

THE DEATH OF REASON

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Mathematics education in Victoria is in poor shape. Such criticism is common, and hardly contentious. But it is not generally recognised how deep, how fundamental are the problems. There is a pervasive misunderstanding of the very nature of mathematics and, therefore, how we might teach it, or why we should even bother.

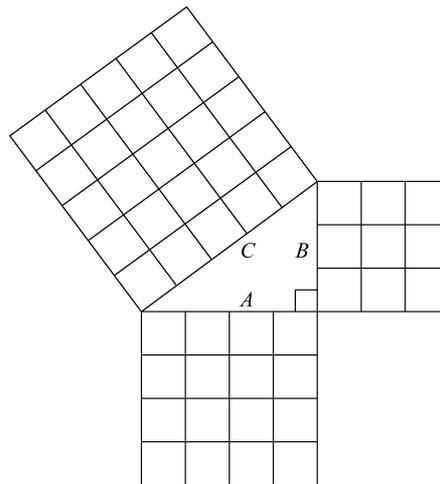
Questioning the worth of teaching mathematics may appear to be blasphemy, the slighting of a critical life skill. Sure, mathematics is one of the three R's, and it is true that we all need some understanding of 'Rithmetic, of percentages and the like. But, glory be to calculators, little is needed beyond that which is taught in primary school: just enough to know which buttons to push and to be able to interpret the resultant answers. Calculators are indeed a blessing for the performance of practical arithmetic, for simply getting the answer. Arithmetic aside, secondary school mathematics is impractical. Ask yourself, when was the last time that you made any use of geometry, or algebra, or trigonometry, or calculus, or even of probability or statistics (although an appreciation of the last two would be prudent if you frequent the Crown Casino).

Why teach this useless mathematics? A common answer is that it is preparation for tertiary education and beyond, training for those students with some interest in technical issues or thoughts of entering a technical profession. It is a weak answer: the percentage of secondary school students with any thought of these professions is quite small; and, even for those students, the advent of computers and mathematical software has dramatically diminished the amount of mathematical knowledge needed in almost every such profession. The fact is, the vast majority of people has no need for the vast majority of secondary school mathematics, in either a personal or professional capacity.

Clearly the purpose for teaching mathematics cannot be primarily to convey facts and formulas which are rarely used and almost immediately forgotten. The true purpose is to teach the *reasoning* by which these facts can be established. The reasoning coheres the knowledge, giving one the ability to recall it later, and expand upon it, if need be. But more than

that, it is the training in logical thinking, learning to reason about *anything*, which can be the genuine reward for studying mathematics.

To illustrate how mathematics can be approached in this manner, and how sadly lacking is current practice, consider the following example, the Pythagorean Theorem. You may remember little of this other than, perhaps, that it has something to do with triangles, or that it is accompanied by the mantra "A squared plus B squared equals C squared".



The diagram indicates what is intended: since A^2 (i.e. A times A) is the *area* of a square with side-length A , the Pythagorean Theorem states that for a right-angled triangle, the areas of the two smaller squares add up to the area of the large square. In a succinct formula,

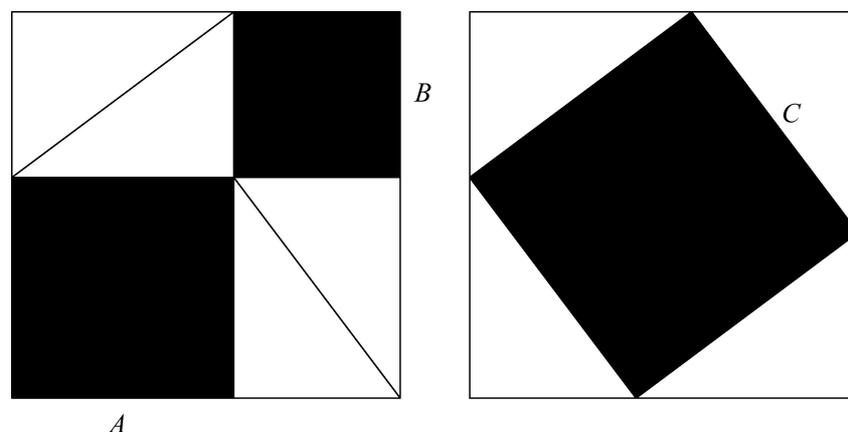
$$A^2 + B^2 = C^2 .$$

Why teach the Pythagorean Theorem? Ironically, since it is seldom presented as such, it is one of the few pieces of secondary school mathematics for which you might find an everyday use. If you are landscaping your garden then the easy way to create accurate, perpendicular lines is to take a 12 meter piece of rope and to stretch it into a 3-4-5 triangle: the Pythagorean Theorem (well, its converse) guarantees that the large angle will be a right-angle. However, the fundamental reason to teach the Pythagorean Theorem is because of its central role in Euclidean geometry, that monumental body of work immortalised in Euclid's *Elements*.

The *Elements* is the most successful textbook of all time, for centuries at the heart of a Western mathematical education. However, its popularity was not because of some universal love of geometry: triangles and their

kin have always been of limited interest and use to the general student. The real lesson was the process by which these geometric truths were obtained. The *Elements* is a brilliant, extended display of reasoning, beginning with a small number of accepted truths and *proving* all that follows, creating an immense structure of implied certainty. The Pythagorean Theorem is a critical part of that structure: as declared by its name, it is a *theorem*, and as such it can be proved.

Words such as “theorem” and “proof” can be off-putting, but here is a famous, beautiful demonstration of the Pythagorean Theorem. In the diagrams below we have pictured two ways of filling the same square:



Summing the areas within each square, we see that

$$A\text{-square} + B\text{-square} + \text{four triangles} = C\text{-square} + \text{four triangles.}$$

“Subtracting” the four triangles, we have proved our theorem!

The above argument can be made tangible by using paper and scissors to construct specific figures, but notice that the argument applies to *any* right-angled triangle. It is an archetype of mathematical reasoning: from our basic understanding of angles and triangles and squares, the Pythagorean Theorem follows as a logical consequence. The clarity and universality of its statement, and the simplicity of its proof, are surely stunning.

One would hope that mathematics was being taught with emphasis upon its meaning and its beauty. Sadly, in Victoria, this is seemingly not the case. The current guide to the teaching of mathematics up to Year 10 is the Curriculum and Studies Framework II, as guided by the Victorian Essential Learning Standards. It is a fact-dominated approach, with a clear push to use “technology” in the teaching of every topic: this is very good for showing the students *what* is true, but not *why* it is true. The

current VCAA plan to incorporate sophisticated CAS-calculators will make things much worse, dramatically increasing the territory of these educational cane toads.

The Pythagorean Theorem is the only theorem that I could find referred to by name in either CSFII or VELS (one would have thought that at least the Fundamental Theorem of Arithmetic was, well, fundamental), and it is treated poorly. The clear emphasis is upon the use of the Theorem, with its proof being at best an afterthought. More generally, VELS seems to make absolutely no mention of the goal of proving theorems: it only refers to the “consideration of *evidence* to support theorems” (emphasis added). This may suggest new exercises for those expensive calculators, but it misses the point horribly.

The contempt of the Euclidean tradition displayed in the curriculum is faithfully reflected in Victorian textbooks. Some are better than others, but none that I have seen are good. (Most have now adapted to the VELS by making a few cosmetic changes, and by incorporating meaningless motherhood statements in the teachers’ versions). Consider one Year 9 text, not the worst, published recently and staidly written with the CSF II guidelines in mind. It contains two exercises amounting to proofs of the Pythagorean Theorem, but with no indication that this is in fact their purpose, no indication that these are anything other than two more dull exercises. The lack of signposting indicates well enough the writers’ lethargic interest, but there is stronger evidence: to get from the statement of the Theorem to these hidden proofs, the long-suffering student must wade through a swamp of plug-in-the-formula exercises involving 163 triangles. This is fantastically boring, and it sends a clear message: facts matter and reasoning does not.

Undoubtedly, in spite of these poor guidelines and materials, there are teachers who present mathematics with purpose and integrity. But, for those of us trying to teach mathematics at university, the general effect is evident. The students we see, presumably the stronger students and the ones for whom this material is most relevant, enter university viewing mathematics as no more than a collection of facts, facts upon which they have the most tenuous grasp. They arrive with little appreciation of mathematics, with little understanding of how to reason mathematically, and with no real understanding that this is even a possibility and that there is something lacking in their approach. I doubt that their respect for reasoning in other disciplines is significantly higher.

Albert Einstein said: “Any fool can *know*. The point is to *understand*.” To treat mathematics simply as knowledge is to forsake the teaching of reasoning. This presents mathematics as boring and ugly and pointless, something to be endured rather than appreciated. Foolish, indeed.