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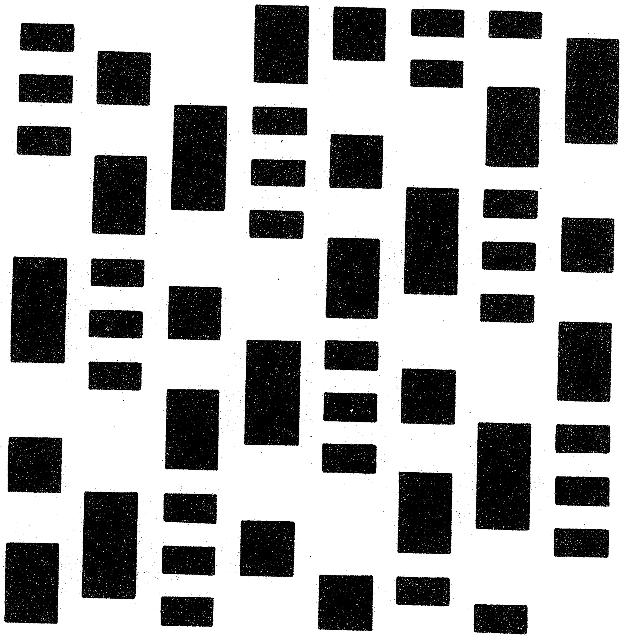

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Function is a mathematics magazine addressed principally to students in the upper forms of schools. Today mathematics is used in most of the sciences, physical, biological and social, in business management, in engineering. There are few human endeavours, from weather prediction to siting of traffic lights, that do not involve mathematics. *Function* contains articles describing some of these uses of mathematics. It also has articles, for entertainment and instruction, about mathematics and its history. Each issue contains problems and solutions are invited.

It is hoped that the student readers of *Function* will contribute material for publication. Articles, ideas, cartoons, comments, criticisms, advice are earnestly sought. Please send to the editors your views about what can be done to make *Function* more interesting for you.

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Articles, correspondence, problems (with or without solutions) and other material for publication are invited. Address them to:

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Alternatively correspondence may be addressed individually to any of the editors at the mathematics departments of the institutions shown above.

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Registered for posting as a periodical - "Category B"

This issue of *Function* might almost be called our international edition, with articles coming from Austria, the Netherlands and the U.S.A. We have had several requests for an article on Pythagorean triples and are glad to be able now to provide one. Professors Lidl and Pilz give a somewhat unusual application of mathematics - to sociological analysis, and Professor Sherbert tells of a nice economic analysis. The delightful article on the waterjet appeared anonymously in the Netherlands journal *Pythagoras*. We thank Ms A.-M. Vandenberg for her translation.

We remind our readers that they too can submit articles. Over the years, we have had some very good articles by readers still at school.

CONTENTS

THE FRONT COVER

Bob Griffiths, Monash University

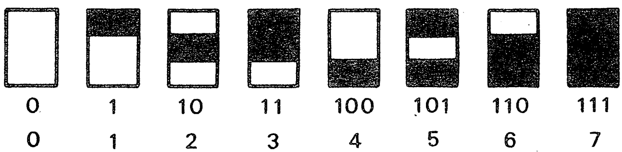
The cover design is based on an 8 × 8 *Latin square.*

0	1	2	3	4	5	6	7
5	6	7	0	1	2	3	4
2	3	4	5	6	7	0	1
7	0	1	2	3	4	5	6
4	5	6	7	0	1	2	3
1	2	3	4	5	6	7	0
6	7	0	1	2	3	4	5
3	4	5	6	7	0	1	2

Each different number appears exactly once in each row and column in a Latin square. (Can you work out a formula for the number in row i, column j in this particular Latin square?)†

Latin squares are used in experimental design in agricultural trials. For example, eight varieties of wheat might be planted on plots in this way. Because of the systematic placement of varieties over the whole square it is possible in a statistical analysis to separate out the effects of differing fertility in the plots and differences between the varieties.

In the design each number is represented in its binary form by a vertical block of three rectangles. Dark rectangles are units and clear rectangles are zeros.



Boundaries between the columns are separated, but not between rows. The whole square is drawn upside-down on the cover, as it looks best this way!

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†See p.30 for the answer.

THE COBWEB THEOREM IN ECONOMICS†

Donald R. Sherbert, University of Illinois at Urbana-Champaign

In the market place the supply of a product and the demand for it are closely related to the price. In general the higher the price the lower the quantity demanded by consumers. A typical relation between price and quantity demanded is illustrated by the demand curve shown in Figure 1. As is usual in the discussion of this curve in Economics, the vertical axis is taken to measure price and the horizontal to measure quantity demanded. We have, in a typical situation, a downward-sloping *demand curve*.

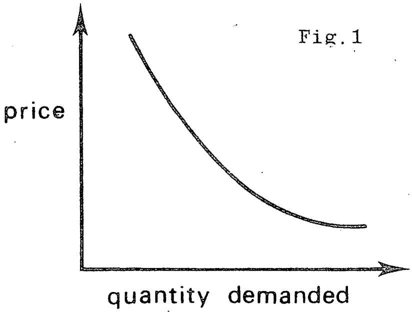

Fig. 1

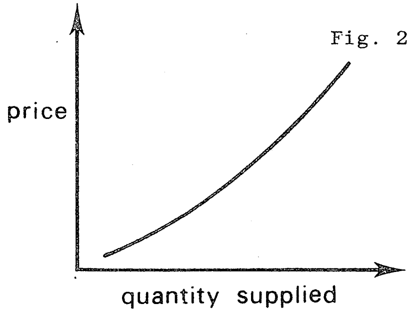

Fig. 2

Similarly those who are supplying the product will try to supply more if the price is higher so that a typical graph of quantity supplied against price will look something like the curve in Figure 2. In many industries this also reflects the fact that it costs less per unit to make a product when the total amount produced is greater. This curve is referred to as the *supply curve*.

If these two curves are plotted on the same diagram, as in Figure 3, then their intersection represents a so-called *equilibrium point*, at which price the quantity supplied to the

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†For other articles on difference equations (or recurrence relations) see *Function*, Vol.1, Part 2, Vol.2, Part 5 and Vol.3, Part 2.

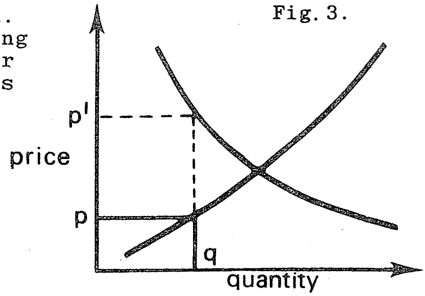


Fig. 3.

market is exactly what is demanded. When this does not happen, resulting pressures occur on the market. For example if the quantity supplied is less than at the equilibrium point then consumers will be willing to pay more than the equilibrium price. On Figure 3 if q is the quantity supplied in the expectation (supply curve) of price p, then the higher price p', obtained from the demand curve, would represent the price that the market would pay. In reaction to this suppliers would try to produce more.

With goods which come to market in a regular time-cycle, such as wheat, or pigs for bacon, there will be a regular time-lag between the decisions the suppliers take about the amount to produce for the market in response to the amount paid on one market day, and the next market day when they bring this new amount produced for sale.

We can set up a mathematical model that approximates this dynamical situation and use this to analyse the behaviour of the market. Let us simplify the situation by assuming that both the demand curve and supply curve are *straight lines*. In fact the argument we are going to give applies in much the same way in the general case. Let us also assume, for simplicity of speaking, that the goods we are considering, such as fat-stock lamb, come to the market once a year.

Let p denote price and q denote quantity of goods. The straight-line, downward-sloping, demand curve will then have an equation of the form

$$p = a - bq, \tag{1}$$

where a and b are positive constants.

The corresponding equation of the supply curve is of the form

$$p = kq \tag{2}$$

where k is a positive constant. Putting the straight-line supply curve in this form corresponds to the natural assumption that zero price brings zero supply.

Suppose that in year $n - 1$ the goods fetched a price p_{n-1}. In response to this, in year n, as determined by the supply curve, a quantity q_n, which it is hoped to sell at this price, is brought to market; by equation (2) q_n is given by

$$p_{n-1} = kq_n . \tag{3}$$

In fact the price realised, p_n, is determined by the demand curve; so by equation (1) we have

$$p_n = a - bq_n . \qquad (4)$$

Substituting from (3) in (4) we get

$$p_n = a - \frac{b}{k} p_{n-1} , \qquad (5)$$

an equation connecting the prices in successive years. Equivalently, (3) and (4) give also

$$kq_{n+1} = a - bq_n, \qquad (6)$$

connecting the quantities brought to market in successive years. These equations (5) and (6) give the yearly fluctuations in supply and price.

Equations such as (5) and (6) are called *recurrence relations*. Let us solve (5). To do this it is simplest first to rearrange (5) as

$$p_n - c = - \frac{b}{k}(p_{n-1} - c), \qquad (7)$$

which is equivalent to (5) if

$$c + \frac{bc}{k} = a,$$

i.e., if

$$c = \frac{ak}{b + k}. \qquad (8)$$

From this form we have immediately,

$$p_n - c = \frac{-b}{k}(p_{n-1} - c) = (\frac{-b}{k})^2(p_{n-2} - c) = \ldots$$
$$= (\frac{-b}{k})^n(p_0 - c). \qquad (9)$$

The quantity $p_0 - c$ is a constant; let us call it C. Thus from (8) and (9) we have

$$p_n = \frac{ak}{b + k} + C(\frac{-b}{k})^n. \qquad (10)$$

The long-range behaviour of the price thus depends on the size of b/k. There are three cases to consider.

Case 1. If $b/k < 1$, i.e. $b < k$, then $p_n \to \frac{ak}{b + k}$ as $n \to \infty$, because the geometric sequence formed by $C(-b/k)^n$ tends to zero. The market price then tends to stabilize as in Figure 4.

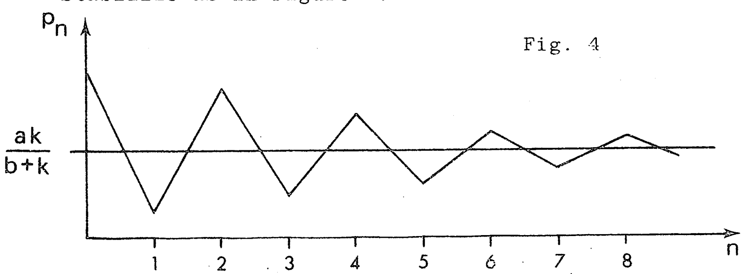


Fig. 4

Case 2. If $b/k > 1$, i.e. $b > k$, then the oscillations of p_n become larger and larger, because the terms of the geometric sequence formed by $C(-b/k)^n$ increase in magnitude with n and alternate in sign. The market is unstable, but the model fails when the price becomes negative. See Figure 5.

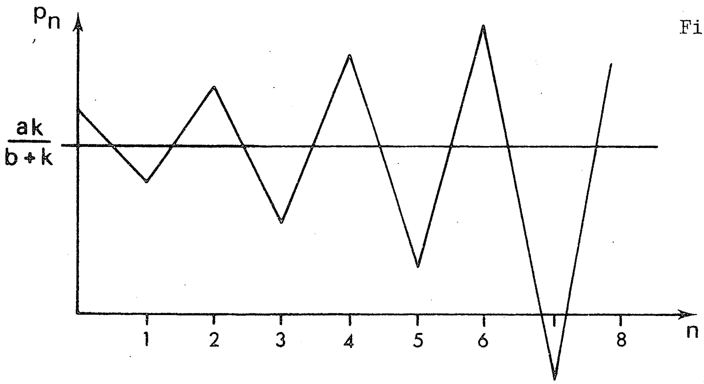


Fig. 5

Case 3. In the unlikely event that $b/k = 1$, i.e. $b = k$, p_n oscillates between the two values $\frac{1}{2}a + C$ and $\frac{1}{2}a - C$. In this case, the market is classified as unstable. See Figure 6.

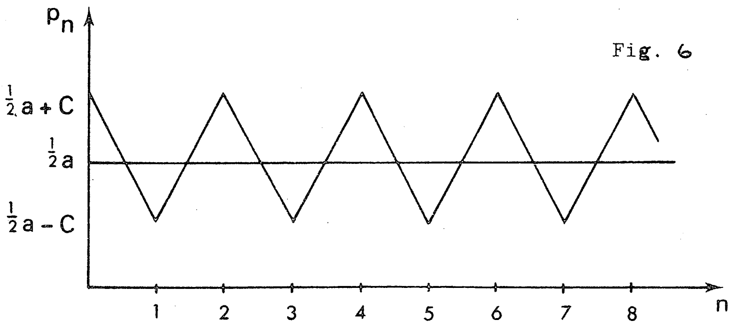
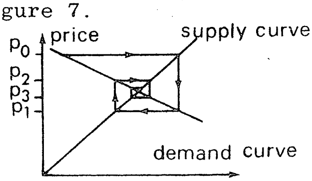
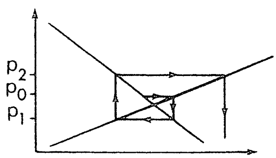


Fig. 6

The lagged adjustments can be dramatically displayed by plotting these changes, together with the supply and demand curves, on a single diagram. Cases 1 and 2 are illustrated in Figure 7.





Case 1. $b < k$. Stable Fig.7 Case 2. $b > k$. Unstable

The suggestive appearance of these pictures is the reason that the above analysis is referred to as the Cobweb Theorem in Economics. Note that b is the slope of the demand graph and k is the slope of the supply graph.

Thus *if supply adjusts more radically than demand to price changes, then the market will tend to stabilise* (Case 1), *while the reverse situation leads to instability* (Case 2). In Case 3, the cobweb is reduced to a rectangle that is retraced over the years.

The cobweb theorem first appeared in 1930 and was discovered independently by three workers; Schultz, Tinbergen and Ricci, although in all cases it appeared incidentally to the main analyses. All three of these researchers wrote in German. The English title "cobweb theorem" is due to Kaldor (1934), who also drew attention to the general status of the theorem. This article has considered only the simplest case. More general discussions replace Equations (1) and (2) by more general forms.

For a more technical discussion of some aspects of the economic analysis, see, for example, a paper, simply titled "The cobweb theorem", by Mordecai Ezekiel in the *Quarterly Journal of Economics* (1938).

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RAMPANT MATHOPHOBIA

The following story indicates the lengths to which mathophobia (the fear of mathematics) will drive supposedly responsible and reasonable people. It occurs in the course of an article by Dr M. Holcombe of the Queen's University of Belfast and published in the *Bulletin of the Institute of Mathematics and its Applications*.

"A mathematics department in a Californian college examined the way that the Pasadena Court Administration organised the call-up of jury members. The mathematicians constructed a simple procedure for deciding how many jury members were needed according to the number and nature of court cases that day. This scheme would have saved tens of thousands of dollars every year as well as preventing hundreds of people hanging around the court waiting to be called for service, instead of going to work. The court administrators could not accept that the mathematics could be correct, they did not believe that the theory of probability existed and so the scheme was never used."

Dr Holcombe states that this is "typical of the attitudes of many people in power. Even when the mathematics has been done for them, they are still suspicious mainly because they feel insecure with the subject."

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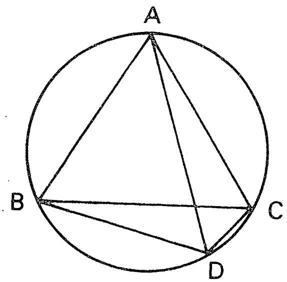
A LITTLE OLD-FASHIONED PROBLEM†

J.N. Crossley, Monash University

In 1979, when I visited England again, one of my former research students, Peter Aczel, now a lecturer at Manchester University, posed me a problem in elementary geometry. However, it was not one of the usual kind: prove this angle equals that, or this line equals that line. It was of the kind that arises in the study of similar triangles, namely the establishing of a proportionality. Let $ABDC$ be a cyclic quadrilateral, i.e. a quadrilateral whose vertices lie on a circle. The problem was to show that $BD + DC$ is *proportional* to AD when triangle ABC is isosceles with $AB = AC$. (See Figure 1 below.) D is an arbitrary point on the arc BC.

Fig. 1



Exercise 1. Establish the above result.

If you cannot do that (and I could not do it without a hint), try the simpler case with ΔABC equilateral. In this case the question becomes:

Exercise 2. If ΔABC is equilateral, show that $BD + DC = AD$.

I put this problem as an exercise (under the same title as this article's) in the Monash Mathematics Department Weekly Newsletter. Within a day or two I had three distinct, short, pretty solutions. One solution used trigonometry (hint: join A,B,C,D to the centre O of the circle) and the other two required choosing an extra point and drawing a line. That is to say, they required a construction. One of these goes roughly as follows: let E be the point on BC such that $\measuredangle BAE = \measuredangle DAC$ as in Figure 2.

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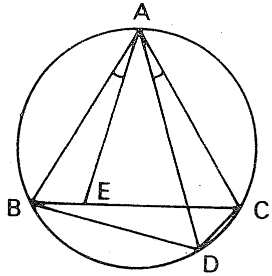
†For some related material, see the cover story of *Function*, Volume 5, Part 3.

Fig. 2



Then the triangles ABE, ADC are equiangular (hence similar) so

$$AB:BE = AD:DC$$

whence $$AB \cdot DC = BE \cdot AD \qquad (1)$$

where the dot indicates multiplication.

Again if we add the angle EAD to each of the equal angles BAE and DAC we get equiangular (and, therefore, similar) triangles BAD and EAC.

Therefore $$AC:EC = AD:BD,$$

whence $$BD \cdot AC = EC \cdot AD. \qquad (2)$$

Adding (1) and (2) we obtain

$$AB \cdot DC + BD \cdot AC = BE \cdot AD + EC \cdot AD = BC \cdot AD \qquad (3)$$

But $AB = AC = BC$ so

$$CD + BD = AD$$

as required.

In fact equation (3) holds for any triangle ABC and point D provided only that all four points lie on a circle. Equation (3) is generally known as *Ptolemy's theorem*. Ptolemy was from Alexandria (northern Egypt) and was certainly alive between 125 and 141 A.D. His most important work is generally known through its Arabic name the *Almagest* and this book was *the* astronomy book for centuries. Although it is almost certain that the theorem was not Ptolemy's discovery, nevertheless the proof we have just given is the proof in the Almagest. He used it in the process of calculating (what were equivalent to) sine tables. Ptolemy's theorem gives the equivalent of $\sin(\theta - \phi) = \sin\theta\cos\phi - \cos\theta\sin\phi$.

Exercise 3. Derive this result from equation (3).

(For further details see T.L. Heath, A History of Greek Mathematics, Oxford University Press 1921, Vol.II, pp. 273 ff.)

However, Ptolemy's "solution" requires the construction of the point *E*. So I asked for a solution of Exercise 2, which does not use a construction.

I received quite a number. All depend either on the use of similar triangles, as you might have guessed or, as you might also have guessed, on Ptolemy's theorem. One of my colleagues asked if I had discovered Ptolemy's theorem! But only one person solved Exercise 1. Can you do it? If so, please send your solution to me, c/- the Editor, by August 31st. I will either report on the solutions received or provide a solution in the next issue but one.

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ROBOT'S CUBE

"Once again, robots have taken a time-consuming task out of the hands of people. "Robbie Rubik", a project designed and built by the engineering students in the Tau Beta Pi honor society of the University of Illinois, actually solves the Rubik's Cube puzzle in just two-tenths of a second - putting even the most ingenious humans to shame. Admittedly, Robbie's mechanical hands work a bit slower than its computer brain; it takes the short, squat robot up to seven minutes to physically maneuver the cube's 54 colored squares into the correct order.

To solve the immensely popular puzzle, Robbie was programed with 3000 lines of computer instructions adapted from a published solution book. Although the robot is equipped with a sophisticated electronic eye designed to sense and differentiate the color of individual squares, Robbie is currently operating somewhat in the dark due to technical problems. Until the eye is perfected, the robot requires some human assistance: the color sequence of the squares must be punched into its computer keyboard. Within a second, the computed solution appears on a display screen. The solution can require a series of as many as 145 different moves, which Robbie performs by using pneumatic controls to flip the cube over and a mechanical spindle to rotate the six sides until the colors are lined up. While Robbie can rely on its computer program to solve the problem every time, the robot still appears to get emotional over the frustrating cube - its nose lights up while it works."

Newsweek, 17.5.82, p.5,
submitted by A.-M. Vandenberg.

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THE PERFECT DRAW

The Age (15.5.82) reports on p.33 that the VFA draw for the following day was: 1st to play 2nd, 3rd to play 4th, and so on all the way down to 11th vs. 12th. The probability of this unusual draw is $1/(11 \times 9 \times 7 \times 5 \times 3 \times 1) = 1/10395 = 0{\cdot}0000962$.

This assumes that when the draw was made, no account was taken of the relative strength of the teams.

IS THE ENEMY OF A FRIEND THE FRIEND OF AN ENEMY ? A NOTE ON SOCIAL NETWORKS[†]

Rudolf Lidl, University of Tasmania
Günter Pilz, Universität Linz

Sociology is the study of human interactive behaviour in group situations. In many societies, some underlying structure is what is of interest to the sociologist. Such structures are often revealed by mathematical analysis. This note indicates how algebraic techniques may be introduced into studies of this kind.

Imagine an arbitrary society, such as your family, your circle of friends, your school colleagues, etc. In such societies, provided they are large enough (at least 3 members), coalitions can be formed consisting of groups of people who like each other, who have similar interests or who behave similarly. How can we recognize such coalitions or formations of "blocks"? One way of finding out is asking, either directly or by questionnaires. Evaluation of the results of such questionnaires is very messy and usually it is not possible to see a pattern of coalitions. For instance, we number the members of a society by 1,2,... and ask the member with number i for his/her opinion about member j (e.g. praise, fear, love, esteem, etc.).

This article is based on a study of this type, carried ou by S.P. Sampson who gained his Ph.D. degree for it from Cornel University (U.S.A.). Sampson's rather lurid title was "Crisis i a Cloister". This was the story.

In an American monastery the degree of integration of new comers to the monastery was studied. Numbers were allocated t the monks and novices according to the number of years they

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[†]Based on a section of the book *Angewandte Abstrakte Algebra* (*Applied Abstract Algebra*) by the same authors and published by Bibiographisches Institut Wissenschaftsverlag, Mannheim (1982).

have been serving in the monastery. Each of the 18 members was asked to assign 3 or 2 or 1 points according to the esteem in which they hold a person ("no answer" was also permitted as well as the same point value for several people). The result is shown in the following table. (To interpret the table, look at a row labelled *i*, for the entries that occur in that row. Each entry gives the number of points allocated by member *i* to the member whose number heads the column containing the entry. Thus member 4 gives members 6, 10, 11 the points 1, 2, 3 respectively and no points to any other members.)

Person asked about

Person asked	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1					1			3				2						
2	3				1			2										
3													2				1	1
4						1				2	3							
5				3					1		2							
6				3							2							
7		3										1			1	2		
8				3	1	2												
9	1			3				2										
10																		
11					2				3									
12	1	2												3				
13					3		2				1							
14	3	2										1			2			
15	1	3												2				
16		3					2					1			2			
17	1	2											1					3
18	2	3															1	

The subdivision into blocks makes the table easier to read. As we might expect, this block design is not very useful as it stands.

In the mid 70's a Harvard University computer program was developed to change the order of 1,2,...,18 in a way which makes it possible to observe a separation into "strong" and "weak" blocks in the table. These algorithms, applied to our example of a monastery, yield:

	10	5	9	6	4	11	8	12	1	2	14	15	7	16	13	3	17	18
10																		
5			1		3	2												
9					3		2		1									
6					3	2												
4	2			1		3												
11		2	3															
8		1		2	3													
12									3	2	1							
1		1					3	2										
2		1					2		3									
14								1	3	2		2						
15									1	3	2							
7								1		3		1		2				
16								1		3		2	2					
13		3				1							2					
3									3						2		1	1
17										1					1	2		3
18										2						3	1	

When a formation of blocks has been obtained by these algorithms, the "strong" blocks (the ones with "many" elements) are encoded as 1, the "weak" blocks are encoded as 0. (We define a block as "weak" if the sum of the elements in a block is smaller than half of the on-average expected sum within the block.)

From the second table above we thus obtain the matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} .$$

We can interpret this as follows: the first and second group of monks and novices have high regard for themselves, the third group regards itself and group 2 highly.

The question "whom do you like?" resulted therefore in the encoded matrix

$$M_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} .$$

The opposite question "whom do you not like?" resulted in

$$M_2 = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} .$$

In general, for the questions $R_1, \ldots, R_s$ about the relation between individuals in a group one obtains corresponding matrices $M_1, \ldots, M_s$ with elements either 0 or 1, like M_1 and M_2. The product of these matrices has an interesting interpretation. If we take M_1 as measuring the relation (R_1, say) of "friend" and M_2 that of "enemy" (R_2, say) then we can interpret the relation R_1R_2 as "enemy of a friend" and R_2R_2, " enemy of an enemy" by matrix multiplication as follows. We multiply matrices in the usual fashion, but use the following rules for adding and multiplying the 1's and 0's,

$$\begin{array}{c|cc} + & 0 & 1 \\ \hline 0 & 0 & 1 \\ 1 & 1 & 1 \end{array} \qquad \begin{array}{c|cc} . & 0 & 1 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 1 \end{array} .$$

Then the matrix product M_2M_2 is

$$M_2M_2 = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix},$$

which is *not* the matrix M_1. Thus, the enemy of an enemy is not necessarily a friend in this society. The matrix corresponding to "friend of an enemy" is

$$M_1M_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix},$$

while the matrix corresponding to "enemy of a friend" is

$$M_2M_1 = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix} \neq M_1M_2 .$$

So in this society "friend of an enemy" is different from "enemy of a friend".

The question of whether the enemy of an enemy is a friend is equivalent to whether $M_2M_2 = M_1$ and the "equation": "friend of enemy = enemy of friend" is equivalent to the property that the matrices M_1 and M_2 commute, i.e. that $M_1M_2 = M_2M_1$. We can have different answers to these questions according to the different societies we are investigating.

Let us introduce the term *role structure*, which abstracts from particular concrete situations, like the one in the monastery, or in a commune.

Let G be a community partitioned into blocks $B_1,\ldots,B_k$, and let $R_1,\ldots,R_s$ be relations on G with corresponding $k \times k$ matrices $M_1,\ldots,M_s$ over $\{0,1\}$. The *role structure* corresponding to G, $B_1,\ldots,B_k$, $R_1,\ldots,R_s$, $M_1,\ldots,M_s$ is defined as the set which is obtained by taking all possible products of $M_1,\ldots,M_s$.

In our example, let

$$G = \{1,2,\ldots,18\} = \{10,5,9,6,4,11,8\} \cup \{12,1,2,14,15,7,16\}$$
$$\cup \{13,3,17,18\} = B_1 \cup B_2 \cup B_3$$

and let R_1 and R_2 be as above. Forming all possible products of M_1 and M_2, such as $M_1M_1M_1$, M_2M_1, M_2M_2, etc. leads to the set of matrices

$$\left\{ \begin{pmatrix} 1&0&0\\0&1&0\\0&1&1 \end{pmatrix}, \begin{pmatrix} 0&1&1\\1&0&1\\1&1&0 \end{pmatrix}, \begin{pmatrix} 0&1&1\\1&0&1\\1&1&1 \end{pmatrix}, \begin{pmatrix} 0&1&1\\1&1&1\\1&1&0 \end{pmatrix}, \begin{pmatrix} 0&1&1\\1&1&1\\1&1&1 \end{pmatrix}, \begin{pmatrix} 1&1&1\\1&1&1\\1&1&1 \end{pmatrix} \right\}.$$

This set of six elements is the role structure of the monastery.

The setting up of role structures such as this allows the comparison of different societies. Let G_1 and G_2 be two different societies whose role structures are determined. It turns out to be possible (using properties of certain algebraic structures known as *semigroups*) to define a "distance" between the two role structures. This is taken to be a measure of the difference between G_1 and G_2 from the structural point of view.

Such comparisons were made by the sociologists S.D. Boorman and H.C. White in a 1976 study published in the *American Journal of Sociology*. They found that (despite all ideological differences) the role structure of the monastery and a commune were remarkably similar. The case of a group of biomedical researchers deviated rather more clearly and that of a set of managers in a large company was completely different.

It is also interesting to observe the situation at various points in time. Here the block partitions and the corresponding matrices may or may not change. In our example of a monastery such observations had been taken over certain time intervals. The monastery rapidly changed. Soon monks numbered 2, 3, 17 and 18 were excluded; then number 1 left of his own accord. Within a few days, numbers 16, 15, 14, 7, 13 and 8 followed. One month later number 10 left. This may be interpreted in terms of the blocks obtained above.

Boorman and White suggest that methods like these could be used in other areas. Official organizations like the public service could, they suggest, be so studied. Possibly also such methods could find applications in legal situations involving the clarification of complicated conflict situations. We agree that the approach has considerable promise.

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THE WATERJET CURVE†

When a tap is only slightly turned on, we often observe a beautifully curved form at the point where the water leaves the tap. When the tap is turned on further, this curve gradually disappears. But the jet of water keeps getting gradually thinner on its way down, until the water breaks into droplets.

We shall try to determine if there is more to be learnt from these jet shapes. The description given here of the falling motion of the water leads to a description of the edge of the jet in terms of the graph of the function $y = c/x^4$.

Jet Strength and Jet Velocity.

In describing a jet, we use two closely related concepts: *jet strength* and *jet velocity*. We can take as units m^3/s for the first, m/s for the second. This readily indicates the distinction.

The jet strength is the volume of water flowing out in unit time; the jet velocity is the distance travelled by a water droplet in unit time. Between the two there is a relation:

$$\text{jet strength} = \text{jet velocity} \times \text{cross-sectional area.}$$

Since:

* the velocity of a falling droplet increases during the fall, and
* the jet strength in a constant jet is the same for all heights (the amount of water passing any point in a second is the same everywhere),

the above relation implies that the cross-section of the jet decreases on the way down.

---

†From *Pythagoras*, a Netherlands counterpart of *Function*. We thank Ms A.-M. Vandenberg for translating the article, which we have also somewhat abridged. The article is printed under an exchange agreement between the two journals. The jet described is known technically as a "free surface jet", as opposed to a jet of air (say) which has no free surface.

Translation into Formulae.

We introduce some symbols (see Figure 1). At the circular opening, the radius is x_0, and the vertical velocity of the water is v_0 across the entire section. After a fall h, the radius is x, the velocity v. We call the constant jet strength i.

Gravity ensures a uniformly accelerated falling motion with constant acceleration g. The laws of fall now give a relation between h and v.

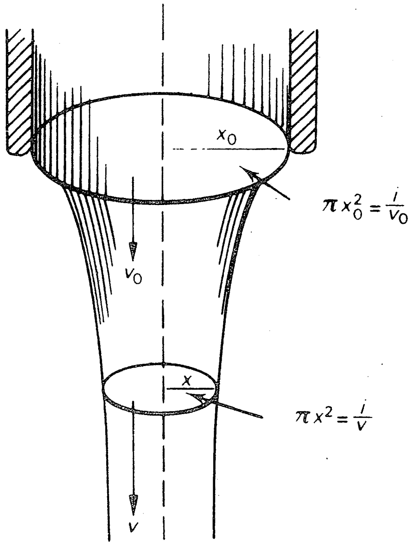


Figure 1

$$h = \frac{v^2 - v_0^2}{2g} . \qquad (1)$$

Because $i = v_0 \cdot \pi x_0^2 = v \cdot \pi x^2$, this becomes

$$h = \frac{i^2}{2g\pi^2}\left(\frac{1}{x^4} - \frac{1}{x_0^4}\right) \qquad (2)$$

The Theoretical Shape of the Jet.

Equation (2) shows that the shape of the jet can best be described by a system of coordinates with an origin some distance above the plane of the outlet, and with the y-axis pointing vertically downwards. Using the abbreviations $c = i^2/2g\pi^2$ and $y = h + (c/x_0^4)$, we find that Equation (2) becomes

$$y = \frac{c}{x^4} .$$

In Figure 2, overleaf, graphs of this relation are drawn for various values of c, that is, for various jet strengths i.

Is the Theory Correct?

Does the shape predicted theoretically actually conform to reality? A question mark is indeed justified here, for there are a number of things we have not taken into consideration.

* The effect of the force of adhesion, the "sticking" force between the water and the edge of the tap has been ignored.

* The cohesive force of the water, the tendency of every fluid to minimise its boundary surface is neglected. As the jet becomes very thin, it appears that a breaking-up into separate droplets is more advantageous in this respect. The "thinning" of the jet after this shows up as a gradual increase in the distance between the droplets.

* The vertical velocity is not strictly constant over the cross-section. The outermost layer of water is slowed down by friction, first with the wall of the pipe, later with the air.

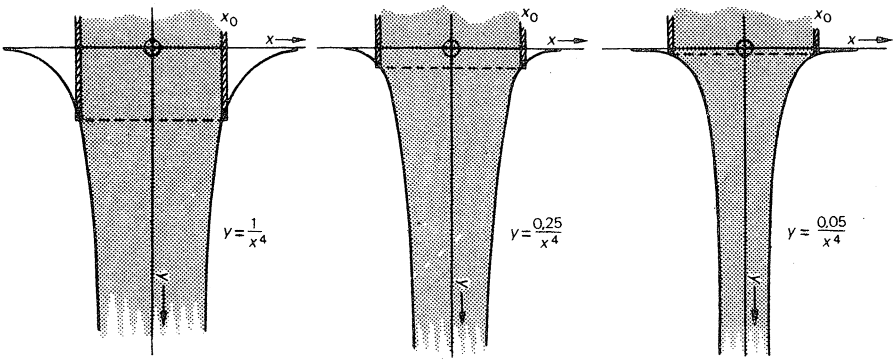


Figure 2

Experimental Test

The validity of Equation (2) can be tested by carrying out measurements on a real water jet, e.g. on the photographs of Figure 3 opposite.

With a small magnifying glass measure as accurately as possible the jet's radius at the top edge and also somewhat lower down. The lines on the scales were drawn one centimetre apart in the experiment, so the actual height between the two points can be determined (in mm.). As $g = 9{\cdot}8$ m/s (the acceleration due to gravity), only the jet strength remains unknown. When the photographs were taken, this was also determined with a litre flask and a stop-watch. However, the values of i can actually be determined from the photographs and we leave this as an exercise for the reader.

The technique to be used is as follows. For each of a reasonable number of heights h_n, measure the radius x_n and convert both readings to millimetres. Next compute x_n^{-4} for each n and graph the number pairs (h_n, x_n^{-4}).

The formula predicts a straight line. Find out from your figure for which points this is most accurate; draw (insofar as it is possible) a straight line through these and continue your calculations using that line.

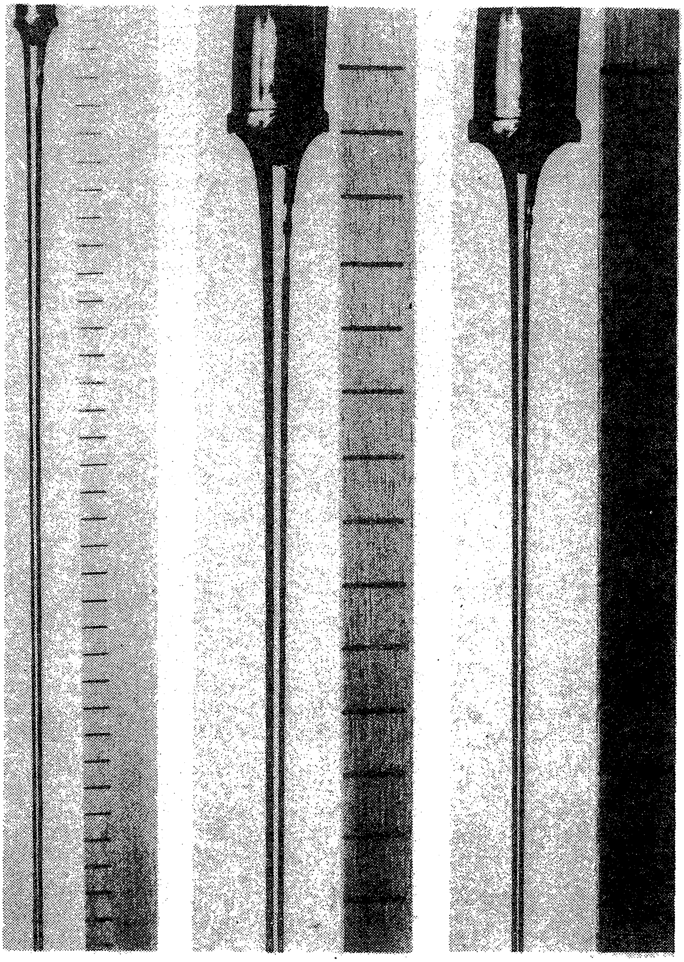


Figure 3

Three descending water jets. The scale lines are placed 1 cm. apart. You could set up the experiment for yourself- it's not difficult.

PYTHAGOREAN TRIPLES

F. Schweiger, Universität Salzburg

I would think that one of the few theorems known outside the classroom is that of PYTHAGORAS: Let a,b denote the lengths of the two legs of a right-angled triangle, and let c be the length of its hypotenuse. Then

$$a^2 + b^2 = c^2.$$

We may rephrase this in more every day language by letting a,b,c be, respectively, the length, breadth and diagonal of a rectangle. (This avoids the technical term "hypotenuse", which is borrowed from the Greek and translates as "the line subtending the right angle", but also means "the string of the bow". The word "diagonal" is also of Greek origin - it means "going from one corner to another"; in this case, the word has entered every-day usage.)

The *converse* of this theorem is also true:

> If three sides of a triangle fulfil the relation $a^2 + b^2 = c^2$, then the triangle is right-angled.

The converse of the theorem is interesting because it was probably the source of the Pythagorean theorem and in particular of the search for Pythagorean triples. A Pythagorean triple is a triple (a,b,c) of natural numbers such that the relation $a^2 + b^2 = c^2$ holds.

The most famous of these triples is (3,4,5), which can be found in very old texts. For example, it marks the beginning of the scientific dialogue between the brother of the Chinese emperor and a scholar in the ancient Chinese text† ZHOUBI SUANJING (周髀算經)(dated between 100 B.C. and 100 A.D.).

Again, Problem One of an Ancient Egyptian text known as Berlin Papyrus No.6679 (c. 2000 B.C.) reads: "A square and a second square, whose side is one-half and one-quarter [i.e. 3/4] that of the first square, have together an area of 100. Show me how to calculate this." The solution is (8,6,10), which, apart from a permutation, is just twice the triple (3,4,5).

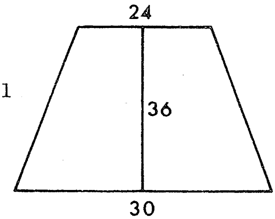
---

†This book's title is also written Chou Pei Suan Ching. The best translation of the title is probably The Arithmetical Classic of the Gnomon [set-square] and the Circular Paths of Heaven.

Why did these early mathematicians become interested in Pythagorean triples? We can speculate on various possible answers to this question.

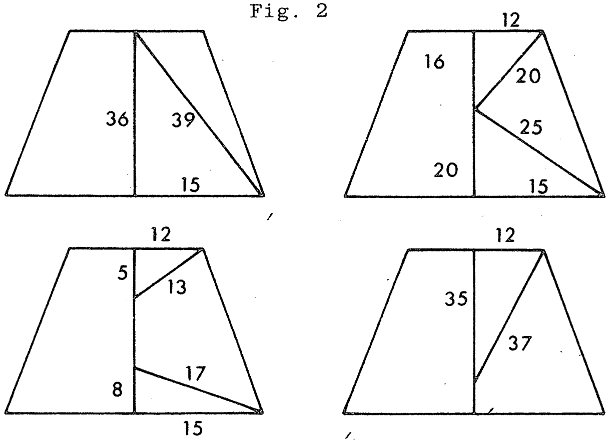
(1) Earlier measurement could only be made by expressing the length of a given distance as a multiple of the length of a given measuring rod. Therefore all lengths were expressed by natural numbers. A right-angled triangle or a rectangle (seen together with its diagonal) are basic geometric shapes so that the ones which could be measured "wholly" are specially attractive to anyone who loves geometry. In the Old Indian text ĀPASTAMBA ŚULVASŪTRA (about 500-300 B.C.) the following triples (and some of their multiples) are mentioned: (3,4,5), (5,12,13), (8,15,17), (12,35,37). Besides this, in even older texts, the dimensions of certain quadrilaterals (used in connection with building altars) are mentioned. Let us take as an example the isosceles trapezium given in Figure 1.

Fig. 1



A lot of Pythagorean triples can be found in this trapezium by dissecting it as shown in Figure 2.

Fig. 2



(2) Since the converse of PYTHAGORAS' theorem holds, knowledge of a Pythagorean triple easily leads to the construction of a right angle. The Greek historian Herodotus reports the Ancient Egyptian practice of tightening cord or rope during the building of temples. A rope of length 12 can be tautened into the form

of a right-angled triangle by measuring off sub-lengths of 3, 4 and 5. One speculation is that this was the purpose of the rope-tightening.

(3) Special Pythagorean triples can be used to picture a (generally valid) proof of PYTHAGORAS' theorem by counting areas. The arrangement reproduced in Figure 3 (corresponding to $(a - b)^2 + 4ab = (a + b)^2$) has been attached to the text of ZHOUBI SUANJING by later commentators.

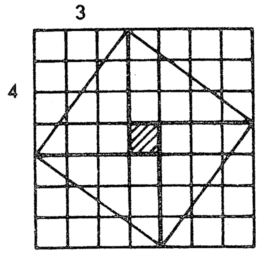


Fig. 3

The proof is built up from a square divided into 7×7 (= 49) squares. In this case, $a = 4$, $b = 3$, and the algebraic identity is clearly diagrammed.

(4) Pythagorean triples can be used to construct a lot of enjoyable mathematical problems the solutions of which are all given in whole numbers. Let us mention one problem taken from the Chinese text JIUZHANG SUANSHU (九章算術 - meaning Nine Chapters of the Mathematical Art), published about 100 B.C. to A.D. 100: "There is a bamboo of height 10. The top is broken and reaches the ground a distance of 3 from the root. The question is: What is the height of the break? The answer is: $\frac{91}{20}$." In this problem the triple (60,91,109) is hidden. Similar problems can be found in Ancient Babylonian texts.

(5) The relation $a^2 + b^2 = c^2$ is a beautiful track in the jungle of the natural numbers. The amazing plentitude of relations such as

$$3^2 + 4^2 = 5^2$$

$$5^2 + 12^2 = 13^2$$

$$8^2 + 15^2 = 17^2$$

$$12^2 + 35^2 = 37^2$$

naturally leads to the question of how to find Pythagorean triples.

The Greek philosopher PLATO is said to have known the formulae

$$a = 2A, \; b = A^2 - 1, \; c = A^2 + 1,$$

for A an arbitrary integer. PYTHAGORAS himself is said to have known a slightly different set of formulae:

$$a = 2A + 1, \quad b = 2A^2 + 2A, \quad c = 2A^2 + 2A + 1.$$

The formulae:

$$a = 2AB, \quad b = A^2 - B^2, \quad c = A^2 + B^2$$

were known to DIOPHANTOS (between 150 and 350 A.D.) as well as to the Indian mathematician BRAHMEGUPTA (born 598 A.D.). They can also be found in Arabic manuscripts. It is known that these formulae essentially give all Pythagorean triples.

The story of Diophantine equations (those whose solutions are integral), which is related to Pythagorean triples, has been greatly developed since the time of DIOPHANTOS, after whom they are named. Here is one example: Give the solutions (in integers) of

$$a^2 + b^2 = c^2$$

$$ab = 12d^2$$

(the latter equation means that the area of the rectangle with sides a and b is 12 times a square). The triple (3,4,5) gives the solution $d = 1$, the triple (49,1200,1201) gives $d = 70$. The next triple was found by FERMAT (1601 - 1665), and involves the triple given by $A = 2738 = 2 \times 37^2$, $B = 529 = 23^2$. This gives: $a = 289\,6804$, $b = 721\,6803$, $c = 777\,6485$ and $d = 131\,9901$.

In conclusion, we note the following discovery, due to VIÈTE (1540-1603). Let (a_1, b_1, c_1) and (a_2, b_2, c_2) be Pythagorean triples, and write

$$a_3 = a_1a_2 - b_1b_2$$

$$b_3 = a_1b_2 + a_2b_1$$

$$c_3 = c_1c_2.$$

Then (a_3, b_3, c_3) is a new Pythagorean triple. Note a connection with complex numbers. Put

$$z_1 = (a_1 + ib_1)/c_1, \quad z_2 = (a_2 + ib_2)/c_2,$$

where $i^2 = -1$.

Then

$$z_1z_2 = \{(a_1a_2 - b_1b_2) + i(a_1b_2 + a_2b_1)\}/c_1c_2.$$

VIÈTE's formulae thus anticipate the multiplication of complex numbers. You may care to prove VIÈTE's result for yourself. The complex number formulation allows an elegant proof.

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B.L. van der Waerden, On Pre-Babylonian Mathematics I-II. *Archive for History of Exact Sciences* 23 (1980).

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AMERICAN DEVELOPMENTS

The results of an extensive 1980 survey of U.S. Mathematics departments have just been published. Such surveys are carried out at 5-yearly intervals by the Conference Board of the Mathematical Sciences (CBMS). The survey shows that in the five years reviewed, mathematics enrolments in four-year (i.e. degree granting) undergraduate institutions increased overall by 33%. The largest component of this increase lay in the areas of computing and closely related fields, where enrolments rose by 196% over the five years. This was perhaps to be expected. The second largest component was, however, no cause for congratulation. Remedial mathematics course enrolments rose by 72% to the point where one mathematics enrolment in every six is in the remedial area. After these, the next area of increase was Calculus (up 30%). The CBMS reports that one reason for the increases is "the surge of student interest in such practically-oriented majors as engineering and business, where employment prospects have recently been excellent".

There was, over the five-year period, a decline in the number of students specialising in mathematics. This was only partially offset by a large increase in numbers of computer science majors.

Although teaching loads increased by 33%, the staff increase was more modest: 13%. Of the teaching staff, 14% are women. The previous figure was 10%.

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WOMAN PRESIDENT

The President-Elect of the American Mathematical Society is Professor Julia Robinson of the University of California at Berkeley. A world authority in Symbolic Logic and Number Theory, Professor Robinson is a member of the (U.S.) National Academy of Sciences. She will be the first woman president of the American Mathematical Society.

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"[The basic concept of probability] is a complicated and deep subject. The debate concerning it will, I am sure, go on forever, In the meantime, do not forget one massive fact about probability theory - it works."

Lady Luck,
Warren Weaver, 1963.

LETTER TO THE EDITOR

An Unusual Square Root.

To find $\sqrt{51}$.

The sequence of odd numbers is 1, 3, 5, 7, 9, 11, ... Subtract these in turn from 51. 51 - 1 = 50; 50 - 3 = 47; 47 - 5 = 42; 42 - 7 = 35; 35 - 9 = 26; 26 - 11 = 15; 15 - 13 = 2.

7 odd numbers can be subtracted, so the integral part of the answer is 7.

The remainder is 2. Divide this by the next even number after the last odd number subtracted, i.e. 14. So $\sqrt{51} \approx 7\frac{2}{14} \approx 7\cdot143$. From tables $\sqrt{51} = 7\cdot141 \ldots$.

This method depends largely on the fact that a square number is the sum of a series of odd numbers.

If the square root of x is required, $x = y^2 + \delta x$, where δx is the remainder, and the answer is given as $\sqrt{x} \approx y + \delta x/2y$. Now

$$(y + \delta x/2y)^2 = y^2 + \delta x + (\delta x/2y)^2.$$

Hence the error in the square is $(\delta x/2y)^2$. In our example, this error is $\{2/(2 \times 7)\}^2 = 0\cdot0204$, which may be checked directly.

Garnet J. Greenbury,
123 Waverley Road,
Taringa, Queensland.

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PROBLEM SECTION

We begin by completing the solution to an outstanding problem.

COMPLETED SOLUTION TO PROBLEM 5.4.2.

This problem, from a modern Chinese source, concerned the scheduling of women drawing water from a village pump. We saw in *Function* Vol.6, Part 1 that if only one pump is available, the total number of woman-hours waiting time is minimised by the following process.

Let t_i be the time to be taken by woman number i and then ensure that $t_1 \leqslant t_2 \leqslant \ldots \leqslant t_{10}$ for the ten women.

Now let two pumps be available. The first two women, the quickest, begin. When the first woman finishes, the third woman takes her place at pump number one. When the second woman finishes, the fourth woman takes her place at pump number two, and so on. This process clearly generalises.

SOLUTION TO PROBLEM 5.5.2.

To settle a point of honour, three men, *A*, *B* and *C* engage to fight a three-cornered pistol duel. *A*, a poor shot, has only a 30% chance of hitting his target; *C* is somewhat better, his chance of a hit being 50%; *B* never misses. *A* however has first shot. *B*, if he survives, fires next; then *C*; then *A* again, etc. However, if a man is shot, he takes no further part in the contest either as a marksman or a target. What should *A*'s strategy be?

Solution by J. Ennis, Year 10, M.C.E.G.S.:

If we assume that each man will try to maximise his chances of survival in a rational fashion, then we may reason as follows. *B*, when his turn comes, will fire at *C* rather than *A* (if *C* is still involved) as *C* is more likely than *A* to kill *B*. But suppose *A* hits *C*, then *B* fires at *A* and *A* does not survive. If *A* hits *B*, *A* and *C* continue to fire at each other until a hit is scored. The probability that *C* hits *A* is $\frac{1}{2}$; the probability that he hits on his next shot is $\frac{1}{2} \times \frac{7}{10} \times \frac{1}{2}$ (i.e. he misses the first, is missed in turn and then scores). The probability that *A* is hit is the sum of all such probabilities:

$\frac{1}{2}\left(1 + \frac{7}{20} + \frac{7^2}{20^2} + \ldots\right) = \frac{10}{13} \approx 0 \cdot 77$. Thus *A*'s chance of survival, if he first hits *B*, are about $0 \cdot 23$.

Suppose however *A* misses. Then *B* will kill *C* at the next turn and *A* will have a $0 \cdot 3$ chance of killing *B* before *B* kills him. Thus *A* should fire in the air.

Of course, if *B* and *C* do not try to maximise their chances of survival, the situation is different; if, for example *B* has a special grudge against *A*, then *A* should try to shoot *B* on his first turn.

Here are some further problems to try - please send us your solutions for inclusion in *Function*.

PROBLEM 6.3.1.

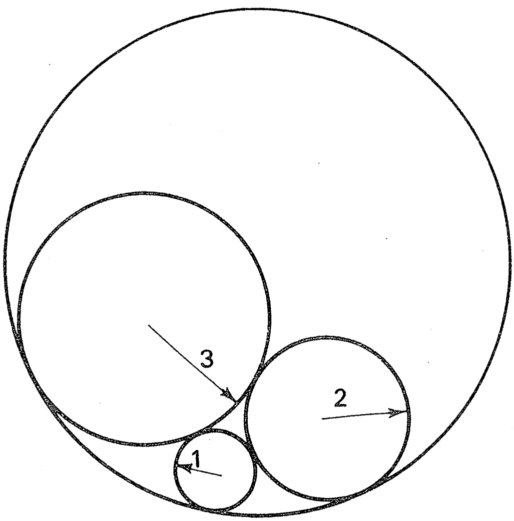
A student lives on the bus route between the university and his girl friend's house. The buses run regularly, but the student gets up at quite random hours. When he leaves home, he takes the first bus that comes, irrespective of its direction of travel. On average he visits his girl friend twice as often as he goes to the university. Explain how this can be.

PROBLEM 6.3.2.

Al Capone is holed up and wishes to communicate with his confederate Squizzy Taylor, who is similarly disadvantaged. Al writes his message and seals it in a strong box which he padlocks. He is forced to entrust this to a crafty but unscrupulous courier, who on no account is to see the message. This means that Al has to keep the key, of which no copy exists. How does Squizzy read the message?

PROBLEM 6.3.3 (Submitted by J. Ennis, Year 10, M.C.E.G.S.)

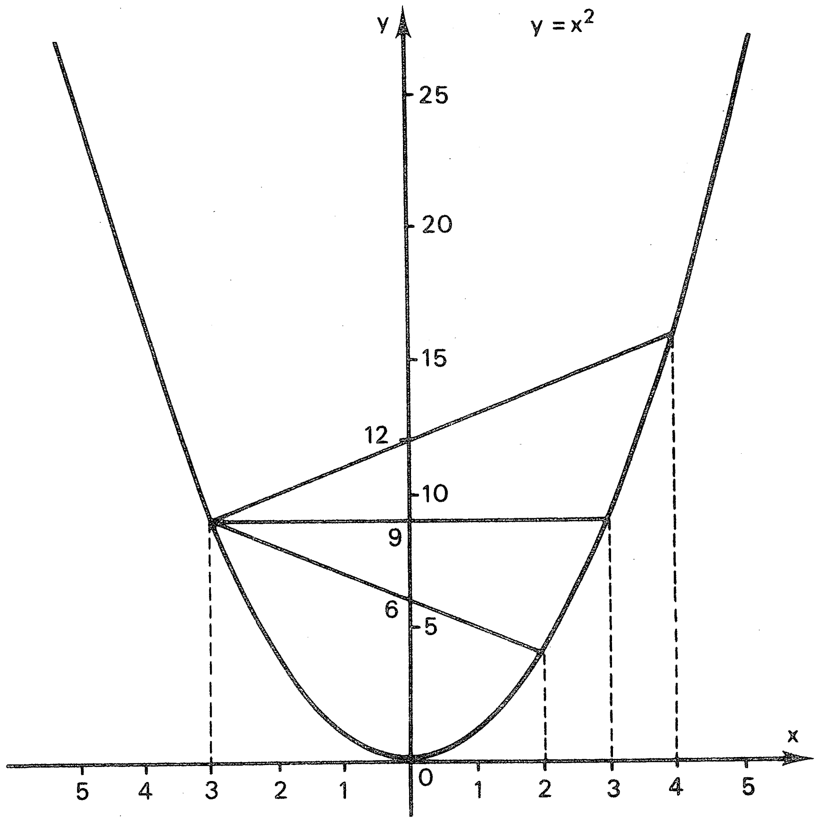
Each circle in the diagram is tangent to the other three.



What is the radius of the large circle?

PROBLEM 6.3.4. (Submitted by Garnet J. Greenbury, 123 Waverley Road, Taringa, Queensland.)

Consider the following graph of $y = x^2$ (overleaf). Note that $3 \times 4 = 12$, $3 \times 3 = 9$, $3 \times 2 = 6$, the values of the intercepts on the y-axis. Explain why this is so.



PROBLEM 6.3.5.

This problem consists of the exercises set in Professor Crossley's article (pp. 8 - 10).

JUMPER'S CHANCES

According to Peter Singer (*Age* letters 14.5.82), a horse starting in a jumping race has a 1/88 chance of being killed in that race. He then went on to "deduce" that in 50 races, its chance of death is 56·5%. (He converted 1/88 to a percentage, rounded down to 1·13% and multiplied by 50.)

Even assuming that the 1/88 figure is correct (which seems unlikely - it comes from an earlier letter by a Mr Barber, whose incomprehensible figures Professor Singer chose to interpret in this way), this calculation is incorrect. The correct method of calculation was pointed out by Ewan Coffey (*Access Age* 15/5) and by Paul Sheahan (17/5) of Geelong Grammar School.

Here is how the calculation should be done.

If the jumper has a 1/88 chance of death during a race, then its probability of surviving that race is 87/88. Assuming that the same probability applies to each race, its chance of surviving n consecutive races is $(87/88)^n$. This means that its probability of *not* surviving n consecutive races is $1 - (87/88)^n = 1 - (1 - 1/88)^n$. While this is approximately $1 - (1 - n/88) = n/88$ for small n (the expression used by Peter Singer), it is not a good approximation for larger n. Thus in 50 races the chances of death are $1 - (87/88)^{50}$ or 43·5%, not 56·5%.

Singer's argument becomes manifestly absurd in the case $n = 100$. The chances of death are then correctly calculated as 68·1%, whereas Singer's approach gives 113%!

One reason for doubting the accuracy of Singer's figures is that Barber's corresponding figure for flat races has the chance of death per start (on Singer's interpretation) as 1/298. In a flat race career of 50 races, the horse would have a 15·5% chance of death. This figure is regarded by the racing men (including mathematicians and statisticians) we have consulted as absurdly high.

Problems of this nature can be handled by the Poisson distribution. This relies on the approximation

$$(1 + \frac{x}{n})^n \approx e^x,$$

valid for large n. Thus to calculate $\left(1 - \frac{1}{88}\right)^{50}$, we could write

$$\frac{1}{88} \approx \frac{0 \cdot 568}{50}$$

and so get

$$\left(1 - \frac{1}{88}\right)^{50} \approx \left(1 - \frac{0 \cdot 568}{50}\right)^{50} \approx e^{-0 \cdot 568} \approx 0 \cdot 567.$$

On this approximation, the chances of the jumper's death are $1 - 0 \cdot 567$ or 43·3%, very close to the more exact 43·5%.

THE RAINBOW SERIES

The following equalities may readily be checked.

$$1 - \frac{1}{2} = \frac{1}{2}$$

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} = \frac{1}{3} + \frac{1}{4}$$

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} = \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$$

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} = \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} .$$

They are instances of a general result that is not very difficult to prove, and which we leave as an exercise to the reader, namely

$$1 - \frac{1}{2} + \frac{1}{3} - \ldots - \frac{1}{2n} = \frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{2n} .$$

Now it is known that as $n \to \infty$, the left-hand side of this expression tends to $\log_e 2$, i.e. the natural logarithm of 2. It thus follows that

$$\lim_{n\to\infty}\left(\frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{2n}\right) = \log_e 2. \qquad (*)$$

The series on the left differs from more usual series in that as n gets larger, not only does the end of the series tend to disappear "over the horizon" as it were, but the start also.

This was published in a pamphlet entitled *The Rainbow Series and the Logarithm of Aleph Null* by W.P. Montague in 1940. (The result was almost certainly known three centuries before this, but evidently Montague rediscovered it.) Montague was a professor of Philosophy at Barnard College, U.S.A., and an amateur mathematician. The name "rainbow series" is his term for the left-hand side of Equation (*), so named because, like the rainbow, both its ends lie over the horizon.

Regrettably, Professor Montague's learning was deficient in many things. Rainbows do not have the character attributed to them and the left-hand side of Equation (*) only disappears if one attempts actually to put $n = \infty$ there. The later developments, in which Professor Montague "discovers" a new class of infinite numbers ("the pygmy members of a giant race") are quite wrong. (For a correct account of infinite numbers, see *Function*, Vol.2, Parts 1,2.) However, I learned of Equation (*) through his pamphlet and like the name he attached to it.

M.D.

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The formula for the Latin square (p.2): entry in row i, column j is $(j + 4 - 3i) \pmod 7$.

ICME 5

The next International Congress on Mathematics Education (of which there have been four held so far) is to take place in Australia. These conferences are organised by a body known as the International Commission on Mathematical Instruction. This body was invited by the Australian Academy of Science to hold the 1984 conference at the University of Adelaide. Chairman of the International Program Committee for the congress is Dr Michael Newman of the Australian National University. Chairman of the National Organising Committee is Dr John Mack of the University of Sydney (and one of *Function*'s editors). There are four other committees, whose work, along with that of the two mentioned, culminates in the large congress of 24 – 30 August, 1984.

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UPDATE ON GALOIS

Function, Vol.3, Part 2 carried a story on the life of the brilliant mathematician Evariste Galois. Much more is now known about Galois' life. Recent articles in *Scientific American* (November 1981, April 1982) and in *American Mathematical Monthly* (February 1982) give a lot more detail than was known even a few years ago. Galois' tragic life has been much mythologised, but Dr Lausch, author of the *Function* article, omitted the fictional elements that have accrued around Galois' life. The lengthy accounts mentioned above tell a fuller story and could be read by *Function*'s readers without difficulty.

Some people hold that fictional lives have a place in the history of mathematics. The above articles show that the truth is preferable.

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A STRANGE PATTERN

It is usual to ask students to memorize the exact values of the sines of certain angles - namely 0°, 30°, 45°, 60°, 90°. Professor Moss E. Sweedler of Cornell University points out that the values are respectively:

$$\frac{\sqrt{0}}{2}, \frac{\sqrt{1}}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}, \frac{\sqrt{4}}{2}.$$

(*American Mathematical Monthly*, Jan. 1982.)

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OUR OLYMPIANS

This year, due to restrictions by the host country (Hungary), we can only send four competitors to the International Mathematical Olympiad. Last year we sent eight, one of whom won a Bronze Medal.

The photograph above shows the 1982 team. From left, they are: Mr J.L. Williams (Team leader), Alan Blair (15) of Sydney Grammar School (N.S.W.), Ken Ross (17) of Mt Scopus Memorial College (Vic.), David Chalmers (16) of Unley High School (S.A.), Dirk Verlegen (16) of Elizabeth Matriculation College (Tas.), Professor G. Szekeres (Deputy Leader). (Photo. courtesy P.O'Halloran, Australian Mathematics Olympiad Committee.)

Best wishes to our team.

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99·9999 9999% CERTAIN

To check the laws of arithmetic, perform this computation on your calculator

$$(1 \div 3) \times 3.$$

This should convince you that the 3's cancel.

(*American Mathematical Monthly* Jan., 1982, from Professor Moss Sweedler of Cornell University.)

MONASH SCHOOLS' MATHEMATICS LECTURES, 1982

Monash University Mathematics Department invites secondary school students studying mathematics, particularly those in years 11 and 12 (H.S.C.) to a series of lectures on mathematical topics.

The lectures are free, and open also to teachers and parents accompanying students. Each lecture will last for approximately one hour and will not assume attendance at other lectures in the series.

Location: Monash University, Rotunda Lecture Theatre R1. The Rotunda shares a common entry foyer with the Alexander Theatre. For further directions, please enquire at the Gatehouse in the main entrance of Monash in Wellington Road, Clayton. Parking is possible in any car park at Monash.

Time: Friday evenings as below; 7.00 p.m. to 8.00 p.m. (approx.).

Program: The remaining talks are:

July 2 "Chaos - Fluctuations in Populations". Dr G.A. Watterson.

July 16 "Formation of the Solar System". Dr A.J. Prentice.

July 30 "Two Circles Intersect at Four Points!". Dr C.F. Moppert.

Enrolment: There will be no enrolment formalities or fees. Just come along!

Further Information: Dr C.B.G. McIntosh: (03) 541 2607