

Drawing Homer with epicycles (July 6, 2018)

This notebook is really just a tweak of the following contribution by “anderstood” on the Mathematica stackexchange:

<https://mathematica.stackexchange.com/questions/171755/how-can-i-draw-a-homer-with-epicycloids>

It supplements a discussion my Mathologer video “Epicycles, complex Fourier and Homer Simpson’s orbit”.

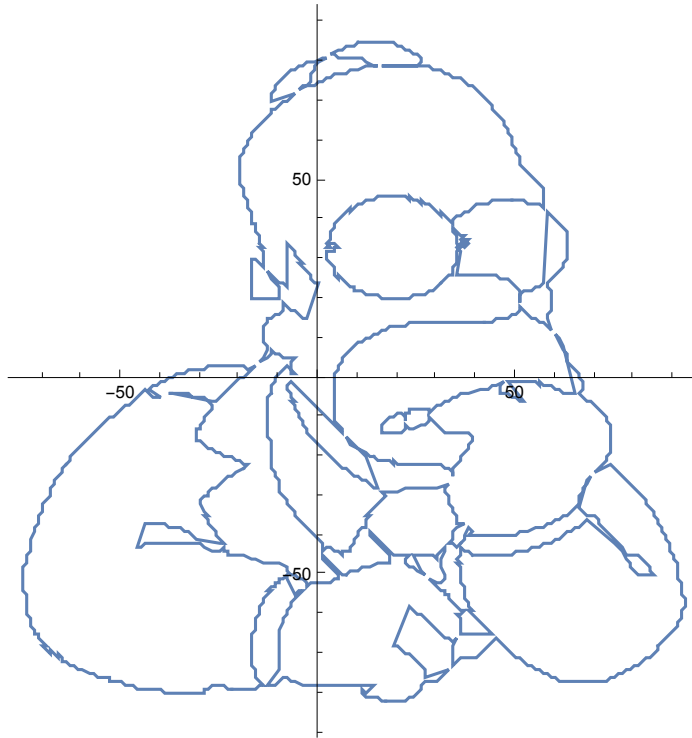
Here we go. First we read in the single pixel outline of Homer that I mention in the video and make a list of the coordinates of the black pixels in this picture (pts is this list).

```
img = Import["http://www.qedcat.com/misc/homer1_clean_pixel.png"];  
pts = PixelValuePositions[img, Black, 0.05];
```



We shift the coordinates so that the origin of our coordinate system comes to rest roughly in the centre of the image. Then we order the list by creating a (close to shortest) roundtrip through the points using the command **FindShortestTour**. The rest of the file works exactly as described by anderstood (except I also fixed up the wrongly placed centres of the epicycles).

```
pts = Map[{-90, -90} + # &, pts];  
elephantPlot = ListPlot[pts, AspectRatio → Automatic];  
shortest = Last@FindShortestTour@pts;  
pts = pts[[shortest]];  
ListLinePlot[pts, AspectRatio → Automatic]
```

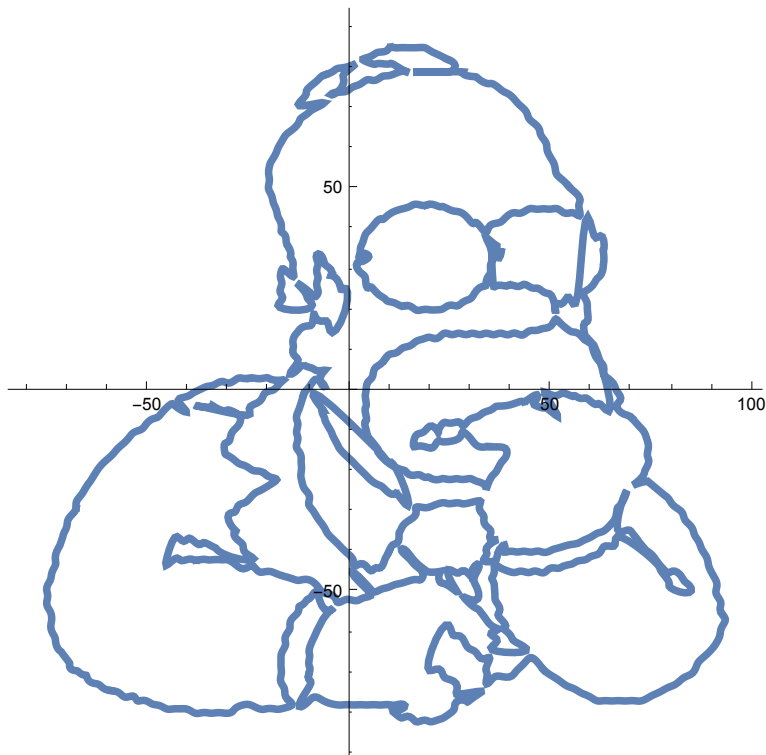


The following is a preview of what 900 epicycles will eventually draw.

```

SetAttributes[toPt, Listable]
toPt[z_] := ComplexExpand[{Re@z, Im@z}] // Chop;
cf = Compile[{{z, _Complex, 1}}, Module[{n = Length@z},
  1 / n * Table[Sum[z[[k]] * Exp[-I * i * k * 2 Pi / n], {k, 1, n}], {i, -m, m}]];
z = pts[[All, 1]] + I * pts[[All, 2]];
m = 450
cn = cf[z];
{f[t_], g[t_]} = Sum[cn[[j]] * Exp[I * (j - m - 1) * t], {j, 1, 2 m + 1}] // toPt;
ParametricPlot[{f[t], g[t]}, {t, 0, 2 Pi},
  PlotStyle -> Thickness[0.01], AspectRatio -> Automatic]
450

```



Here we go. I actually only include a preview of the animation in this notebook to keep the file size reasonably small.

If you've got access to Mathematica just generate the animation yourself. If your computer is struggling with

900 epicycles, just lower the $m=450$ above to something smaller. Apart from that, if you are just interested in making a silhouette just

use *anderstood's* code together with my center position fix (spot the difference :)

```

r = Abs /@ cn;
theta = Arg /@ cn;
index = {m + 1} ~Join~ Riffle[Range[m + 2, 2 m + 1], Reverse[Range[1, m]]];
p[t_] = Accumulate@Table[cn[[j]] * Exp[I * (j - m - 1) * t], {j, index}] // toPt;
circles[t_] = Table[Circle[p[t][[i - 1]], r[[index[[i]]]]], {i, 1, 2 m + 1};
anims =
  ParallelTable[ParametricPlot[{f[s], g[s]}, {s, 0, t}, AspectRatio -> Automatic,
    Axes -> {False, False}, Epilog -> {circles[t][[2 ;;]], Line[p[t]], Point[p[t]]},
    PlotRange -> {{-100, 100}, {-100, 100}}, ImageSize -> 500],
    {t, Subdivide[0.1, 4 Pi, 400]};
ListAnimate@anims

```

