Review of Victorian Year 9 Mathematics Texts

MathsWorld Year 9
Macmillan, 2006
Maths Zone Year 9
Heinemann, 2008
Essential Mathematics Year 9
Cambridge, 2007
Maths Quest Year 9
Jacaranda, 2006
Mathematics 3A and 3B
ICE-EM, 2007 and 2008
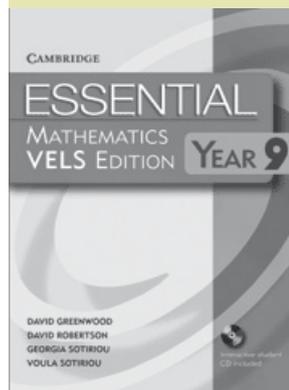
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1. INTRODUCTION

In any Victorian school, the mathematics coordinator will determine the texts to prescribe. They must do so with no official guidance, with no accreditation or review process having approved or ranked or critiqued the competing texts. This is a regrettable state of affairs: if the belief is that market forces will ensure a range of quality texts, then we do not share this belief. We feel the currently available texts are strong evidence to the contrary.

We offer here a comparative review of the Victorian Year 9 mathematics texts, and the national *ICE-EM* texts [International Centre of Excellence for Education in Mathematics]. This review is necessarily brief, and we do not pretend that it can replace a proper accreditation process. But we hope that by presenting this review we can alert teachers both to the occasionally clear differences in quality between the texts, and to some of their common failings.

Our approach is to consider a selection of topics, and to summarise the approach of each text. We then conclude with some observations, and some recommendations for those selecting a text, to write one, or to approve one.

Some initial remarks are called for. First, what we have written below of the Victorian texts is substantially negative. We do not shy away from this. We are simply of the opinion that *all* of the Victorian Year 9 texts have serious and systemic problems.

There is then the question of fairness. We have been faced with the choice of either writing generally about the texts, or of selecting specific examples to critique. For definiteness, we have tended towards the latter, which then leaves us open to charges of bad faith cherry-picking. To any such charge, we offer two defences. Firstly, it is easy to demonstrate that the examples chosen are representative of much more common failings: we have left many more cherries to be picked. Secondly, we note that these texts have gone through many editions, and presumably through many revisions: as such, we feel there is no excuse for the existence of any such cherries.

2A. IRRATIONAL NUMBERS

Irrationality is a difficult and unavoidable concept, making it an excellent measure of the integrity of a text. All the Victorian texts handle the topic poorly. *Maths Quest* and *Maths Zone* have almost nothing, seemingly hoping the topic will just go away. *Essential Mathematics* has little more, though it does express clearly that the real numbers are split exactly into the rational and irrational; but, they then absurdly suggest that a calculator can be used to determine the irrationality of numbers.

MathsWorld does better. They have a section devoted to irrationality and the algebra of surds. They emphasise that rational approximations are

Calculators can only approximate and conceal irrationality of numbers

22/7 is a rational number, and approximates irrational π

just that. And, they make clear that rational approximations are the way to get a handle on irrationals. There is some nicely presented stuff here. But, there are also clangers: their Venn diagram of the real numbers is vague, and wrong; they ask the student to determine the irrationality of $\sqrt{10}$ without a hint of guidance; and, they suggest using a calculator to see that $\sqrt{3}$ and $3^{\frac{1}{2}}$ are "equivalent". This last is truly appalling, for the use of the calculator, for the needlessly nervous use of "equivalent", and above all for the logical obtuseness: the two are *equal* because $3^{\frac{1}{2}}$ is defined that way.

ICE-EM does much better. They naturally introduce surds using Pythagoras's Theorem, and later do the algebra of surds. They note that π is irrational, *and* that this is hard to prove. They have the courage to cover the material, and they do it clanger-free.

Most strikingly, *ICE-EM* includes a proof of the irrationality of $\sqrt{2}$. The presentation of the proof could have been very much better, with more geometric motivation, but here this is nitpicking. To simply include any proof is way beyond the other texts, which don't even say a correct word about how we *know* anything is irrational. *ICE-EM* is the only text which presents irrationality as something to be investigated, rather than merely accepted.

2B. ARITHMETIC

Prime numbers and integer factorisation essentially disappear after year 8. This is regrettable, since the subject is so alive, and can easily be shown to be alive, with a bounty of accessible and engaging material. It is also lays the proper foundation for factorisation of polynomials, introduced without motivation in all Victorian texts.

Probably the texts are not responsible for the lack of number theory; more likely, the culprit is the syllabus, with the later emphasis upon calculus, and the everywhere de-emphasis of proof. But, the Victorian texts seldom miss a chance to miss a chance: we cannot find a single interesting word about prime numbers. For example, with prime factorization one can state easily and exactly which roots will be rational: but of this, seemingly not a word.

If primes are missing, indices are a mess. All the Victorian texts state the index laws for "numbers", without any concern for whether the base is positive, or whether the exponent is a natural number. So, the Index Laws are stated in broad (and false) generality, but the motivating examples are always the obvious cases. These purported Laws are then used to "prove" identities such as $a^{-3} = \frac{1}{a^3}$. (True to form, *MathsWorld* merely claims the two forms are "equivalent").

This is all logically backwards. In reality, we *define* negative powers as reciprocals, exactly so the Index Laws for natural number exponents (which *are* easy) remain true. Beyond the logical absurdity, the texts' sloppiness leads to some outright falsehoods: "any number raised to the power of zero equals one" (*Maths Zone*); $\sqrt{h^2} = h$ (*Maths Quest*).

MathsWorld's treatment is better, but, again, *ICE-EM* is way ahead in scope and presentation. All definitions are motivated by ensuring that the index laws remain valid, and there is an excellent introduction to rational exponents. The validity of the Laws is even set as a challenge exercise.

2C. ALGEBRA

Essential Mathematics, *Maths Quest* and *Maths Zone* all have a poor discussion of algebra. This is exemplified by their treatment of the critical expression $(a + b)(c + d)$. The expansion of this expression is *not* obvious, but *Essential Mathematics* simply states the formula, without a word of justification. And, neither *Essential Mathematics* nor *Maths Quest* gives the natural geometric motivation of the formula using rectangular areas.
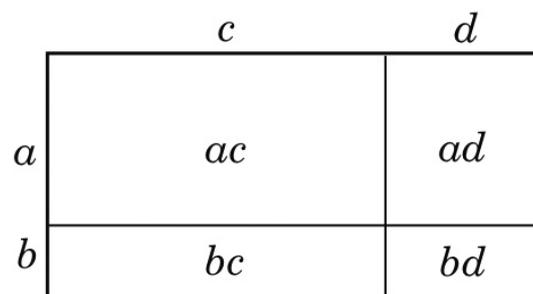
*Simple geometric motivation for the expansion of $(a + b)(c + d)$*

What diagram is a model for $(a + b)(c + d)(e + f)$

By contrast, *ICE-EM* and *Mathsworld* motivate standard expansion and factorisation formulae by means of elegant cut-and-paste illustrations. *Maths Zone* attempts to do the same, but they are less successful.

Finally, *Maths Zone* compounds the sin of confusion with the sin of absurdity. All the Victorian texts suffer from the burden of contrived and boring problems, but *Maths Zone* wins the Gold and Silver medals: we never thought to see a shop display $x^2 + 2x - 48$ types of cheese; and, a farmer may well have $2c$ cows and $3h$ horses, but why he might want to find the square of the sum of his animals is simply beyond us.

2D. PYTHAGORAS

Each of the texts has a chapter on Pythagoras's Theorem, but we feel none of the treatments are satisfactory. None emphasise the blunt and clear and essential message that a theorem is only a theorem because it is *proved* to be. (Although *ICE-EM* treats the Theorem with respect in its Year 8 text, the proof warrants a proper review in Year 9). In the place of proof and reason, the Victorian texts have seemingly endless applications of Pythagoras, the majority contrived or boring.
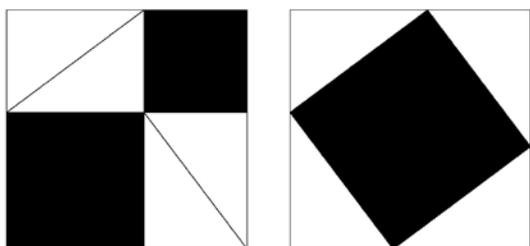
Pythagoras proved: the two smaller squares + four triangles = the large square + four triangles.

We simply cannot understand why any text would fail to begin with, accompanied by glowing words, the famous, simple and gorgeous windmill proof. *MathsWorld* gives this proof early on, though in a less than elegant manner, and labelled belittlingly as a "Try This" exercise. *Maths Zone* gives a much better presentation, and also gives the harder Perigal proof, though too late in the piece.

Essential Mathematics begins their chapter with Perigal's proof, but it is labelled only as an "illustration", and the presentation is confusing. Much worse, they end the chapter with a labelled "proof", using a geometry package to measure select triangles. This is an awful use of technology, and a fundamental misrepresentation of the idea of proof.

Maths Quest begins with the tedious measuring of particular triangles, followed by Perigal as a paper and scissors exercise. We have no idea what these specific computations are meant to achieve. *Maths Quest*, *Essential Mathematics* and *ICE-EM* all apply Pythagoras in 3D, but no one clearly states it or proves it.

2E. GEOMETRY

Similarity and congruence are treated confusingly by all the Victorian texts. *Maths Zone* contains only one brief and seemingly irrelevant section on similarity of triangles. *Essential Mathematics* contains no definition of similarity, only providing a collection of tests, valid only for triangles; turning to the glossary, one finds that two shapes are similar if they have the "same shape, but different size": in an attempt to be helpful, they end up being pointlessly vague, and wrong (since a shape is of course similar to itself). *Maths Quest* exhibits the same vagueness, of "identical shape, different size", and claiming nervously that congruent figures "often" result from reflections, rotations or translations.

Similarity and congruence can only be explained properly with some reference to transformations. This is the clear and correct approach taken by *ICE-EM*. The geometry chapters feature a detailed introduction to congruence, enlargements and similarity. This treatment includes clear and correct definitions and interesting exercises.

None of the Victorian texts seem to recognize that π is not given by God, but actually arises from similarity. It is just not obvious that circumference/diameter is the same for every circle. In fact, circles are approximated by polygons, and similarity tells you how dilating a polygon will change its perimeter. Here, then, is an excellent opportunity to engage the students

> Pythagoras's Theorem is a keystone in school mathematics, and deserves a clear proof: a dynamic generalisable dissection of a square is easy!

Similarity arises through ideas of scale factors for lengths, and comparability of angles

with Archimedes, and the centrality of his idea of approximating circles by polygons. The opportunity has been missed.
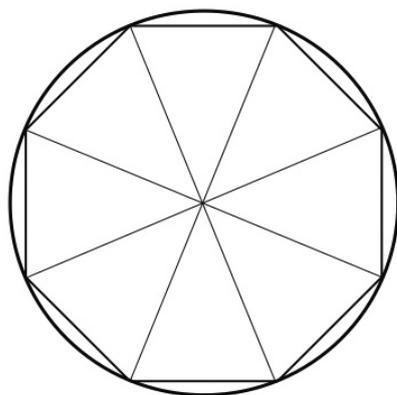
Polgyons, as Archimedes' key to understanding circles, and thus π.

2F. TRIGONOMETRY

The basis of trigonometry is similarity. Given the above, this means we're in trouble. *MathsWorld* and *ICE-EM* make the crucial connection between similarity of triangles and the trigonometric ratios, but the other texts omit this justification: trigonometry is reduced to SOHCAHTOA. The values of sin, cos and tan then magically appear from the calculator, without a word that somehow, somewhere, someone needs to have calculated these values.

ICE-EM and *Mathsworld* take the opportunity to link topics, using Pythagoras's Theorem to establish trigonometric ratios for the special angles, 30°, 45° and 60°. *Essential Mathematics* does the same, but less comprehensively. Neither *Maths Quest* nor *Maths Zone* make the effort. In place of serious exploration and linking of important ideas, students are presented with a billion right triangles for which they are asked to find x. The application questions are contrived or dull, or both. *Maths Zone* asks "What angle does the seesaw make with the ground?": we ask in response, who on Earth cares?

3. STYLE

There was never a better illustration of McLuhan's "The Medium is the Message" than the current crop of Victorian texts (*Essential Mathematics*

and, to a lesser extent, *MathsWorld* excepted). The medium is page after page of garishness, cartoons and colour and arrows and thought balloons, and on and on. The message of all this noise seems to be that the actual mathematics is too dull to be presented openly and honestly.

We imagine that the publishers justify this clumsy pandering by reference to the needs and demands of Generation Pick-Your-Letter. We don't believe it. And, we don't believe there is a student in the State who is fooled by what's being offered into liking mathematics more.

Such styling is not merely ugly, it is a pedagogical own-goal. When every page is screaming for attention, careful emphasis is impossible. No room remains for clear chapter and sub-chapter titles, or for clear highlighting of critical formulas and examples and arguments.

Essential Mathematics is a notable exception. The presentation is clear and uncluttered, and pleasing to the eye. We can only regret that *Essential Mathematics* tends to be weaker in actual content than the other texts.

By comparison, *ICE-EM* seriously errs in the opposite direction. By worrying too much about integrity, *ICE-EM* is unnecessarily grey and humourless. There is a proper role for diagrams and colour, and cartoons. It is critical to present mathematics correctly, but it is also critical to present it as beautiful and fun. The inclusion of lightness and emotion and humour is a good thing, as long as it is used in moderation, to enhance the mathematics, and to clarify and give life to the mathematics. *ICE-EM* would have benefited greatly by not having taken itself so completely seriously. But, if *ICE-EM* is too Radio National, it is still infinitely preferable to the Las Vegas alternative. [We don't have space here but an interesting comparison can be made with the Pearson Longman texts prepared for Singaporean schools. These are stylishly lighter and more fun than *ICE-EM*, while still being mathematically quite solid.]

4. CONCLUSION

In a famous essay, the great mathematician and educator Georg Polya wrote:

Necessarily, strong understanding of similarity underpins conceptual understanding of trigonometry

Mathematics is, or should be, something to THINK about

I am concerned here with mathematics in the high school curriculum and I have an old fashioned idea about its aim: first and foremost, it should teach those young people to THINK.

We agree wholeheartedly with Polya, and consequently our advice to teachers is obvious: use *ICE-EM*. We feel that is the only available text that consistently treats mathematics as something to THINK about, as something other than a lifeless list of God-given rules. It is mathematically literate. [If you absolutely must choose a Victorian text, then we would hesitantly recommended *MathsWorld*: it appears to have more material presented in an honestly mathematical fashion, and it does less wrong, than its competitors.]

Some may be concerned that *ICE-EM* does not incorporate the use of calculators, let alone CAS. Of course, this is no coincidence, as *ICE-EM* refuses to be distracted by this fad. In any case, we regard this as a non-issue. First of all, we are not at all convinced of the pedagogical value of any of the calculator activities and exercises in the Victorian texts: they mostly seem to be spooned on top, are usually distracting, and at best seem pointless and boring. Secondly, students pick up calculator and CAS skills easily, and if such material is required or demanded, it can easily be covered as a supplement to the main text. [We hold out hope that VCAT will reverse its shameful decision to force CAS upon Victorian schools.]

We also have some predictable but critical advice for the authors and publishers of the Victorian texts: hire a mathematician. What is most striking about the Victorian texts is not the specific clangers of the type we have indicated, but that generally these texts give no global sense of mathematics or mathematical thinking: topics are seldom linked; exercises are seldom inventive or challenging; definitions are often tied in logical knots; the history is seldom interesting or accurate. The Victorian texts simply do not reflect or offer a mathematical culture. We feel the current text authors just do not demonstrate the mathematical background to produce a mathematics text unassisted. Thus, we feel Victorian publishers simply must acquire the services of good, concerned and attentive mathematicians (yes, they do exist).

Finally, we have advice, a plea, for the Powers that Be: please, stop shirking your responsibility, and institute some form of accreditation or review process for Victorian texts. Until Victorian publishers acknowledge, or are forced to acknowledge, the poor state of their texts, there can be no improvement. And until then, we feel there is simply no reasonable alternative: use *ICE-EM*.

References

Polya, G. (1963). On Learning, Teaching, and Learning Teaching, *American Mathematical Monthly*, 70, 605-619.

Meet the Reviewers

Marty Ross is a mathematician, who has lectured at Melbourne, Monash and La Trobe Universities. David Treeby is a teacher at Ivanhoe Girls Grammar School, and a member of the MAV Council. Neither author has had any role in the preparation of or marketing of any of the reviewed texts.

While calculators, CAS software and other digital devices may provide speed and efficiency, they provide a poor avenue to understanding and proof.