

Mathematics

# What is the best way to lace your shoes?

The two most popular ways to lace shoes have historically been to use ‘criss-cross’ or ‘straight’ lacing — but are these the most efficient? Here we demonstrate mathematically that the shortest lacing is neither of these, but instead is a rarely used and unexpected type of lacing known as ‘bowtie’ lacing. However, the traditional favourite lacings are still the strongest.

The  $2n$  eyelets of an idealized shoe are the points of intersection of two vertical lines and  $n$  equally spaced horizontal lines in the plane. The two columns of eyelets are one unit apart, and two adjacent rows of eyelets are a distance  $h$  apart. An  $n$ -lacing of our shoe is a closed path in the plane that consists of  $2n$  line segments, the end points of which are the  $2n$  eyelets.

For any given eyelet, we require that at least one of the two segments that ends in it should not be contained in the same column as that eyelet; this condition ensures that every eyelet genuinely contributes towards pulling the two sides of the shoe together. Virtually all lacings that are actually used satisfy this condition.

We call a lacing ‘dense’ if neither of the

two segments ending in any eyelet is contained in the same column as the eyelet — that is, a dense lacing zigzags back and forth between the two columns of eyelets as, for example, do the traditional lacings (Fig. 1a, b). Finally, we assume that  $n$  is at least 2.

Using standard combinatorial techniques, we find that the number of  $n$ -lacings is

$$\frac{(n!)^2}{2} \sum_{k=0}^m \frac{1}{n-k} \binom{n-k}{k}^2$$

where  $m = n/2$  for even  $n$ , and  $m = (n-1)/2$  for odd  $n$ . The number of dense  $n$ -lacings is

$$\frac{n!(n-1)!}{2}$$

The length of an  $n$ -lacing is the sum of the lengths of the segments that it consists of. Using the symmetries of the configuration of eyelets, it is possible to design a powerful list of local shortening rules and to use these to identify the bow-tie  $n$ -lacings as the shortest  $n$ -lacings (Fig. 1c–e). Furthermore, by generalizing earlier results<sup>1–4</sup>, we can show that the criss-cross  $n$ -lacing is the shortest dense  $n$ -lacing, even if the eyelets are not fully aligned. Note that it is also possible to identify the longest dense  $n$ -lacings for general  $n$ .

When you pull on the ends of a shoelace, it acts like a pulley. Ideally, the tension along the shoelace is a positive constant,  $T$ . This tension gives rise to a total tension,  $T_{\text{hor}}$ , of the pulley in the horizontal direction; that is, the direction in which the two sides of the shoe are being pulled together. This total tension,  $T_{\text{hor}}$ , is the sum of all horizontal components of  $T$  along the different segments of the lacing. The strongest  $n$ -lacings are then  $n$ -lacings that maximize  $T_{\text{hor}}$ .

The unique dense 2-lacing is also the strongest 2-lacing. Note that the shortest  $n$ -lacing is independent of the distance  $h$  between two adjacent rows of eyelets. In contrast, for  $n > 2$ , the strongest  $n$ -lacing does depend on  $h$ . We can show that there is a positive value,  $h_n$ , such that the strongest  $n$ -lacings are: the criss-cross  $n$ -lacing, for  $h < h_n$ ; the criss-cross  $n$ -lacing and the straight  $n$ -lacings, for  $h = h_n$ ; and the straight  $n$ -lacings, for  $h > h_n$ .

For many real shoes with  $n$  pairs of eyelets, the ratio of the distance between adjacent rows of eyelets and the distance between the columns is very close to  $h_n$ . This means that no matter whether you prefer to lace them straight or criss-crossed, you come close to maximizing the total horizontal tension when you pull on the two ends of one of your shoelaces.

And what is the strongest way to tie your shoelaces?<sup>5</sup> Most people place one half-granny knot on top of another (it is not

essential to consider the loops here), which results in either a notoriously unstable granny knot or a very stable reef knot, depending on whether the two half-knots have the same or opposite orientation. As we have seen, hundreds of years of trial and error have led to the strongest way of lacing our shoes, but unfortunately the same cannot be said about the way in which most of us tie our shoelaces — with a granny knot.

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COMMUNICATIONS ARISING

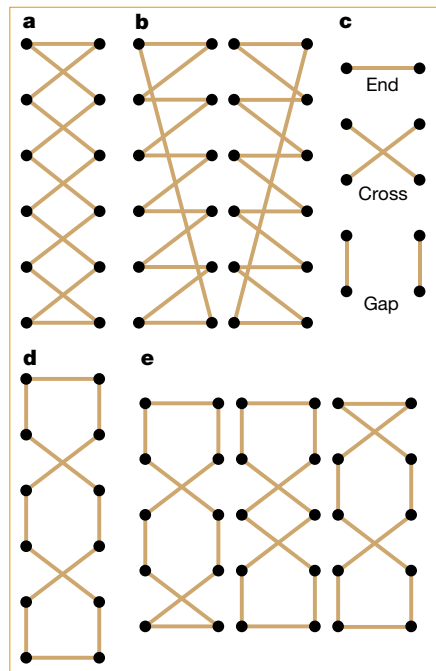
Laser-Raman spectroscopy

## Images of the Earth’s earliest fossils?

Fossil remains of the most ancient, minute forms of life on Earth and other planets are hard to recognize. Schopf *et al.*<sup>1</sup> claim to have identified the biological remnant material known as kerogen in microscopic entities in rock by using Raman spectroscopic analysis. On the basis of a substantial body of published evidence, however, we contend that the Raman spectra of Schopf *et al.*<sup>1</sup> indicate that these are disordered carbonaceous materials of indeterminate origin. We maintain that Raman spectroscopy cannot be used to identify microfossils unambiguously, although it is a useful technique for pinpointing promising microscopic entities for further investigation.

We believe that Schopf *et al.* have over-interpreted their Raman spectra in attributing biogenicity to the extremely ancient, fossil-like objects that they analysed, as already addressed by Brasier *et al.*<sup>2</sup> We disagree with the underlying assumption by Schopf *et al.* that Raman spectroscopy is sensitive to a distinctive carbon signal of organic matter (kerogen), and with their conclusion that “measurement of Raman point spectra (Fig. 3) [together with optical microscopy and Raman imaging] substantiates the biological origin of the oldest putative fossils”.

Contrary to the inference by Schopf *et al.*<sup>1</sup>, laser-Raman microprobe spectroscopy does not reveal the chemical composition (as defined by geochemists) of a sample, but rather provides information about the molecular bonds of the constituent structural units. For instance, it can distinguish between carbon bonds in carbonate, in condensed benzene rings, in graphite and



**Figure 1** The strongest and the shortest shoe lacings. **a, b**, The most popular  $n$ -lacings, the criss-cross  $n$ -lacing (**a**) and the two straight  $n$ -lacings (**b**) are also the strongest  $n$ -lacings (here  $n$  is the number of pairs of eyelets). **c–e**, The shortest  $n$ -lacings are the bow-tie  $n$ -lacings. They are made up of ends, gaps and crosses (**c**). If  $n$  is even, there is exactly one bow-tie  $n$ -lacing (**d**) consisting of the two ends at the top and bottom,  $n/2$  gaps and  $n/2 - 1$  crosses. If  $n$  is odd, there are exactly  $(n+1)/2$  different bow-tie  $n$ -lacings (**e**) consisting of the two ends,  $(n-1)/2$  gaps and  $(n-1)/2$  crosses.