;\quad \frac{n_3}{d_3} = \frac{(3 \times 19)}{(4 \times 19) + 12} = \frac{57}{88}\,;\quad \frac{n_4}{d_4} = \frac{(3 \times 88)}{(4 \times 88) + 57} = \frac{264}{409}\,.$$

We thus obtain an infinite sequence which tends to the limit of $(\sqrt{7} - 2)$.

$$\frac{3}{4},\ \frac{12}{19},\ \frac{57}{88},\ \frac{264}{409},\ \frac{1227}{1900},\ \frac{5700}{8827},\ \frac{26481}{41008},\ \frac{123024}{190513},\ \dots\,.$$

[†]Also known as Cohen's First Theorem. See *Function, Vol.5, Part 2*.

Thus the eighth approximation to $\sqrt{7}$ is $2 + \frac{123024}{190513}$ which gives $\frac{504050}{190513}$. Use this rational number as the parameter pair $(m,p) = (504050,\ 190513)$ where $m^2 + p^2$, $m^2 - p^2$ and $2mp$ is a Pythagorean triad.

$m^2 + p^2 = 290\ 361\ 605\ 669$

$m^2 - p^2 = 217\ 771\ 199\ 331$

$2mp = 192\ 056\ 155\ 300$

The quotient gives $\sqrt{7} = 2{\cdot}645751$ to 6 decimal places.

The eleventh parameter pair (50540729, 19102600) generates the next triangle in the series in which the quotient gives $\sqrt{7} = 2{\cdot}645751311$ to 9 decimal places.

Obviously the preceding triangle in the series will be generated by the fifth parameter pair $2 + \left(\frac{1227}{1900}\right)$, that is $(m,p) = (5027,\ 1900)$.

$m^2 + p^2 = 5027^2 + 1900^2 = 28\ 880\ 729$

$m^2 - p^2 = 5027^2 - 1900^2 = 21\ 660\ 729$

$2mp = 2 \times 5027 \times 1900 = 19\ 102\ 600$

The quotient gives $\sqrt{7} = 2{\cdot}6457$ to 4 decimal places.

Observe that $\frac{21660729}{3} = 7220243$ and $4 \times 7220243 = 28880892$.

The hypotenuse $= 28880729$.

The error, the difference between these, is sixty three. If $7 = n^2 \pm 1$, then the difference would be only one. But $7 \neq n^2 \pm 1$, where n is an integer. That is why finding the Pythagorean triads generated by $\sqrt{7}$ is so much more difficult than finding those generated by $\sqrt{n}$ where n is of the form $r^2 \pm 1$, r being an integer. $\frac{3 \times 19102600}{21660729} \cong 2{\cdot}6457$ accurate to 4 decimal places giving $\sqrt{7} = 2{\cdot}646$ accurate to 3 decimal places.

Just as a matter of interest, you might like to know that the fourteenth parameter pair

$$(m,p) = (5067682250,\ 1915603851) \text{ generates}$$
$$m^2 + p^2 = 2935\ 0175\ 2993\ 9069\ 2701$$
$$m^2 - p^2 = 2201\ 2631\ 4745\ 3943\ 2299$$
$$2mp = 1941\ 3316\ 1945\ 8868\ 9500$$

This gives $\sqrt{7} = 2{\cdot}6457513111$ accurate to 10 decimal places.

S.J. Newton,
348A Bourke Street,
Darlinghurst, N.S.W.

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PERDIX†

This column will be a regular feature of *Function* and will bring news of mathematical competitions of all kinds that occur in Australia and, when it seems of interest, of international competitions.

Competitive mathematics has in recent years become very popular. The Australian Mathematics Competition for the Westpac awards (formerly the Wales awards) has now involved over one million competitors. In 1983 over quarter of a million competitors, from Australia, New Zealand, French Polynesia, Papua New Guinea, Fiji, Solomon Islands, Western Samoa, Tonga, Kiribati, Christmas Island, and Tokelau, took part in the competition. The International Mathematical Olympiad has been held each year since 1959 and an increasing number of countries has been sending teams to vie for Olympic honours. A team of six can be sent by any participating country. Six problems are given to the contestants, three on each of the two competition days.

Australia has formed its own Australian Mathematical Olympiad Committee which each year selects a team to represent Australia. Membership of the 1984 team has just been announced. The members are:

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†Perdix, the author of this column, was the nephew of Daedalus. Daedalus was a Greek sculptor, architect, inventor, creator and solver of puzzles, whose name was associated with so many achievements in ancient Greece, that the word Daedalus took the meaning of "cunningly wrought". Indeed as time passed an increasing number of sculptures and inventions were attributed to him: he personified cleverness, inventiveness, ingenuity. Daedalus' sculptures were said to be able to walk. Daedalus constructed the famous maze for Minos of Crete that was used to entrap Athenian maidens. He fell out of favour with Minos and escaped from Crete by constructing wings, made from large feathers sewn together and finished with small feathers attached by wax, which enabled him and his son Icarus to fly. Icarus ignored his father's advice and, exalting in his power of flight, flew too close to the sun: the wax on his wings melted and he plunged into the sea and perished. Hang-gliders and astronauts of the Ancient World! Daedalus was so proud of his achievements that he could not bear the thought of a rival. His sister placed her son Perdix under his charge to be taught the mechanical arts. Perdix was a quick learner and gave striking evidences of his ingenuity. Walking on the sea shore he picked up the spine of a fish. Imitating it, he took a piece of iron and notched it on the edge, and thus invented the saw. He made the first pair of compasses. Daedalus saw a potential rival in Perdix and killed him by pushing him off a high tower. Other versions of this legend say that the goddess Minerva caught Perdix as he fell and changed him into a partridge. (Perdix is the latin for partridge.)

Alan Blair, Year 12, Sydney Grammar School
Jonathan Ennęs, Year 12, Melbourne Grammar School
Matthew Hardman, Year 11, Knox Grammar School, Sydney
Andrew Jenkins, Year 12, North Sydney Boys High School
John Kramer, Year 12, Sydney Grammar School
Michael Peake, Year 12, Prince Alfred College, Kent Town, South Australia

Reserve

Boudewijn Roukama, Year 12, Mount Lawley Senior High School, Mount Lawley, Perth

In May a ten-day training school for members of the team is being held in Sydney. Also selected to attend the training school are:

Adrian Chen, Year 10, Prince Alfred College, Kent Town, South Australia
John Graham, Year 11, St Ignatius College, Riverview, Sydney
Andrew Hassell, Year 11, Christ Church Grammar School, Claremont, W.A.

Major support for the training camp is being provided by IBM Australia Ltd.

The Australian Olympiad team will travel abroad under the leadership of Mr J.L. Williams, and with deputy leader Mr G.R. Ball, both of the department of Pure Mathematics, at Sydney University. They will be running the training camp with the help of several other mathematicians, including Professor J.C. Burns, Professor G. Szekeres, and Mrs E. Szekeres.

Training and choosing possible Olympiad team members for 1985 is now under way. Arrangements are made to do this in each State. The Victorian State organiser for 1984 is Mrs JUDITH DOWNES, 46 Hill Road, North Balwyn, 3104 (Telephone: 859 4837) who will be happy to provide advice to those interested. It is hoped that separate training arrangements can be made at each of Monash, Melbourne, La Trobe, and Deakin universities. This column will bring you more information when it comes to hand.

Competitions in solving mathematical problems are not an innovation of this century. In the Middle Ages, kings, princes and emperors used to hold competitions open to all comers and mathematicians used to come from all over the world to try to win prizes. Most mathematicians respond to the (competitive) challenge of trying to solve a problem before others do. A lot of scientific research is similarly very competitive.

Apart from bringing news about competitions this column will also from time to time discuss problem solving, and try to give some hints about how to go about dealing with difficult (and not so difficult) problems.

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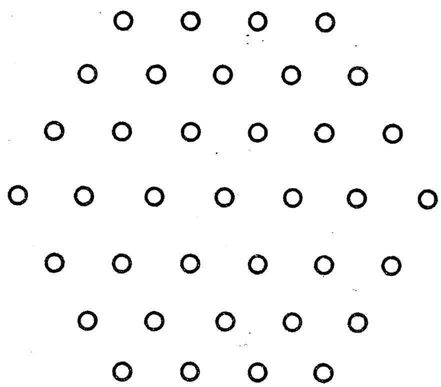
PROBLEM SECTION

We have had some solutions sent in to us but we will hold these over till next time. Meanwhile, here are some more problems.

PROBLEM 8.2.1.

A number of tubes are bundled together into a hexagonal form:



Thus bundle contains 37 tubes. In fact the number of tubes in the bundle can be 1, 7, 19, 37 (as shown), 61, 91, ... If this sequence is continued it will be noticed that the total number of tubes is often a number ending in 69.

What is the 69th number in the sequence which ends in 69?

The above puzzle is meant to be solved, by readers of The *Australian*, by constructing a computer programme. We invite readers of *Function* to find a mathematical solution. In particular, find a formula for the number of tubes in a hexagon with n tubes along each edge. Then determine all n for which the number of tubes ends in 69.

PROBLEM 8.2.2. (Submitted by J.L. Bairstow.)

There are five regular solids: the tetrahedron, the cube, the octohedron, the dodecahedron and the icosahedron. Each may be inscribed in a sphere so that all its vertices lie on the sphere. Calculate the radius of the sphere in each case.

PROBLEM 8.2.3.

(a) Show that in any party of six or more people, there is at least one set of three people all acquainted with each other or at least one set of three people no two of whom are acquainted.

(b) In a party of six people must there be either four mutual acquaintances or four mutual strangers?

PROFESSOR CHERYL PRAEGER

Late in 1983, the University of Western Australia appointed Dr Cheryl Praeger to a Chair in the Department of Mathematics. She is the only woman professor of Mathematics in Australia at the moment and only the second in the history of this country. (The other was Hanna Neumann - see *Function*, *Vol*.3, *Part* 1.)

Professor Praeger was born and educated in Queensland, gaining her B.Sc. (First Class Honours) and her M.Sc. from that university, and winning their university medal in 1969. This was followed by a University of Queensland Research Award and a Commonwealth Scholarship to St Anne's College, Oxford, where she was awarded her doctorate.

Between 1973 and 1975, she was a Research Fellow at the Australian National University in their Institute of Advanced Studies. She joined the University of Western Australia in 1976, initially as a lecturer, but was later promoted to Senior Lecturer. She has also spent brief periods at the universities of Virginia and Tel Aviv, at New Hall, Cambridge, and at the Prince of Songkla University (Haadyai, Thailand).

Her research interests lie in various aspects of the theory of finite groups, which has recently (1981) been the subject of a major result - the full classification of all "finite simple groups". She writes that this task involved the combined efforts of hundreds of mathematicians around the world over 30 years, and has been described as "one of the most remarkable achievements in the history of mathematics".

"The mathematical concept of a group developed last century in connection with the problem of solving polynomial equations. Groups are used to describe the symmetry of a system and as such are central to mathematics. One can think of a group as being made up of 'building blocks', called simple groups, 'glued together' in some way. In the case of finite groups, the set of simple groups is uniquely determined. One of the basic steps to understanding the structure of finite groups was therefore the classification of finite simple groups."

Her own use of group theory has included the study of permutation groups, groups arising from graph theory and other combinatorial questions, the use of group theory in experimental design with applications in agriculture and enumeration problems in weaving and textile manufacture.

In private life she is married to a statistician working in private enterprise and has two children, aged 5 and 2. She also holds the A.Mus.A. in piano and performance and plays the organ at her local church.

She writes that two early influences encouraged her. First, her success in Mathematics competitions and at Mathematics camps. Second, her years at Brisbane Girls Grammar School gave her role models in the very supportive and competent women Mathematics and Science teachers there. She was also greatly influenced by Hanna Neumann whom she met after winning a vacation scholarship to A.N.U. in 1968. Such encouragement she sees as most important for girls taking up mathematics.

Mathematics, she says, is actually a very convenient job for a woman, for it does not depend as heavily as other careers do on environment. It can be done in many different times and places, although it is important to have access to good library facilities and it is necessary to keep up contacts with other mathematicians.

Asked if she had a message for today's schoolgirls, she replied that they should have confidence in their own abilities and should not allow themselves to be intimidated out of it.

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MATHEMATICS TALENT QUEST, 1984

The Mathematical Association of Victoria with the help of 18 sponsors (including Monash University) organises an annual Mathematics Talent Quest. First held in 1982, it grew in 1983 to 1800 entries representing involvement of almost 3000 students from 20 schools.

Section 3, involving students in Years 10, 11, 12, and Section 4 (for computer entries) are those most likely to interest readers of *Function*. Entries can be from individuals, groups of 2, 3 or 4 students, or classes (at least 50% of the class participating).

You should ask your teacher for information on how to enter and for his help in processing your entry, as all entries must be sent through the school the entrant attends.