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Liouville's number, the easiest transcendental and its clones

Mathologer
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Published on 27 May 2017

Today's video is all about convincing you that Liouville's number is really a transcendental number. For this video I am elaborating on a proof for this fact that you won't find in any textbooks. I am keeping my fingers crossed that people will agree that this is the most accessible proof of the transcendence of any

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Mathologer 3 weeks ago

As promised here is my proof that Liouville's number is transcendental that did not fit in my last video. I did not have much time for Mathologer over the past three month. However things look much better timewise in the coming months now that the first semester here in Australia and most of my teaching for the year is done :)

Reply • 61

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Omi Slash 3 weeks ago (edited)

Mathologer there is a "schrodingerness" with all of your videos. They are understandable but to fully "grok" them take multiple views, and then I found out I don't really understand them. I have yet to reproduce your proofs from memory to another soul, a required step to show that I have fully "grok" the idea.

Thank you very much for the wonderful headaches.

Show less

Reply • 3

Mathologer 3 weeks ago

+Omi Slash Working hard on more headaches for you :)

Reply • 7

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Sam Hawke 3 weeks ago

Mathologer your videos are soooo good!!! This was my first ever transcendence proof! I really appreciate your amazing content :)

Reply • 1

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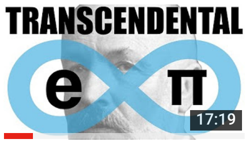

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Mathologer
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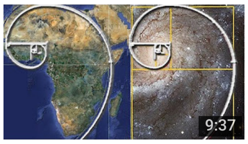

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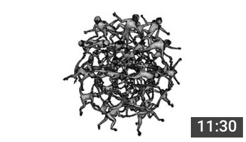

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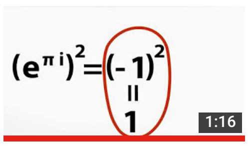

e to the pi i = -1 paradox
Mathologer 2
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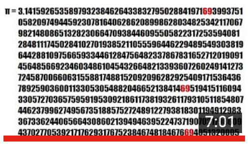

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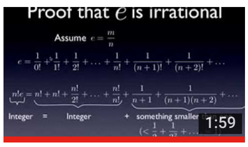

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4,859 views

Wau: The Most Amazing, Ancient,

https://www.desmos.com/calculator/rgjt4ori5t
Reply • 5

Vaia Patta 2 weeks ago
I'd like to translate this video to Greek as soon as I have time. How do I make the subtitles and where do I send them?
Reply •

Zander Rossman 2 weeks ago (edited)
Peter LeRoy Barnes So is this Liouville pi in different bases?
Reply •

Peter LeRoy Barnes 2 weeks ago
You can set k equal to any number, and anything without a terminating expansion in the base you put in will give you the approximations for some Liouville-like transcendental number. However, it is only capable of showing the decimal expansion, even though you're using a different base for the equations.
Reply •

Brewer021 3 weeks ago
As a "high school Mathologer fan" (and not even from an English-speaking country) I can confirm this is very understandable
Reply • 173

Hide replies

Mathologer 3 weeks ago
Perfect :)
Reply • 21

Juan T 3 weeks ago
Mathologer I'm also a "high school Mathologer fan" from a non English speaking country (Uruguay, actually), and I agree, very understandable content.
Reply • 6

Skytern 3 weeks ago
Same from Argentina
Reply • 7

Jarrett Haroldsen 3 weeks ago
I used to be a middle school Mathologer fan, but, as of yesterday, I too am a high school Mathologer fan.
Reply • 8

apburner1 3 weeks ago
I is b frum da cuntry of D'troyt an I don b unnerstannin dis.
Reply • 3

Hugh Paynter 3 weeks ago
burkard comes to my high school and does talks it's fuckin lit af
Reply • 5

3 weeks ago
same from Brazil :)
Reply • 6

Oliver Hees 2 weeks ago
Jarrett Haroldsen I'm still a middle school Mathologer fan, but in less than a month will also be a high school Mathologer fan (and I found this proof easy to follow).
Reply • 2

NicosMind 3 weeks ago
And to think. Once upon a time i thought this channel was simular to Numberphile. In comparison Numberphile talks about math this channel actually does math. Now thats a big difference
Reply • 130

Hide replies

bene7553 3 weeks ago
Skip navigation
NicosMind You might like 3blue1brown
Reply • 22

Naman Gupta 2 weeks ago
bene7553 3b1b is damn fantastic.

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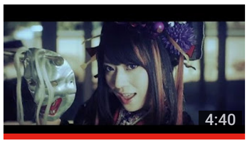

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avex
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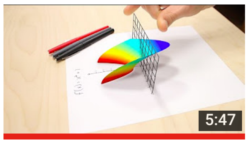

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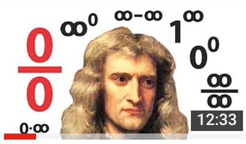

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Mathologer
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Mathologer
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Felipe Hindi 2 weeks ago (edited)
Numberphile is still interesting, but yeah, Mathloger, Vheart, PBS Infinite Series* and 3Blue1Brown are better.

*Thanks therealEmpyre for reminding me of that one.

Reply • 1

therealEmpyre 2 weeks ago
@Felipe Hindi: Add Infinite Series to that list.

Reply • 1

Felipe Hindi 2 weeks ago
Oh yeah, definitely! This one is kind of new, but it's already one of the best math channels in YouTube. I had forgotten.

(PBS Space Time is also excellent, by the way)

Reply •

therealEmpyre 2 weeks ago
Space Time is where I found out about Infinite Series. The list can continue (in no particular order): PBS Idea Channel, PBS It's OK to be Smart, Veritasium, 2Veritasium VSauce, VSauce 2, VSauce 3, Smarter Every Day, Numberphile2, Sixty Symbols, Deep Sky Videos, Periodic Table of Videos, NottinghsmScience, Computerphile, Minute Physics, Minute Earth, BBC Earth Lab, BBC Earth Unplugged, CGP Grey, Kurzgesagt, Astronomy Cast, Fraser Cain (Guide to Space), Life Noggin, Paul M Sutter (Ask an Astronomer), SciShow, SciShow Space, SciShow Psych, Physics Girl, Space Fan News, Space with Sara, Steve Mould, TheBackyardScientist, Tom Scott, The Slow Mo Guys.

If you add to that all the musicians and comedians and vlogs I am subscribed to, I spend way too much time watching You Tube.
Show less

Reply • 3

Felipe Hindi 2 weeks ago
Wow, I still don't know thirteen of those. Thanks for the recommendations!

Reply •

Sigge Stjärnholm 3 weeks ago
"I have discovered a truly marvelous proof of this, which this youtube video is too short to contain" - Mathologer about his proof of clones measure of 0

Reply • 69

Mathologer 3 weeks ago
Yes, yes, there are always too many nice things to say but saying them would make the video at hand lose focus. Anyway it's really a nice little proof which gives a lot more insight into the nature of Liouville's number and transcendental numbers in general :)

Reply • 22

Sigge Stjärnholm 3 weeks ago
Mathologer's Last Theorem ;)

Reply • 29

Martin Heermance 3 weeks ago
It took me a minute to realize that was Pinocchio experiencing the effects of his self-contradictory statement.

Reply • 28

Hide replies

Mathologer 3 weeks ago
Currently one of my favourite t-shirts :)

Reply • 8

Mario CCat 3 weeks ago
Mathologer Please make a video about your awesome collection of awesome shirts!

Reply • 4

Mathologer 3 weeks ago
That would be a long video (I've now got about 200 :)

Reply • 17

Steffen Widmaier 3 weeks ago
Would be an awesome mechandise store...

Reply • 6

Reply • 15

Reckless Roges 1 week ago
Well spotted. Should have changed to a 6.
Reply •

Jan Pokorný 3 weeks ago
Mathologer as a german - 16:28 : NEIN NEIN NEIN NEIN NEIN NEIN NEIN NEIN NEIN NEIN!
Reply • 15

Mathologer 3 weeks ago
:)
Reply • 7

CaesarsSalad 3 weeks ago
I don't get it. Why does L being a solution of a polynomial imply that it's "correct" approximations also are solutions? The approximations are only correct up to a point.
Reply • 12

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lifetimeoflearning 3 weeks ago
The approximations raised to some power have finite length (ignoring trailing zeroes). They exactly match L raised to the same power for as far as the approximations go. Since the polynomial evaluated at L is 0 or 0.0000..., and the approximation exactly matches L for as far as it goes, the polynomial evaluated at
Reply • 7

CaesarsSalad 3 weeks ago
I had stopped the video before he showed that the polynomial will also be correct up to the point where the trailing zeroes start. This was not obvious to me and I didn't get why this was a proof so I stopped in frustration.
Reply •

Tehom 3 weeks ago
Perhaps it would have been more exact to say that the approximations "agree with the decimal expansion of L (or whatever) in the first N digits".
Reply • 1

CaesarsSalad 3 weeks ago (edited)
@Tehom No, I got that. That wasn't the problem.
Reply •

Tehom 3 weeks ago
Oh, I see. Sorry for misunderstanding.
The approximations, except a finite number of early ones, are also solutions because we're looking for values where the polynomial equals zero. If you plug one of them into the polynomial, it equals zero - the part before the cutoff digit equals zero because it agrees with the original which is zero, the part after the cutoff equals zero because we established that the approximation to whatever power is still all zeroes after that point.
Show less
Reply • 4

Jake Reilly 3 weeks ago (edited)
The point is, for ANY given polynomial you can find a solution which will agree from some point (n) onwards since the gap between the digits becomes arbitrarily large as n tends to infinity. Therefore you always have an infinite number of solutions no matter what your choice of polynomial is.
Reply • 1

CaesarsSalad 3 weeks ago (edited)
@Tehom This is not a good explanation. It's not obvious (to me) that the fact that any power of L will have infinitely "correct" approximations implies that this will also be true for a polynomial. This is exactly the step that gave me trouble and your post doesn't explain that at all.
Reply •

Tehom 3 weeks ago
Well, the video already covered that reasoning so I don't know what more I can tell you.
Skip navigation
Reply •

CaesarsSalad 3 weeks ago (edited)
@Jae Reilly I got that too. It's just that Mathologer acted like the proof was done when it was still missing an important step in my opinion. He said that squares and cubes and so on of L will have infinitely many "correct" approximations and then

this step but I had already stopped watching because it looked like I was supposed to understand the proof at this point and I didn't.
Show less
Reply • 3

CaesarsSalad 3 weeks ago
@Tehom Yes he did. I wrote in another answer that I hadn't watched the video to completion because he only explained this step AFTER he acted like the proof was complete. I thought I I didn't get the proof because this wasn't obvious to me and got frustrated. I do understand the proof now, I'm mainly arguing that the ordering
Reply •

erik satter 3 weeks ago
@Ceasarssallad Moving from powers to polynomials:

If you have a correct approximation Lk of L such that (Lk)^1 "=" L^1, (Lk)^2 "=" L^2,..., (Lk)^n "=" L^n (were "=" means correct approximation), then sums of these will be correct as well; for example (Lk)^1+(Lk)^2 "=" L^1+L^2 since (Lk)^1 "=" L^1 and (Lk)^2 "=" L^2. Since all coefficients of the polynomials are integers any polynomial can be written as a long sum of powers (for example 3*L^2 = L^2+L^2+L^2). Therefore the approximation will be correct for any polynomial of degree n or less.

Is this the sort of middle step you were missing?
Show less
Reply •

niCole Bright 3 weeks ago
@erik satter how do you prove that the part you chopped off when by approximating to the nth digit does not add to more than 10^-n? The answer to this is just to take a number where the gap is arbitrary large so that the parts taken off by approximation are too small to make a difference. (This holds true because of
Reply •

Zander Rossman 2 weeks ago
A ten digit approximation of 0.00000000000000000000000000000000000000000000000... is 0.0000000000 = 0
Reply •

erik satter 2 weeks ago (edited)
@niCole Bright I see, my argument breaks down at the summation. However we do not need any more assumptions (though what you say is implicit in our previous assumption).

Let Coeff(p) be the sum of the absolutes of all coefficents in p i.e. if p(x) = (c1)x + ... + (cn)x^n then Coeff(p) = |(c1)|+...+|(cn)|. Since polynomials are finite there exists an integer M such that Coeff(p)<10^M. Let (Lk) be an approximation of L with k accurate decimals such that (Lk)^2 "=" L^2 ,..., (Lk)^n "=" L^n. Then the error terms add up to at most 10^M*10^-k and we can achieve any desired accuracy for our polynomial. For ten decimals accuracy for p(L) it's sufficient to take k=M+10.

Addressing this in the video by showing why the example polynomial has the accuracy mentioned would make the "proof" easier to follow.
Show less
Reply •

Anton Mordberg 3 weeks ago
I suppose that we can change f(n) = n! function (that assigns ones to decimal positions f(1), f(2), ... for our handmade transcendental number) to any other function that grows fast enough to make sure that zero gaps becomes at some point large enough to cover needs of any chosen power (as in the proof). The question is, what is the slowest function that fits the proof? I mean, you can change n! by n^n in the proof and not so many changes will be needed because n^n grows faster then n! and we still have infinitely many non-periodical ones. On the other hand, we cannot take a function like c^n as it does not grow fast enough and something like L^c will never have a good cutting approximation.
Show less
Reply • 11

Hide replies

Mathologer 3 weeks ago
Yes, you are right, as long as the function that pins down the position of the 1s grows fast enough everything I talk about in this video will still work. Also nice question about how "slow" we can make this function. Have to give this some thought :)
Skip navigation Reply • 7

Daniel Darroch 3 weeks ago
I also considered this problem. Clearly n^n and n! are similar in that they grow like e^O(n log(n)), since n^n = e^(n ln(n)) and Stirling's approximation gives n! ~ e^((n+1/2) ln(n) - n). If we construct our number using some f(n), then the number

If our f(n) function grows asymptotically like f(n) ~ e^(g(n)), where g(n) grows faster than linear, say g(n) ~ n*h(n), where h(n) is increasing, then the condition f(n+1)-n*f(n) >= 1 is met asymptotically. And this seems to be about the limit of this particular proof of transcendence. So even functions like f(n) = ln(n)^n = e^(n*ln(ln(n))) will work with this proof method.

But there must be more complicated methods which can prove transcendence with even slower growing functions, since according to the Wikipedia article on transcendental numbers the Fredholm number, which uses a similar sum with the function f(n) = 2^n is also transcendental.
Show less

Reply • 4

Daniel Darroch 2 weeks ago
The proof only requires that f(n+1)/f(n) is unbounded. So if f(n) ~ e^(g(n)), then f(n+1)/f(n) ~ e^(g(n+1)-g(n)), so f(n+1)/f(n) is unbounded if g(n+1)-g(n) is unbounded. If we let g(n) ~ n*h(n), then g(n+1)-g(n) ~ n(h(n+1)-h(n))+h(n+1). So if h(n) is increasing and unbounded, then n(h(n+1)-h(n)) >= 0, so g(n+1)-g(n) ~ h(n+1), so g(n+1)-g(n) is unbounded.

So even if we pick h(n) to be an extremely slow growing function which is still increasing and unbounded, like the inverse Ackermann function, we still have that f(n+1)/f(n) is unbounded and so the proof technique succeeds in proving that the corresponding constructed number is transcendental.
Show less

Reply •

Zander Rossman 2 weeks ago
I think a proof for position function of f(x) = 2^n would be difficult as L^3 will never be accurate, nor L^4 etc.
It is of course true that more complicated methods exist. According to the title, All other methods are more complicated than this!

Reply •

Michel ten Voorde 3 weeks ago
Nicely done Mathologer. Now, how about an easy to understand proof of the transcendence of e, π and (just kidding) γ?

Reply • 5

Hide replies

Mathologer 3 weeks ago
The one I really want is a REALLY accessible proof that pi is irrational :)

Reply •

Michel ten Voorde 3 weeks ago
You've seen this? http://projecteuclid.org/euclid.bams/1183510788

Reply • 1

Mathologer 2 weeks ago
Yes, I know this and quite a few other proofs. They are all really nice and ingenious but none of them is the one from the BOOK as far as I am concerned :)

Reply •

Michel ten Voorde 2 weeks ago
I see what you mean. Have to admit, I'm not that charmed about it either. But who knows, maybe it just doesn't get any more accessible than that. It's that FLT feeling again: it ought to be easy, but just isn't. Good luck in your search anyway. Stay waiting for that γ irrationality proof though. ;)

Reply •

WHoZ 3 weeks ago
What about the sum of n!/(10^(n!)), n starts at 1 to infinite?
0.120006000000000000000024000000000000000000000000000000000000000000000000000000000000000000000000000000000000000120...

Reply • 5

Andrey Chen 3 weeks ago
Liouville's original argument is slightly more loose, although more complicated. His inequalities still hold in this number and this is still a Liouville number, by his exact same argument. https://en.wikipedia.org/wiki/Liouville_number
Mathologer's argument also holds, although much more work on estimates is

Reply •

sivad parks 3 weeks ago
I don't know if this is an actual reason, but includes sets of two plus digits like 24 or 120

Reply •

computer graphics, where you have to pack several numbers into a 64-bit variable and leave enough gaps that you can do multiplication and still get an accurate result without the bits bumping into each other.
Reply • 4

Miguel Sanchez 3 weeks ago
Gee! Everything touches everything in mathematics. I'm a computing engineer you made remember one of my first c programs to draw a line on the screen. Thanks!
Reply • 1

barış aytekin 3 weeks ago
Will you make another video on set theory and axiom of choice? btw I'm a high school Mathologer fan too and I also agree on the fact that your videos are really understandable for anyone who had taken math lessons during high school.
Reply • 4

Mathologer 3 weeks ago
Axiom of Choice is on my to-do list but not really high up :)
Reply • 4

Cheuk On Li 3 weeks ago
at 11:42, should it be 10^-120 at yellow?
Reply • 3

Mathologer 3 weeks ago
Yes, an annoying typo that but luckily does not matter in terms of the proof :)
Reply •

orochimarujes 2 weeks ago
Thanks, I thought I was going crazy.
Reply •

Evenly 2 weeks ago
once I thought anyone who watches these videos are complete nerds

but then i realise im watching this video while typing this comment. So...yeah
Show less
Reply • 2

Mathologer 2 weeks ago
:)
Reply •

Ashbakhaaz 3 weeks ago
Isn't 0 an exception (as said at 16:45) just because of the (arbitrary) way we write numbers?
0=0.999999... - 1
0.999999... - 1 can also be written -0,00...(infinite 0s)...001, which for sure doesn't work here; however...
If 0.999999... - 1 could be written -0.999999... (in which case, the number we usually write -0.9999999... would be the unwritable one rather than 0, or rather would be written -0,00...(infinite 0s)...001 just like we can write 0 now), then wouldn't 0 also have a transcendental Liouville clone? And it seems to me like this alternate mathematic system exists, even though it's unused, right?
Show less
Reply • 2

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Treufuß 3 weeks ago
He uses the decimal expansion to create the "clone set". 0.999...-1 is not the decimal expansion of 0. And 0.000...001 does not have any meaning I am aware of. However it is as well not the decimal expansion.
Reply • 1

Yee 3 weeks ago
Skip navigation
So first, the way we write numbers isnt arbitrary. There is a logical pattern behind it. Your 'alternate mathematics system' is really just a different way of writing fp numbers. Its not that it couldnt be used, but it makes less sense in the way youd write negative numbers.

Also Treufuß is correct in saying that 0.00..1 doesnt have any meaning in this context

Tetraedri_ 2 weeks ago
Actually 0 also has two equivalent decimal expansions: 0.0000... and -0.0000...
Problem is, neither of them will generate trancendental numbers.
Reply •

Yee 2 weeks ago
Those arent different decimal expansions.
Reply •

Mark Callaghan 3 weeks ago
i was hoping to seea video of hamsters eating little tacos at the end , but your way is fine too
Reply • 2

Mathologer 3 weeks ago
:)
Reply •

Watt Nu 3 weeks ago
I think the easiest way to see that this set has measure zero is to notice that none of it's members can be a normal number. but this of course only works if one knows that almost every real number is normal
Reply • 2

Hide replies

Destroctive Blade 3 weeks ago
What's a normal number
Reply • 1

Benjamin Przybocki 3 weeks ago (edited)
Destroctive Blade A number is normal if every digit appears infinitely often.
Reply •

NotaWalrus 3 weeks ago (edited)
Not exactly, a number is normal if every digit appears as much as every other without a pattern (Formally defining this requires limits), as does every single finite string of digits.
Reply • 3

Destroctive Blade 3 weeks ago
don't worry I understand limits so can you give me the more formal definition ?
Reply •

Benjamin Przybocki 3 weeks ago
Destroctive Blade It's on the Wikipedia page for "normal number".
Reply •

Destroctive Blade 2 weeks ago
ok I will look it up
Reply •

NotaWalrus 2 weeks ago
Basically you need to use limits because you need a tenth of the decimal expansion to be zeroes, a tenth to be ones, etc, but it doesn't actually make sense to talk about a tenth of infinity, so what you say is that at the limit of the decimal expansion every finite string of some length is equally represented.
Reply •

Destroctive Blade 2 weeks ago
oh so you take an interval (not exactly but you get the point) and you say that as that interval tends towards infinity , the accurence of each number tends towards a 1/10 of the interval
Reply •

Benjamin Przybocki 2 weeks ago
Destroctive Blade Yes, but you also look at the distribution of strings of digits. For example, every two digit pair should tend toward a frequency of 1/100 (in base 10).
Skip navigation Reply •

Destroctive Blade 2 weeks ago (edited)
and every 3 digit combination should tend towards a frequency of 1/1000 I get it thx though it should be hell to prove isn't it ?
Reply •

a Basic Tool in probability theory

Reply •

Destroctive Blade 2 weeks ago
hhhh I am still in high school so I am yet to see that

Reply •

Anirudh Sivakumar 3 weeks ago
i kno this vid is gonna be worth the wait ;)

Reply • 2

bsanieschool 1 week ago
your vid e yos make mne anUUTTTT

Reply • 1

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bsanieschool 1 week ago
I AGREE
-john

Reply •

bsanieschool 1 week ago
very same !1!11!11111111

Reply •

bsanieschool 1 week ago
they make me HAVE SEX

Reply •

Coullio 2 weeks ago
Can you do a video on Godel's incompleteness theorems? I only learn maths for fun so all the videos either skip steps and it goes way over my head or they go into mathematical detail at all (cough, numberphile). You always seem to find the happy balance.

Reply • 1

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Mathologer 2 weeks ago
I think Numberphile just posted one. Could you watch it for me and tell me whether it is any good (i.e. more than just the usual entertaining fluff when it comes to them covering complicated topics?)

Reply •

Coullio 2 weeks ago
Mathologer I did just watch it. It explains the concept that Godels theorems /might/ show that not everything is provable, but does not go into the mathematical rigour. It doesn't explain how Godel code numbers work or even show any of the theorems. It merely states the interpretation of the theorems.

Reply •

Mathologer 2 weeks ago
They may have some additional videos floating around. Did you also watch those. Anyway, definitely a great one to do properly at some point :)

Reply •

Danil Dmitriev 2 weeks ago (edited)
The good thing about those videos on Numberphile is that they explain the main idea and the implications of the theorem very clearly and interestingly. For instance, after talking about the theorem (very peripherally), they described an amazing way to prove the truthfulness of the Riemann hypothesis with the help of Godel's idea. They said that if we prove that Riemann hypothesis is undecidable (unprovable) within the system of mathematics where it is formulated, then it must be true. In the extra footage, there is a lot of talking about connection of Godel's theorem to various areas of mathematics, consistency of mathematics, some related fun stories, etc.

What is not so good about those videos is that they simplify things so much that it is actually not clear at all what Godel did. As Coullio mentioned above, the videos don't explain properly how Godel coding works, they just say that it is a way of assigning a number to any mathematical statement within a system of axioms.

Moreover, in the beginning of the first video it is said that verbal paradoxes are fine, since we don't expect every verbal statement to have a truth value. But in the part where they explain the proof ot the theorem they use a verbal self-referencing statement to get a contradiction. Unfortunately, they didn't explain what is different about the context in which they got this contradiction and the context of ordinary verbal paradoxes, which they talked about before.

We can encode any mathematical statement in Godel numbers, and we ask a question whether all true statements within a system of some axioms can be proven from them. A statement is provable from the axioms if (this is the oversimplifying analogy that they used) its Godel number is kind of divisible by the axioms' numbers. (I don't know how this works) Then consider a statement "This statement cannot be proven from the axioms". As any other statement, this one can be encoded as a Godel number, and it is either true or false. If it is false, then it is provable, so it must be true – contradiction. So, the statement must be true, which means that we have found a true statement that is not provable from the axioms in this particular mathematical system.

Of course, I am not sure whether there is a good way to not oversimplify the proof of the theorem and also make it accessible, but I'm just saying that this explanation seems too "hand-wavy"...
Show less

Reply •

Mathologer 2 weeks ago
I still remember spending quite a bit of time as a student going over Godel's proof. It really took a while to digest. I've been thinking on and off on trying to do a video about it. The main problem is that to do it right I don't think there is much of a shortcut in terms of setting things up very carefully, so properly a 30 -40 minute

Reply •

Danil Dmitriev 2 weeks ago
Maybe there is a way to divide the discussion of this topic in several pieces, like you did with transcendental numbers? I don't know much about Godel's incompleteness theorem and its proof, unfortunately (my knowledge is pretty much exhausted by the content of the corresponding Wikipedia page...), but maybe there are some other interesting results or ideas that are floating around this proof? If so, it may then be possible to divide a 30- or 40-minute video into, say, two videos of 15-20 minutes length, which is more than manageable in terms of viewing. Just a suggestion :)
Show less

Reply •

Mathologer 2 weeks ago
We'll see, lots of other things I want to do first :)

Reply •

MuffinsAPlenty 2 weeks ago
From looking at the comments, there were a couple of questions people seemed to have about the numberphile video.

One of the big ones was about the difference between "proof" and "truth" since Marcus du Sautoy (the mathematician du jour) used the phrase "true but unprovable" without explaining the difference. A lot of people were under the impression that true is equivalent to provable, because they've never seen Tarski's definition of truth or Gödel's definition of proof within a first-order axiomatic system. Other people who are more knowledgeable about first-order logic appealed to Gödel's Completeness Theorem saying that truth is equivalent to proof, but didn't realize that Marcus du Sautoy meant "true in the standard model" when referring to "true", as opposed to meaning "true in every model".

As was already mentioned, a lot of people were skeptical about how Gödel was able to encode a variation of the liar's paradox into the natural numbers, since no details were given about this. Essentially, there were no details about the proof given at all.

Again, as was already mentioned, Marcus du Sautoy mentioned that if the Riemann Hypothesis were independent from the axioms, it would be true. He didn't go into many details about this, simply saying that if it were false a counterexample would exist. A lot of people thought that this argument applied *in general* to any mathematical statement, missing that the Riemann Hypothesis is special in that regard. So it was lacking in why this is true specifically for the Riemann Hypothesis as opposed to other mathematical statements.

And on this last point, people thought this was contradictory. If you can neither prove nor disprove it, then it's true, which constitutes as a proof, meaning you can prove it. This is because the video never made clear that different axiomatic frameworks can be used. It never discussed the idea of meta-logic about an axiomatic framework vs. formal logic within an axiomatic framework.

And of course, there were a decent number of cranks trying to use Gödel's Incompleteness Theorems to push their preferred philosophy, ranging from claims that the law of the excluded middle is false to arguments about how mathematics is a religion.

So yeah, feel free to keep these thoughts in mind if you want to make a video on the topic :P I expect it would be your second most crank-filled comment section ever (behind your 9.999... = 10 video, of course).
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Reply •

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1.11000100000000000001... is exactly equal to the square of the truncation 1.1100010000000000001, including every last one of the infinite digits?

Reply • 1

Hide replies

Mathologer 3 weeks ago

The digits of the square of the truncation are exactly the leading digits of the square of L. There are more non-zero digits of L squared beyond those leading digits that belong to the squared truncation :)

Reply •

therealEmpyre 3 weeks ago

It looks to me as if your proof that L is transcendental depends on the powers of the truncations being exactly equal to the powers of L, so that if L solves the polynomial then all the truncations (after a certain point) do too, leading to the contradiction. But if they are just very close, even arbitrarily close, then they are not solutions and therefore not leading to the contradiction that your proof is depending on.

I am not questioning that L is transcendental. I am just questioning that this proof proves it. Maybe it is a valid proof and I am just missing something important, even after watching the video second time. If so, I'll just have to live with it. Either way, this video got a like, and I am not unsubscribing.

Show less

Reply •

Mathologer 3 weeks ago

Cool, but let me try to put it another way which may work better for you.

Let's say f(x) is the polynomial under consideration and let's say we know that T is a truncation of L such that the digits of f(T) are exactly the first digits of f(L).

Now we assume that L is a solution of f(x)=0, that is, f(L)=0. This means that all digits of f(L) are 0s including all the ones at the beginning that come from f(T). But that means that f(T) =0. And so T is also a solution of f(x)=0. Does this work for you?

(Just in case, maybe I should also say that since T terminates in 0s so will f(T)).

Show less

Reply • 1

therealEmpyre 2 weeks ago

That helped a lot. Thank you for taking the time to explain it to me.

Reply •

MrWbu 2 weeks ago (edited)

I had the same objection, and you answer does help, but I have a follow-up question. Why is it safe assume that the leading digits of f(T) will match the leading digits fo f(L)? This seems to be based on the fact that the leading digits of T match the leading digits of L. Is that always true that for if T matches L to a degree then

Reply •

Зенон Дамирович Берёзин 3 weeks ago

maths is borinjg I love to plaay mincecraft and other games like thiss, yea?

Reply • 1

Danil Dmitriev 3 weeks ago

Хи-хи

Reply •

Dustin Rodriguez 3 weeks ago

Two questions: Might you someday do a video about computable and non-computable numbers? I know pi and e are computable, but from my (admittedly amateur) understanding, Liouville's number may not be because it is defined in terms of a decimal expansion? Second question: Where can I get that shirt? It is magnificent and I can't find

Reply • 1

Hide replies

Mathologer 3 weeks ago

Yes, a video on computable numbers, the Halting problem, etc. would be great. Liouville's number is computable and here is the place where I got the t-shirt from :) https://shirt.woot.com/offers/liars-paradox

Reply •

Skip navigation

Pyjamadeus 3 weeks ago (edited)

Good idea I think he should do a video on that too. Liouville's number is computable though; any number defined according to a decimal expansion which follows a pattern that you can tell people about in a video of finite length will be computable. Non-computable numbers are a little more mysterious, it's hard to give an explicit

Reply •

Wikipedia page for computable numbers that numbers defined in terms of their decimal expansion are non-computable. I'd certainly enjoy a video about the Halting Problem and similar CS things, that is what my degree is in. After your video and my curiosity about whether Liouville's number was computable or not I went down a bit of a Wikipedia rabbit hole, learning about Chaitin's constant (probability a random program will halt... which has some very weird and surprising (to me anyway) properties) and algorithmically random numbers and such. Lots of interesting math in CS! Now to buy that shirt if I can...
Show less

Reply •

P **Pyjamadeus** 3 weeks ago

On the contrary I would expect numbers defined in terms of their decimal expansion to always be computable. Depends exactly what that means, but if you know that you can sit down with pen and paper and write down (in theory at least, maybe it will be hard in practice) any finite truncation of the decimal expansion you like then it will be computable. Non-computable numbers will have the property that as you try to write down the decimal expansion you'll realise that you don't actually know how to calculate it, and in fact neither does anyone else.

That's a really cool area of maths you're into there, I'm trying to learn more about it too. Check out Kolmogorov complexity if you haven't already, that blew my mind when I first learnt about it. Closely related to the Berry paradox
Show less

Reply •

Dustin Rodriguez 2 weeks ago

Pyjamadeus I think you might be confusing undefinable numbers and noncomputable numbers. Many noncomputable numbers can be described and precisely defined. For a number to be computable, it must be possible to create a program which can compute the number to any desired degree of precision with a program with a finite number of steps and which halts. Chaitin's constant, for instance, is the probability that a randomly constructed program will halt. It is known that this converges to a real number. Yet, there can be no algorithm to compute it. Calculating the first few bits are possible (I'm not sure how many or if a definite limit is known... it would involve enumerating every possible program and determining if they halt for a large enough number that they contribute bits known to be certain for the overall probability. This can be done in finite time, but the amount is noncomputable itself) but any more would be equivalent to solving the Halting Problem. It's known that the digits of the number are uniformly distributed. It is known that it is an arithmetical number (this surprised me).

I'm not sure what exactly the mention of 'defined in terms of a decimal expansion' means on the Wikipedia page. I think, re-reading it, that it may be referring just to two different ways the idea of computable numbers can be defined. One is in the terms I said earlier, being able to produce a given number of digits, and the other has to do with getting within a certain epsilon of it. Those sound pretty much equivalent to me but must not be.

Kolmogorov complexity was a big part of a course I took in college about algorithm analysis. At the time I didn't understand it at all, but I think I have a handle on it now. Going into college, I understood very little of mathematics. I knew lots of procedures for computing, things like finding derivatives and solving equations, but I didn't understand the deeper principles at play. It wasn't until reading the great book(s) 'Comprehensive Mathematics for Computer Scientists' after graduation that I really started grasping the conceptual underpinnings.
Show less

Reply •

P **Pyjamadeus** 2 weeks ago

+Dustin Rodriguez nah we are totally talking about the same thing I think. You said "for a number to be computable, it must be possible to create a program which can compute the number to any desired degree of precision with a program with a finite number of steps and which halts". I said a number is computable "if you know that you can sit down with pen and paper and write down... any finite truncation of the decimal expansion you like". I believe the Church-Turing Thesis is basically the statement that those two concepts are the same thing. More formally you can define the notion with Turing machines, but if the Church-Turing Thesis holds then there's no algorithm I can perform as a human which can't be emulated by a Turing machine.

When I say "write down a finite truncation of the decimal expansion" I literally mean for any integer m, write down the first m digits of the decimal expansion. So if a number is not computable, there exists some point in the decimal expansion beyond which there does not exist a halting algorithm to calculate the digits, in other words no one knows a way to do it (and in principal it can't be done unless there are models of computation more powerful than Turing machines) .
Show less

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Jivan Scarano 3 weeks ago

3:43 the 6 changed back to a 3.

Reply • 1

Reply • 1

Daniel Fisher 3 weeks ago
At 11:30, should 10^-116 be 10^-120==(10^-24)^5?
Reply • 1

Mathologer 3 weeks ago
Yes, well spotted, that should be 10^-120. Annoying typo but luckily does not matter in terms of the proof :)
Reply • 1

Tim H. 3 weeks ago
So, that clone of the reals is a strict subset of the reals, and so it must contain itself. And the terms produced by including it in itself once are also included. And so on. Prepare for some ZFC mathologer, because that set might have Russel quite unhappy...
Reply • 1

Hide replies

Jo Reven 3 weeks ago
I don't think the term "clone" is being used literally here. That pi "clone" is definitely not pi, for example. Or are you just worried about the usage of the word "clone"?
Reply •

P
Pyjamadeus 3 weeks ago
Actually this isn't that outrageous, all infinite sets contain strict subsets which are 'clones' of the whole set. One can even take this as the definition of an infinite set if you accept the axiom of choice. See Dedekind-infinte set on wikipedia
Reply •

T
Tim H. 3 weeks ago
Jo Reven "clone" is just a less scary word for bijection, which is what this is. And Pyjamadeus is correct that infinite sets have the ability to be in bijection with strict subsets of themselves. I guess I jumped the gun a bit on being concerned.
Reply •

P
Pyjamadeus 3 weeks ago (edited)
Though I was a bit sloppy with my mention of the axiom of choice, should have put it earlier in the comment. For clarity: with the axiom of choice one can show that for any infinite set X there exists a function f: X->X which is injective but not surjective. But given that I'm totally cool with the axiom of choice I just go around
Reply •

T
Tim H. 3 weeks ago
Pyjamadeus As an intuitionist, I literally don't care about AoC. LEM on the other hand...
Reply •

Tony James 3 weeks ago
So if you can create a "clone" of any real number as a transcendental Liouville number, and it would be a real number itself, you could also create a clone of the transcendental clone of the real number. So there is a set of transcendentals within the transcendentals that maps to all the real numbers. And you could do this as deep as you wanted. Oy.
Reply • 1

Mathologer 3 weeks ago
Yes, that's correct and a nice thought :)
Reply •

Rodolpho V.Santoro 3 weeks ago
My proof for "Adding an integer to a transcendental, gives a number that is still transcendental", not only that but adding any algebraic to a transcendental gives another transcendental:
Let a and b be algebraic numbers, t a transcendental.
There are no values of a and t such that a=t, thus a!=t(not equal to).
Adding b to both sides: b+t!=a+b.
a+b is still algebraic*, since b+t!=a+b, b+t is not equal to any algebraic number thus the sum of t+b is transcendental.
*the sum of two algebraic numbers is algebraic
(how good is it? i'm not a mathmatician myself, sorry if i made any mistake).
Skip navigation
Show less
Reply • 1

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Igor Noga 3 weeks ago (edited)

b using some finite algebraic algorithm. Now if you add instruction "subtract a" at the end of this algorithm you get t. Therefore t is algebraic, witch contradicts

Reply • 1

Rodolpho V.Santoro 3 weeks ago
Nice one.I can understand proofs by contradiction, but i just can't do any myself.Whenever i have to prove something, instead of trying to show "x=y" is a contradiction, i show that "x!=y" is.

Reply •

Igor Noga 3 weeks ago
Proofs by contradiction are not always usefull, and it's hard to predict when to use them. BTW I really like your original proof, it's nice and strict. I could imagine it being in a book, while mine is easier to explain to someone

Reply • 1

Fnors 3 weeks ago
It's a nice proof and it is actually a proof by contradiction, since what you are really saying is that b+t cannot be algebraic.
The first problem I see is with how you chose a and t.
You should probably write it as "Let t be transcendental. Then t != a, for any algebraic number a"

Next is the step saying "b+t!=a+b, b+t is not equal to any algebraic number thus the sum of t+b is transcendental". This hides a bit too much (it fixes a choice of number a) and an argument like this can often be wrong, unless you prove that the statement is still true for ALL fixed values of a.
For example, I can say: A is not an invertible matrix, then A != B for any invertible matrix B. But, for some invertible matrix C, A+C can be invertible, even if A+C != B+C, for some B.

A better way to write your proof would be:
Let t be transcendental, and a be algebraic. Now, let b = a + t.
If b is algebraic, then b - a is algebraic, by closure of the algebraic numbers. But t = b - a is transcendental, so it cannot be algebraic. We have a contradiction. Thus, b = t + a is transcendental.

Sometimes, proofs by contradiction are the only way to go, since otherwise you would need to show the proof works for all cases.
Show less

Reply •

sivad parks 3 weeks ago
6:48 is absolutely genius! Great work Mathologer

Reply • 1

Justin Miller 3 weeks ago
I don't understand how you can say that the truncations of L are "spot on". Yes they are correct up to the truncated value, but there are an infinite number of places after that point for which they are no longer correct. Because of that, I would imagine that the polynomial equation could have different values for different truncations, and L could be a solution

Reply • 1

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Mathologer 3 weeks ago
I guess you may have to watch the video again :)

Reply • 4

Justin Miller 3 weeks ago
I would be surprised to find a Mathologer video where that wasn't the case ;-)

Reply • 2

Justin Miller 3 weeks ago
On further reflection, I believe that the point of the video is to show that the polynomial, when evaluated with L and then truncated, would be the same value as the case where you had used the truncated values of L to evaluate the polynomial. That is why there is so much detail in showing algebraic operations such as raising to an exponent, adding terms, and even adding terms multiplied by integers does not affect the value if truncated versions of L are used instead.

The video demonstrates how an example polynomial would in that case have infinite number of solutions, thus indicating that L could not be a solution. No matter what polynomial is chosen (except perhaps some infinite polynomial), there are always some infinite number of truncations that would also be a solution to that polynomial.
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Reply •

Attila Asztalos 3 weeks ago

more rigorous examination, but I hit a brick wall between 07:00 - 08:00 about how that leads to all of them (over some given point) being solutions of the polynomial. After countless re-watches I think I finally got it, but it's still more of a "I can't quite articulate where the disconnect is for me anymore so I just accept the reasoning even though it still feels a bit wrong" rather than grokking it on the fundamental "of course it is so" level on which the rest of the video makes sense for me.
Show less
Reply • 1

EXALTS 3 weeks ago
this is very understable, good job friend
Reply • 1

Mathologer 3 weeks ago
Glad it worked for you :)
Reply •

harikrishnan menon 3 weeks ago
yo
Reply • 1

sylys 6 days ago
Have to admit that your video is way more accessible than Liouville's original paper. Even if understanding french helps, the vocabulary has changed too much for it to make much sense to me.
I am wondering though, do wee need to space the 1's as fast as with a factorial for the
Reply •

Trevor Saunders 1 week ago
I fucking hate and love this guy
Reply •

ChibiRuah 1 week ago
had to double back to understand the punch line of the proof but it makes sense after giving it a second look
Reply •

RapGeneral 1 week ago
Hmmm. Now lets go for transcendental numbers for dummies.
Reply •

Edgar Nackenson 1 week ago
That was some smooth proofery. Now just gotta get on that elementary school proof you brought up in the beginning. Probably not all that possible unless you can give them some familiarity with polynomials, and maybe an introduction to the notion of arbitrary length, but I think a reasonable explanation can be provided once you have those things in place. Actually, thinking on it further, you could probably skip a lot of the polynomial stuff with a bit of hand waving, because as long as you have the notion of arbitrary size in place it doesn't matter all that much what's happening in the interim.

So, you could just be like, "Anything you can do to this number can only make things different up to a certain point, and polynomials are a subset of that." Of course, the part close to the end of the proof, where you talk about the degree of the polynomial and the equality to zero of each shortened version of L, would take some extra jiggering. So, I dunno. You might be right that this proof has to go on the, "Keep out of reach of fourth graders," pile, unless you spend a few months building up to it. Which you can probably do, and it might even be worth it, given that these topics are all useful. The question of how deep you can go with young-folk is super interesting to me.
Show less
Reply •

GPC™ 2 weeks ago
By the way, here's a thing that I have never thought about: 1/3 doesn't yield a 'definitive' result if we try to get it by division. But what's more definitive than looking to 3 things and remove 1/3 of them?
"something strange is going on with the numbers, not only with primes."
Skip navigation
Reply •

SirLightfire 2 weeks ago
I think the whole proof of "the zeros grow faster than the powers of ten" is a more intuitive way of looking at l'hopital's rule. d/(dv) [(x!) / (a^x)] > 1 as x -> infinity and a is a constant

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GPC™ 2 weeks ago
Liouville's 'number' never ends so it is not a number.
0.110001 is a number
0.110001000000000000000001 is a number

0.110001000000000000000001... it's not a number.
Show less
Reply •

Hide replies

GPC™ 2 weeks ago
pi is only a number when we truncate it, if some one is wondering.
etc.
Reply •

Danil Dmitriev 2 weeks ago
Oh, so then things like 1/3, 2/7, or (-1/12) are also not numbers? :)
Reply •

GPC™ 2 weeks ago
Anything without an end.
Reply •

Mathologer 2 weeks ago
Actually given any rational number other than an integer I can choose a base with respect to which "it won't end". This means that only the integers are numbers, right? :)
Reply • 3

GPC™ 2 weeks ago
now you're talking.
Thank you.
Reply •

FaRo 2 weeks ago
One day someone should hand in a math test and have the final answer end with ... 99999.... And then argue with the teacher for hours about the grade.
Reply •

Plinky Snickers 2 weeks ago (edited)
What if you replace all ones by Liouville number's digits ? And what if you repeat the operation as many time as you want ? For the n-th number of this collection, the n!-th first digits (and more) would be 0 (exept the two first if you only take the decimal part), but would it still be transcendental ?
Reply • 1

Mathologer 2 weeks ago
Yes, those iterations would still be transcendental :)
Reply •

ron raisch 2 weeks ago
where can i see the proof for the length of this set, btw, a beautiful proof its such a good and creative one!!
Reply •

Mathologer 2 weeks ago
Have not gotten around to editing this part. Eventually I'll put it on this channel
https://www.youtube.com/channel/UCH74Hc_7WYVzx1GXhLEH6Eg
Reply •

AnarchoAmericium 2 weeks ago
Are the transcendentals a reflexive subcategory of the reals?

Are they the smallest non-trivial reflexive subcategory?
Reply •

Skip navigation

MuffinsAPlenty 2 weeks ago
Hmmm? Are you talking about "reflective" subcategory? I didn't find any results for "reflexive subcategory." Also, in what sense are you viewing the real numbers as a category? Could you describe the objects and morphisms?
Reply •

Reply •

MuffinsAPlenty 2 weeks ago
Haha. I kind of felt the same way when I first found Mathologer. There was a time when Numberphile was my favorite YouTube channel. But now, Numberphile has been surpassed (on my favorites list) by three other mathematics channels: PBS Infinite Series, Mathologer, and 3Blue1Brown. But that's a personal preference.

Each of the channels have strengths. For example, Numberphile is a kind of "pop math" channel, and I don't mean this in a derogatory manner. They usually do a good job of covering lots of interesting and quirky topics and giving people a brief overview of those topics. Numberphile also gives people exposure to lots of different mathematicians. PBS Infinite Series does a similar thing with the "pop math", but with more advanced topics and often more detail. Mathologer has excellent videos in which he gives a clear, accessible, and often novel explanations for topics people may have heard of before but probably have not seen an explanation of before. And 3Blue1Brown does a phenomenal job at viewing ideas from different perspectives than most people have seen before and developing mathematical intuition.

All of the channels like to have some videos which show off interesting things people have probably never seen before, but what I've highlighted are what I consider to be the strengths of each channel.

So welcome aboard! Don't feel bad about seeing videos from "other channels". Please stick around for other great videos from Mathologer, and check out PBS Infinite Series and 3Blue1Brown if you haven't done so already!
Show less

Reply •

Tynan Sigg 2 weeks ago
Could a function other than the factorial function be used to determine the position of non-zero digits in a number with similar results? Would any function growing "faster" than polynomially work, or is that not sufficient?

Reply •

Anas abahaddou 2 weeks ago (edited)
thanks ,i have no words to describe how great are those explanations

Reply •

Mathologer 2 weeks ago
:)

Reply •

TheGloryofMusic 2 weeks ago
What is the Lebesque measure of the set of reals take away the rationals? Since an uncountably infinite set can have zero measure, and the rationals have zero measure, the answer is not obvious to me.

Reply •

Mathologer 2 weeks ago
To make it a bit more tangible let's stick to the interval (0,1). This is an uncountably infinite set of measure 1 and the rationals contained in it are a countably infinite set of measure 0. If you take away the second set from the first you are still left with a set of measure 1. Maybe check out the Wiki article on Lebesque measure for more details. Anyway, all countably infinite subsets of the reals (or of (0,1)) have measure 0. However, the measure of an uncountably infinite subset depends on how exactly it is embedded in the real numbers. In fact, sometimes it is not even be possible to assign certain subsets a measure in any reasonable way (and there are different sorts of measures, all very complicated) :)
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Reply •

Karl Young 2 weeks ago
As a geezer I didn't really want to dust off my functional analysis brain so this proof was nice and worked for me - re. the fact an integer plus a transcendental is transcendental - is that obvious from the fact that plugging that sum into a polynomial (with integer coefficients) just results in another polynomial

Reply •

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Karl Young 2 weeks ago
Sorry hit the wrong key... asI was saying... results in another polynomial (with integer coefficients) which by definition has no solution or am I missing something ?

Reply •

was an algebraic number A that is T+I=A, then we would be able to conclude that T=A-I. However, the difference A-I is a difference between algebraic numbers which is algebraic. And so our assumption that T+I is algebraic gives the impossible

Reply • 1

Karl Young 2 weeks ago
Thanks - straightforward !

Reply •

ffggddss 2 weeks ago
Very nice!
The first thought one (like me, for instance) might have, when faced with the question of how to prove some number isn't algebraic, is, "How in the name of Liouville is that even possible to prove?'

Reply •

ljmtfhgf Fshg 2 weeks ago
i love the T-shirt

Reply •

TheOrganicSquirrels 2 weeks ago (edited)
Very nice, except I'll admit it took me a while to get the main proof. It didn't make sense that if f(L) = 0 then f(L6) and so on would also be guaranteed to be 0, since they are only accurate up to a certain decimal point. But if that number was rational then you would only need an approximation to a certain point since all tailing digits would be the same.

Reply •

johnny 2 weeks ago
Dose prime numbers look less random this way? How dose prime numbers look least random?

If there are infinitely many prime numbers,then we should be able to get infinitely many prime numbers by putting one of the numbers 1,3,7,9 at the end of any real number as the last digit and more and more 0 in between. Like 10–103–1003–10003. 37-3703-37003-370003 and so on. All the way up to infiniety. we can do it with every real number(possible decimal structure).

If we do it with many many real numbers, we might find some numbers that have very different rates or frequencies of providing us prime number when are with different last digits 1 3 7 9. Or even with different alternation of 1 3 7 9.

We are basically linking every real number to prime numbers if it works and not look as random as hell. I think we need to look at prime numbers in different order and find it the least random face possible. non random enough to make furter inefference.

Is there already a name for this operation with prime numbers and real numbers? if so, what are the two numbers that can provide prime numbers most frequently and least frequently?

If it actually work, we can look at prime numbers in as many different way as we can wtih real numbers - actually even more. by putting real numbers in different places and combined it with this prime number feature of them, will a non pattern form?

We can understand more about prime numbers by and in terms of decimal. We can also do the same with something other than decimal.
Show less

Reply •

Gábor Králik 2 weeks ago
This sentence is a lie. → This sentence is true. → This sentence is a lie. → ...
This is paradoxical.

Here´s another speculation:
This sentence is a lie. → This sentence is a JOKE.

Now we have 2 cases:
1.) We consider jokes as lies, but everyone knows that they are lies.
2.) We don´t consider jokes as lies, nor true things.

In case 1.) we have:
This sentence is a lie. → This sentence is a lie. → This sentence is a lie. → ...
which is impossible.

In case 2.) we have:
This sentence is a lie. → This sentence is a joke which is not true. So it´s not true and it´s not a lie. It´s a joke.
This seems legit.

But this doesn´t make the T-shirt any worse.
Show less
Reply •

ben c 2 weeks ago
Great video, proof works for me. So there are uncountable infinity of these. But how many *other* transcendental numbers are there that *don't* have this property of exponentiation being immune to truncation (like pi for example)?
Reply •
Hide replies

Mathologer 2 weeks ago
Unless you have these really long stretches of zeros when you write a number (w.r.t. some base) you won't have this property. Having said that it's quite easy to arrange for this to happen :)
Reply •

ben c 2 weeks ago
Mathologer But pi doesn't have long stretches of zeroes and is transcendental. I mean how many numbers like pi? i.e. transcendental but without long stretches of zeroes.
Reply •

Mathologer 2 weeks ago
How many numbers? Basically "all of them" :) However, the long stretches of 0s translate into those truncations that I talk about being incredibly good rational approximations of L. It then turns out that algebraic numbers cannot be approximated nearly as well using rational numbers as L and a hell of a lot other transcendental numbers (with or without long stretches of zeros). Anyway, all this stuff tends to be very complicated, but most transcendence proofs of specific numbers somehow hinge on finding amazingly good rational approximations to these numbers.
Show less
Reply •

Kasper Rosenlund 2 weeks ago
Okay, I need to get rid of one very important question I have, that might be in everyones interest: Where do you buy/get your t-shirts from? :D
Reply •

Mathologer 2 weeks ago
All over the place, half of them I make myself. Some of the best/funniest ones I have I got from a place called Woot (link to this week's t-shirt in the description :)
Reply •

Kasper Rosenlund 2 weeks ago
Thank you :D - I especially love the hidden nerdy examples!
Reply •

MrWbu 2 weeks ago
Regarding your reply to therealEmpyre: I had the same objection, and you answer does help, but I have a follow-up question. Why is it safe assume that the leading digits of f(T) will match the leading digits fo f(L)? This seems to be based on the fact that the leading digits of T match the leading digits of L. Is that always true that for if T matches L to a
Reply •

agmessier 2 weeks ago
I followed it after thinking through the proof, but I did get stuck at one point. At the 7:30-ish point, where you were talking about how L6, L7, etc. would all fit the polynomial...I didn't quite follow what you meant. I think that's because intuitively I knew that they couldn't all be solutions to the polynomial (i.e. moving the 21 to the other side, how could the various truncations all add up to 21 without approximation?). I finally realized that you were only assuming that they did in order to show a contradiction, and that I should've been thinking only about how the truncations combine as you described earlier. That distinction could've been made more clear.
Show less
Reply •

Zander Rossman 2 weeks ago
11:44 it's actually 10^-120 (4!*5)
Reply •

Mathologer 2 weeks ago

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MusicalRaichu 3 weeks ago
Hey, rational multiples of pi are transcendental, right? that means that trigonometric functions map transcendental some numbers to algebraic numbers. if values like sin 1, cos 2, etc. are transcendental, then these functions also do the reverse. How strange.
Reply •

MusicalRaichu 3 weeks ago
in high school i came up with a similar looking number, 0.101001000100001... = sum of negative triangular powers. you can generalize to sum of integer polynomial powers. i wonder if this class of numbers are transcendental. what about using the fibonacci series for the powers?
Reply •

Mathologer 3 weeks ago
Pretty sure that this number is transcendental. However, you cannot use the arguments used in this video to prove it :)
Reply •

14ercooper 3 weeks ago
This is a very nice way to explain it (and is understandable with high school calculus)
Reply •

ACoral 3 weeks ago
I think polynomial has infinite "solutions", as soon as it is resolved each time only approximately. I mean, if you allow to cut x to some decimal point, then you have to cut your zero in the right part (like 0.0000 , not 0.0000...) and thus you get infinitely many approximate solutions.
Reply •

Salganos 3 weeks ago
That was neat.

(Factorials inevitably grow faster than simple multiples, which is the ultimate growth rate of the precision influenced by a digit through any polynomial.)

((One sentence, at least.))
Show less
Reply •

Mathologer 3 weeks ago
:)
Reply • 1

alexander reusens 3 weeks ago
Great video!
A few questions:
1) can you make a clone from a clone (clone-ception!)
2) why does the 1's have to be place on the factorial places? Can't we make Louiville-ish number by placing 1's at the quadratic places (1,4,9,16,...)? The gaps of zero's will still be arbitrarily large, right? Can't we use any series of numbers that increases faster than a linear series? Or can even linear series be used here?
Show less
Reply •

Mathologer 3 weeks ago
Sure, making a clone from a clone is not a problem. For your second question, the factorial increasing spacing does the job but any spacing that grows even faster will also do. Linearly increasing or quadratic increasing spacing will, I think also give you transcendental numbers. However, you won't be able to use the sort of
Skip navigation
Reply • 1

Marcofrb334 Bastos 3 weeks ago
Hi. i'm big fan of the channel. I have an ideia for a video THE BASEL PROBLEM.

Mathologer 3 weeks ago
Basel problem is on my to-do list :)
Reply • 1

Phi6er 3 weeks ago
This is the best math channel on YouTube. Yeah, you heard me right Numberphile and Infinite Series! Fight me!
Reply •

Attila Kiss 3 weeks ago
Very good!
Reply •

DMSG1981 3 weeks ago
1) I'm so glad to have you back. I already missed the mathologer videos :)
2) Nice video on Cantor sets by PBS Infinite Series: https://www.youtube.com/watch?v=dQXVn7pFsVI
Reply •

The Bearded Math Man 3 weeks ago
Excellent explanation and beautifully made video! Well done on all fronts!
Reply •

andrew sauer 3 weeks ago
This isn't the easiest transcendental to prove. If you want to find a transcendental number, all you need to do is diagonalize over the algebraic numbers. However, this number probably wouldn't have any interesting properties other than being a transcendental number.
Reply •

Mathologer 3 weeks ago
Have a look at the last video :) https://youtu.be/3xyYs_eQTUc
Reply •

Laifs 3 weeks ago
Great!

I'll be waiting for a kindergarten proof that Chaitin's constant is a non-computable definable trancendental real number!

In all seriousness, I think Computable and Definable numbers would be a simple and interesting topic for a video. The proof that all numbers are definable--in a set-theoretic model such as those of ZFC--if being definable is set-theoretic concept, is a simple proof by contradiction; you don't need to know what any of those things mean.
Show less
Reply •

Mathologer 3 weeks ago
Absolutely, a number of people have already suggested to make a video on this, on my list of things to do now :)
Reply • 1

nichonifroa1 3 weeks ago
So gaps of 0's between successive 1's grow infinitely large, which offsets any chance of large integer multipliers (in the polynomial) to 'catch up' on all gaps.
Is that conclusion self-evident? I suppose unless the multipliers are \inf ... How about \infty - 1? I'm just rambling to convince myself I'm not familiar with infinities.
Great video, clear as always :-)
Show less
Reply •

Quartekoen 3 weeks ago
Can someone clarify for me what happens at 7:14? I don't understand.

He says "...the left side would evaluate to 0.00000", which make sense since we're assuming the equation equals 0. However, this says nothing about the actual value of L. He then goes on to take the truncations of L, which he says would all be 0s as well. How can that be since we haven't described L at all? If we're assuming a non-zero value of L that solves the equation, the trunctions would also not all be zeros as he mentions. at 7:37.

Skip navigation

Similarly, if L is zero, then having L6 through Ln all be zeroes as well wouldn't be infinitely many solutions. They would all just be increasingly accurate (more decimals) of the same

I'm assuming he's correct and I'm not understanding it. Thanks.
Show less

Reply •

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hannibal ante portas 3 weeks ago
So before he profed that the clones of l are similar to L in all digits (untill there are only zeros in the clones). Now he uses this. He has a the equation 10x^6-75x^3-190x+21= .If L is a solution 10L^6-10L^3-190L+21=0 is true but if use clones of L (atleast L6 here because its an polynome with the highest power beeing 6), it is true aswell. Because L^6 is too a certain degree the same number as L6^6, which he showed earlier. The same is true for 10L^6-75L^3-190L+21 and 10(L6)^6-75(L6)^3-190(L6)+21 so the solutions of these calculations are the same to certain decimals and after these decimals there are only zeros. So if the solution of the first equation is zero, the second equation is the same up to a certain point and then there only zeros anymore so both equations are zero.
I hope that helped you out.
Show less

Reply •

Mathologer 3 weeks ago
>He says "...the left side would evaluate to 0.00000", which make sense since we're assuming the equation equals 0. However, this says nothing about the actual value of L.

It's important to note that I am not talking about the value of L and its truncations but about the polynomial evaluated at L and at its truncations. We are assuming that the polynomial evaluated at L is equal to zero. Then we know that the leading digits of this number have to be exactly the digits of polynomial evaluated at the truncation (from some truncation onward), etc. :)
Show less

Reply •

Quartekoen 3 weeks ago
Ah, I think I understand. Just like how all digits of the L2, L3, L4, etc... truncations of L^2 match those of L^2, so must the truncations of your 6th-degree polynomial from L6 onward. But if L is transcendental, then that polynomial can't, by definition, equal 0. If it DOES equal 0, then clearly all of its truncations must as well, since it's just 0's all the way down.

This would imply that the different truncations of L would all solve the polynomial, which is, as you said, infinitely many solutions.

Thanks!
Show less

Reply •

Suvi-Tuuli Allan 3 weeks ago
666 \m/

Reply •

erik satter 3 weeks ago (edited)
Adding integers:

Suppose that t is transcendental and n is an integer. Assume that t+n is an integer, then (t+n)-n=t is also an integer which is a contradiction. Thus t+n is transcendental. We use the fact that the integers are closed under addition.

Similarly one can prove that if a is an algebraic number then t+a and t*a are both transcendental.
Show less

Reply •

Mathologer 3 weeks ago
Exactly:)

Reply •

James Lappin 3 weeks ago
why can't you have a number that is 0.0...01? Infinitely small? Limit of zero? Is there such a number?

Reply •
Skip navigation

hannibal ante portas 3 weeks ago
if u have such a number for example name it s: for small what would be 2s, s/2 or s^2 and so on you can`t use this number for equations. So i dont think it is possible to define such a number.

Reply •

Reply •

thermotronica 3 weeks ago
Russels paradox on your shirt
Reply •

kturst s 3 weeks ago
great videos... but can you please use some soothing badgering color instead of white... looking at white is like looking directly at light... hurts my eyes... thank you
Reply •

Imre Pólik 3 weeks ago
A very nice spin on the classical proof that proves that L can be approximated better than real numbers can be.
Reply •

Silly Sad 3 weeks ago
i love your T-shirt
Reply •

Mathologer 3 weeks ago
I got it from here https://shirt.woot.com/offers/liars-paradox
Reply •

Walter Serio 3 weeks ago
i think the proof was very understandable and well explained. however it seems to be entirely based on factorials growing quicker than any polonomial in the long run. in that case, wouldn't it suffice to have a number, that has its digits spaced in exponential gaps?
Reply •

Mathologer 3 weeks ago
Yes, as long as gaps of digits grow fast enough anything goes :)
Reply • 1

mrBorkD 3 weeks ago
(10^(n!))²=10^(2n!)=10^(1*2*3...**n*2) <= 10^(1*2*3...*n**(n+1))=10^((n+1)!) for all n>2
Reply •

mrBorkD 3 weeks ago
this argument applies in general. Let k be the index of the power we're testing for. 10^(kn!)<=10^((n+1)!) for all n>k.
Reply •

Shortcut 3 weeks ago
This channel is giving Numberphile a run for its money. Waddup Brady, You didn have louisville's number yet.
Reply •

Mathologer 3 weeks ago
:)
Reply •

puellanivis 3 weeks ago
I found the "and therefore at some point L_n would have to also be an answer to the polynomial."

The problem is "but each L_n has a small epsilon that keeps it from being identical to L, so I don't see what's going on there."

Then I considered, wait, the polynomial evaluates to zero, that means that at some point all of the digits would have to cancel out. But that would mean that we can eventually find an L_n where the approximation will be correct up to some point, after which, L_n would be zeros, but now all of the polynomial operations performed on it will also produce zeros from that point, and since the polynomial *solves* to zero, that means that at some point, we will end up with an approximation that is "valid" for only x number of digits, but then in the approximation, all the rest of the "invalid" digits will be zero.

Skip navigation

But the solution we're looking for *is* zero. So that means that the "true" solution will have zeros after the "valid" digits anyways, so there is a countably unbounded number of digits necessary to ensure that if the polynomial solves for L, then approximations equal or

Ah... it's hard to describe. Basically, the error shrinks by a rate of O(n!) while the length of digits increases only at a rate O(n^c) (for some constant c related to the power of the polynomial)... and since the error can therefore be made uncountably infinitesimal without reaching an infinite series of digits of accuracy, voilá?
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Reply •

Michael Rothwell 3 weeks ago (edited)
Dear Mathologer, this is a beautiful video, but I'm afraid that the main result about polynomials is not true. Consider the polynomial 1 - L (=0.889998999...) , where the approximations L_n are always wrong in the last decimal (0.9, 0.89, 0.889999 etc). I imagine that this can be fixed by using the following kind of argument: given any ε, an n_0 can be found such that for all n > n_0 we have |L_n - L| < ε*(10^(-n!)) (i.e. the error is as small as you want compared with the last decimal place). This result would extend in a straightforward manner to any polynomial, and the result would follow by showing that if L satisfies the polynomial P(L) = 0, then for an appropriate k, for sufficiently large n 10^k × |P(L_n)| is an integer < 1 so must be zero.
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Reply •

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Mathologer 3 weeks ago
Yes, well spotted, What I say in the video can only be guaranteed to work for polynomial with positive coefficients. To fix things up I think the easiest way to proceed is as follows: Any polynomial equation with a mix of positive and negative coefficients on the left side and the 0 on the right side can be rewritten into one of the form g(x)=h(x) where you only have positive coefficients on both sides e.g. 1-x=0 becomes 1=x, so just move the negative terms to the right side. We get the contradiction by again assuming that L is a solution to an equation like this and then we conclude again that from a certain truncation onward we would necessarily have left g(L_n)=h(L_n), etc.
Show less

Reply • 2

Michael Rothwell 3 weeks ago
Nice fix. Keeps to the spirit of your method, which avoids the kind of technicalities I suggested.

Reply •

Mathologer 3 weeks ago
Yes, but I think it's worth reposting the video with the fix. Thanks for pointing out this flaw in the argument :)

Reply •

Michael Rothwell 3 weeks ago
In that case, may I suggest you go through the "punch line" in more detail. Judging by the comments, a lot of viewers were perplexed by how you get that if P(L)=0 then from a certain point onward P(L_n)=0. You might also want to mention the moral of Liouville number (or better still, where it came from): the result that, in a certain precise sense, any number that can be approximated "too well" by rationals is transcendental. Oh, and don't forget to correct the value of 5x24 at 11:40 :).

Reply •

Mathologer 3 weeks ago
Definitely, posting a fix is not great, but it's the right thing to do and at least also presents an opportunity to finetune parts of the argument that people stumbled across.

Reply •

richard schreier 6 days ago
I hit the same stumbling block as Mr. Rothwell. Your idea of converting p(x)=0 to p_1(x)=p_2(x) with the coefficients of p_1 and p_2 both positive is brilliant. Adding this to the video would make a nice plot twist!

Thanks 1e6 for making these illuminating videos, and kudos for taking the time to respond in such detail to the comments.
Show less

Reply •

Mathologer 6 days ago
+richard schreier I am almost finished editing this update. I'll probably post the update sometime next weekend.

Reply •

Skip navigation

Michael Rothwell 4 days ago
Thanks, I am looking forward to it.

Reply •

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contradiction gives infinitely many DIFFERENT solutions to a polynomial equation. It is not mentioned explicitly why this is the case, but it is a relevant fact and is not hard to see at all :)
Reply •

Mathologer 3 weeks ago
Yes, "different" would be a good word to add :)
Reply •

Jack Lam 3 weeks ago (edited)
Ok. Now we use Louville's Number in the Louville process to create the Louville-Louville Number. And so on.
Reply •

Mathologer 3 weeks ago
Yes, let's have some fun :)
Reply • 1

Kowzorz 3 weeks ago
Does this mean transcendental numbers like in the video can be represented by polynomials of infinite order?
Reply •

Mathologer 3 weeks ago
If you mean to solutions to equations with an infinite series with integer coefficients on one side and a 0 on the other, then the answer is 'Yes" ;)
Reply • 1

d
dylanalexanderful 3 weeks ago
Simply mind blowing stuff. You're like a smarter, older, skinnier and less gay Joe Rogan.
Reply •

Mathologer 3 weeks ago
Had to look up Joe Rogan :)
Reply •

b
beardymonger 3 weeks ago
Thank you!
Reply •

Socrates Alexander 3 weeks ago
We want jpegs of your t-shirts!
Reply •

Mathologer 3 weeks ago
I think what you really want is my t-shirts :)
Reply • 1

3 weeks ago
Y̶O̶U̶
Reply •

Mathologer 3 weeks ago
What language is this?
Reply •

3 weeks ago
YOU
Reply •

Jivan Scarano 3 weeks ago
16:40 You can kind of do it with zero. If you do it with 0.999… and subtract the result from 1 you get the original Liouville number with nines instead of zeroes, and zeroes instead of ones.
Skip navigation
Reply •

anh quốc nguyễn 3 weeks ago (edited)

x=integer. we have an integer + q. COMPLETE THE PROOF
Reply •

Ken Haley 3 weeks ago
That was truly marvelous! Although I don't think I could repeat the demonstration to someone else, I did follow it all the way. I had to take for granted some of the assertions, but those all made intuitive sense to me, so I think it's fair to say that I understood it; even though I'm far from being able to show that the proof is flawless. Nonetheless, I truly
Reply •

Mathologer 3 weeks ago
That's great, glad this worked so well for you :)
Reply •

Tobin South 3 weeks ago
This was actually really fun and approachable! Well done
Reply •

PFC1234 3 weeks ago
I'm third year engineering student and the proof was very understandable. although I think you went too quickly through some important parts
Reply •

Jason Mitchell 3 weeks ago
So your definition of transendental number must have some problem (I'm not a mathametician at all by the way), but you said that a transendental number is one that cannot occur as a solution to a polynomial. So sin can be written as the sum of many polynomial terms and it's solutions are going to be some proportionality of pi, and pi is transendental.

So where am I wrong or did I just split hairs with you on wording?
Show less
Reply •

Mathologer 3 weeks ago
The Maclaurin series for sin that you probably have in mind when you say that "sin can be written as the sum of many polynomial terms" is not a polynomial. Polynomials are finite sums, Maclaurin series are infinite sums :)
Reply • 1

Obinna Nwakwue 3 weeks ago
I hope that this doesn't derail from the topic at hand, but does anyone notice one of the numbers at 1:48?
Reply •

Mike Meyer 3 weeks ago
I can't resist. Adding an integer? hah! How about a proof that applying any algebraic function (such as adding an integer) to a transcendental number is transcendental? Assume that there is a transcendental number t and an algebraic function f such that f(t) is algebraic. But this means there is an algebraic function g such that g(f(t)) = 0. But
Reply •

David Caywood 3 weeks ago
Fantastic stuff Mathologer!!!!
Reply •

Mathologer 3 weeks ago
:)
Reply •

Sami Mas 3 weeks ago (edited)
thanks
does this have to do with why Taylor's series work.
Reply •
Skip navigation

Abound Productions 3 weeks ago
I'm not sure why transcendental numbers are interesting?
Reply •

Reply •

Flobby 3 weeks ago
I love your t-shirt
Reply •

Damien W 3 weeks ago
If L is transcendental the it is not the solution of a polynomial with integer coefficients e.g. x^2 +3x = 10 then you can just change the last number if you add an integer to L
Reply •

Sen Zen 3 weeks ago
1:38 You can't define something in terms of what it's not.
Reply •

Hide replies

Mathologer 3 weeks ago
Of course you can. How about "The odd numbers are all integers that are not even" :)
Reply •

Sen Zen 3 weeks ago (edited)
+Mathologer
Clever, but it's not the same because you have used the concept of an "integer" in that definition, which is a well defined mathematical object, so it's fine.
On the other hand, defining a transcendental number as "a number that is not algebraic" would similarly require a mathematical definition of a general "number". What is a number? That's a question for philosophy, not mathematics.
Show less
Reply •

Mathologer 3 weeks ago
Well instead of "integer" in my example, we use "real or complex numbers" and instead of "even numbers" we use "algebraic numbers" in pinning down the transcendental numbers. And as far as mainstream mathematics is concerned we know what these sets are :)
Reply • 1

Sen Zen 3 weeks ago (edited)
+Christopher Broome
I believe I'm sufficiently knowledgable already about Higher Mathematics, but the statement was not "all the reals that are not algebraic', it was "all numbers are not algebraic", which is not precisely mathematical. Yes I am being picky, but it's important because it means transcendental numbers rely on real numbers, and real numbers rely on infinite choice, which is an axiom in ZFC that I have been doubting as mathematically precise.
Show less
Reply •

Pyjamadeus 3 weeks ago
Well the axiom of choice is known to be independent of the the axioms of ZF, so I guess you can take your C or leave it
Reply •

Sen Zen 3 weeks ago
+Pyjamadeus
But you need C to define Real Numbers, don't you?
Reply •

Pyjamadeus 3 weeks ago
I'm not aware of any such requirement in defining them actually, though you can certainly prove some odd things about them with it.
Reply •

Sen Zen 3 weeks ago
Well how do you define a real number then?
Reply •

Pyjamadeus 3 weeks ago
Skip navigation
A member of any complete Archimedean ordered field - since they're all isomorphic.

If you want an explicit construction of one such field you can take the completion of the rational numbers, or the construction from Dedekind cuts is also quite nice.

Show less

Reply •

Sen Zen 3 weeks ago

+Pyjamadeus
I don't accept the complete Archimedean ordered field definition, because it relies on infinite sets which are not defined. What is an infinite set? How do you define an infinite set?

As for the explicit constructions I'm still not so sure about this, but Cauchy sequences or Dedekind cuts don't actually construct real numbers in the same way as the above axiomatic approach. The difference is they allow for real numbers that you can define algorithmically, but not with infinite choice. For example, you can define the Liouville number algorithmically by 0.000000... and placing 1's in the position of every factorial number (1, 2, 6, 24, 120 etc.). But you can't define a real number by simply choosing an infinite number of digits and putting them together like 1.5485827384920398... because there's no way to represent that with a Dedekind cut or Cauchy sequence. Then you have to throw out all of Cantor's ideas of uncountable infinities, because it invalidates his diagonal argument where he tries to list real numbers in exactly this way.
Show less

Reply •

Pyjamadeus 3 weeks ago (edited)

+Sen Zen
"How do you define an infinite set?" - any set X for which there exists an injective function f:N -> X, where N is the natural numbers. The existence of such a set is guaranteed in ZF (with or without C) by the Axiom of Infinity, which is totally separate from the axiom of choice.

You are definitely referring to something interesting here - seems to me to be related to the idea of computable numbers, those real numbers whose decimal expansion can be computed to any particular precision in a finite amount of time. There of course are only countable many such numbers, meaning that if you accept the existence of the reals and that they are uncountable, then there still must be uncountably many numbers which are not computable, the usefulness/reality of which some question.

I"m not 100% sure where you're coming from regarding the axiom of choice though, at what point in any of these constructions do you think the axiom is required?

EDIT: I awoke with the sudden urge to highlight that I think I've been a bit sloppy in defining an infinite set. The definition I gave would certainly describe a set which is considered infinite in ZF (and one like that is guaranteed to exist by the axiom of infinity), but the idea that such a function exists for all infinite sets might actually require the axiom of choice.... lol anyway you don't need the axiom of choice to define an infinite set, an infinite set exists by definition was really the point to be made there
Show less

Reply •

Sen Zen 2 weeks ago

+Pyjamadeus
I think we're finding a middle ground, though I'm still not exactly sure what my problem with infinite sets is. I know it has to do with functions because one idea of an infinite set is that you can define it by iterating numbers in a function, for example the integers iterated into 2n - 1 gives you all the odd numbers.

But as you said that allows you to build computable numbers, yet the idea of real numbers seem to go beyond that by invoking the idea that you can arbitrarily choose infinitely many numbers and put them into a set, which cannot be done by any algorithm.

Another big problem is what exactly is the definition of a function?
Show less

Reply •

Pyjamadeus 2 weeks ago

Sounds like you're definitely leaning towards intuitionism, which is cool. I'm a more mystical Platonist sort of dude who accepts the objective existence of all mathematical concepts in some sort of infinitely rich mathematical mindscape which we have the pleasure of exploring, but that's not for everyone.

The maths I do I tend to think of as being that part of the mathematical world accessible through the axioms of ZFC (although there a more options), and so given that we can then go on and say "in this setting a function is..." whatever we've defined it as. So indeed, in the context of set theory and ZFC a function between sets A and B is a subset F of AxB such that if (a,b) and (a,c) are elements of F then b=c. That is, it's a special sort of relation where elements of A are related to one and only one element of B. Though someone else might be unhappy with such a definition and require that given an element in A, you can actually construct the corresponding object in B according to some rule like x maps to x^2 or something.

Consider the natural numbers, which I'll denote by N, and the set {0,1}. By the definition of function I gave, I might want to consider the set of all functions from N called the Thue-Morse sequence btw). Now I'd be perfectly happy just to say, no worries I'm happy with that and I can tell that the cardinality of that set is alpeh-1, which is really mystical and cool because I know that means there are uncountably many functions in that set which we can't calculate simply because there can only be countably many algorithms so how about that isn't it neat. Whereas someone more down to Earth than me might say well no unless you can calculate it it's not a function, if it's not at least potentially accessible to the human mind then how can it be said to exist?

But anyway bringing it back in, in the context of ZFC, talking about 2^N (a common name for the set of all functions from N to {0,1}) doesn't require the axiom of choice. There are certainly functions in 2^N which we can't calculate, but the very idea that there is anything in 2^N at all is obvious enough because there are plenty we can explicitly construct , so we don't need to axiom of choice to talk about the "elements of 2^N".

Show less

Reply •

Sen Zen 2 weeks ago

But wait a minute! You defined an infinite set in terms of functions, and you define a function in terms of infinite sets?! That's recursive isn't it, you can't do that?

I actually consider myself a platonist as well, although more precisely I believe only the Natural Numbers exist in physical reality (which allows them to be defined), but then all other concepts such as Integers, Rational Numbers, etc. are constructed from the Natural Numbers, so that makes them abstractions.

Show less

Reply •

P **Pyjamadeus** 2 weeks ago

"God made the integers, the rest is the work of man" - that's some sort of quote. I hope that edit I made pointed out that I didn't think my definition of infinite set was ideal because to show that it's equivalent to the *actual* definition of infinite set requires the axiom of choice, funnily enough. The actual definition, as I'm sure you'll enjoy, is simply a set that's not finite! But the definition of finite still requires functions, so that doesn't get me out of problem. What does get me out of that problem is simply that there is no problem because I defined function as a special kind of relation between two sets but didn't say anything about whether the sets were infinite or not, it really doesn't matter. All you need to define functions this way is the idea of sets and the idea of cartesian products, then with this definition you can say what finite means. So the definition of function comes before that of finite, and then infinite sets are those which aren't finite. One can go on to rightly ask whether infinite sets actually exist, and in the context of ZFC there's just an axiom that says "yeah there's at least one and it contains the natural numbers". And then it seems 'obvious' that infinite sets must contain a countable subsets but actually it turns out the general statement requires the axiom of choice

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Reply •

Sen Zen 2 weeks ago

+Pyjamadeus

Ah but that doesn't work either, and was the original point I was making in the comment I made to start this thread! You cannot define something in terms of what it's not...

Infinite Set = "A Set that is not Finite"

Unless you have a general definition of a set, you cannot define an infinite set in this way. It's like if I created a new type of set, a "transuperset", defined as a set which is neither finite nor infinite.

I think I'm beginning to see the problems now, let's keep going. What exactly is the definition of a "set"?

Show less

Reply •

P **Pyjamadeus** 2 weeks ago

Yes I thought you'd like that!

Ha well as for the definition of a set, we've reached the part of the 'Choose Your Own Adventue' where you get to decide that. Intuitively a set is obviously a bunch of things, but if we're talking about axiomatic set theory then... well let's just say interpreting the symbols is outside the scope of the formal logic lol a set is literally a symbol in the alphabet of a formal language as far as ZFC is concerned.

Not sure what else to say about it other than that. Nice vids by the way you should make more

Show less

Skip navigation Reply •

P **Pyjamadeus** 2 weeks ago

meanwhile numberphile has dropped vid on Gödel's incompleteness theorem! my lucky day

Reply •

Just came from watching the numberphile vid, how about that!

And thanks, you've opened my eyes. I'm a bit preoccupied to make videos right now, but I will definitely return someday. Til then, adios!
Show less

Reply •

upsidedownorangejuice 3 weeks ago
This logical enough to make complete sense
Reply •

Renato Fernandes 3 weeks ago
I really liked this video, very interesting set.
Reply •

MuffinsAPlenty 3 weeks ago
Here's another fun (albeit very abstract) application.

In ring theory, there's a very strange result. Let k be a field. Then the formal power series ring in any finite number of variables over k can be embedded into the formal power series ring in 2 variables over k. In other words, there is an injective ring homomorphism from k[[x1, ..., xn]] to k[[x, y]].

The first step is to prove the following lemma: If you have a set {f1, ..., fn}, in k[[x]] which does not satisfy any homogeneous polynomial in n variables over k, then the k-algebra homomorphism from k[[y·f1, y·f2, ..., y·fn]] to k[[x1, ..., xn]] which sends y·fi to xi an isomorphism, and of course, the domain of this isomorphism is a subring of k[[x, y]].

Once that is proven, you can go on to the main result.

The result is trivial for n = 0, 1, and 2, and by induction, for n larger than 2, it suffices to prove the result for n = 3.

So, you can then show that {1, x, f} does not satisfy any homogeneous polynomials over k for some power series f in k[[x]]. And it suffices to show that there is a power series in k[[x]] which is "transcendental" over k[x]. And to do this, one usually creates a sort of "Liouville power series" f = 1 + x + x^(2!) + x^(3!) + x^(4!) + ... + x^(n!) + ...

And the proof that you gave of the transcendence of Liouville's number also works for showing that f is transcendental over k[x] if you replace 10^(-1) in your proof with x.

And so that's a fun little example of how an idea from analysis can be used in algebra.
Show less

Reply •

Mathologer 3 weeks ago
Very nice :) The "transcendence" result that I am really interested in talking about is this fact there are are integrals of elementary functions that are not elementary. People often mention that e^(-x^2) is an example of such a function but I have not seen a really accessible proof. Would be worthwhile putting something like this

Reply •

MuffinsAPlenty 3 weeks ago
Oh, that would be an amazing topic to discuss! I have to admit that I have never looked into any proofs on this myself. It would be nice to see that.

Reply •

michalchik 3 weeks ago
love the shirt
Reply •

sidg11 3 weeks ago
Hi! Love your videos. Any chance you could do a video on
1 + 2 + 3 ...... = -(1/12)
Reply •

Sempoo 3 weeks ago (edited)
Good graphics - Helvetica still rules!
Reply • Skip navigation

Lukas Franke 3 weeks ago
I love your shirt
Reply •

Reply •

xnick 3 weeks ago
Masterful video. The quality of the presentations and the footage composition is outstanding.

I guess it would've been possible to have the abstract formal demonstration side by side for comparison... but perhaps that would make the video way too long.

I guess that the transcendency of this number hinges in the fact that n! will grow larger that n^m as n goes to infinity, for arbitrary m. If I'm correct, maybe some emphasis on this is in order. n_n
Show less
Reply •

Treufuß 3 weeks ago
Oh, I wonder if someone will start the discussion over 1.2300000... actually being equal to 1.2299999... XD
Reply •

Exact Parrot 3 weeks ago
Treufuß in the reals yes, in the hyper reals no.
Reply •

Treufuß 3 weeks ago (edited)
In that case it depends how to interpret 1.2299999... exactly. I would see it as the series 1*10^0+2*10^(-1)+2*10^(-2)+9*10^(-3)+9*10^(-4)+9*10^(-5)+9*10^(-6)... And afaik no matter if we consider it in the rational, real, complex or hyper real numbers. This series is convergent and has a limit of exactly 1.23.

But if we interpret 1.229999... to mean any kind of number smaller than 1.23 but larger than any *real* number smaller than 1.23 you are correct, this kind of number exists in the hyper real. But in this case, there are infinitely many such numbers and 1.2299999... would then represent a whole set of numbers with this property.
Show less
Reply •

Frank Harr 3 weeks ago
I'd have to listen again.

And again.

And again.

I was a German major. Drill doesn't scare me. :)
Show less
Reply •

William Rutherford 3 weeks ago
In the bit where you show the factorials up aginst the number, the numbers are "1 2 3 24" shouldn't they be "1 2 3 4" or "1 2 6 24" ?
Reply •

Kram1032 3 weeks ago
Ok so.
Where in the world do you get these amazing shirts from?
Reply •

Miguel Sanchez 3 weeks ago
Patterns build with patterns to build patterns completely different from the first ones. Amazing mathness! LOL! Thank you for posting all these videos.
Reply •

asgallant 3 weeks ago (edited)
At around 7:00, you state "all the digits of our approximations will be correct from some truncation onward". I think this is incorrect. It should be "all the digits of our approximations will be correct up to some truncation", which dramatically alters the math here. Given some polynomial P of order k with integer coefficients, all approximations L(n+i) (for some minimum n dependent on k, i >= 0) will agree with L to a truncation at roughly (n + i)! * k digits. P(L) != P(L(n+i)) for all but a maximum of k - 1 values of i (because P can have at most k roots). The assertion that P(L) = 0 would necessitate P(L(n+i)) = 0 for all i >= 0 itself is false. Showing that it must be true if L is polynomial would be a proof by contradiction, but you didn't do that here. What am I missing?

MuffinsAPlenty 3 weeks ago

You're not actually truncating (Ln)^m; you're only truncating L^m. Each Ln has only finitely many nonzero digits, and, therefore, (Ln)^m has only finitely many nonzero digits. But since the nonzero digits of L are so spread out, for each m, all the nonzero digits of (Ln)^m will still agree with the corresponding digits of L^m for sufficiently large n.

So for any polynomial P, for sufficiently large n, P(Ln) will agree with P(L) for the first however many digits, and then the rest of the digits of P(Ln) will be 0. Any digits of P(Ln) which may not agree with P(L) are 0. So if P(L) = 0, it follows that P(Ln) = 0 too.

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Reply •

asgallant 3 weeks ago

I think I get it now. If P(L) agrees with P(L(n)) to some decimal place, and all digits of P(L(n)) after that digit are zero, if P(L) = 0, then all of the digits where they agree are also zero, thus P(L(n)) = 0.

Reply •

ThePower HeX 3 weeks ago

As an 8th grade mathologer fan, I would love to buy a textbook written by yourself, as I believe you make these topics really easy to understand. Do you have any?

Reply •

Ryan Roberson 3 weeks ago

0=1-1 therefore 1-0.9900090000.... is transcendental! 0 is no exception! just gotta sweet talk it a bit.

Reply •

Ryan Roberson 3 weeks ago

hi there!

Reply •

Karl Lind 3 weeks ago

My attempt at proving the little task you gave:

The last non-zero part of (L_n)^2 will be 10^~~(2*n!). Meanwhile, the largest part of L^2 that doesn't match up with (L_n)^2 will be 10^~~((n+1)!+1). (n+1)! + 1 > 2*n! when n is a natural

Reply •

Travis Wolfenberger 3 weeks ago

The language of the was fantastically accessible, but I only got about 3 minutes in before my mind was kind of overloaded with concepts/information--that is, my bailing out of this video is on me, not you

Reply •

chunkyq 3 weeks ago

Does this map from the reals to the transcendentals have any nice properties? It doesn't preserve Lebesgue measure, but it does preserve order. Anything else?

Reply •

Alin-Mihai Dobre 3 weeks ago

Awesome explanation !

Reply •

Maxwell Wibert 3 weeks ago

I love how simple this proof is.

Reply •

Sigmad 3 weeks ago

How do you create videos suitable and fun both for high school kids and graduate math loving adults? No other channel does that.

Skip navigation

Reply •

Anon 3 weeks ago

L6 ^ 6 is only the same as L ^ 6 up to a certain number of digits. If the digits after that point are nonzero then L6 isn't a root...

Reply •

Anon 3 weeks ago
Oh wait. Now I get it. L6 ^ n has a finite number of digits for all n. Of course.

Reply •

Eric Prates 3 weeks ago
simply fantastic.

Reply •

Deldarel 3 weeks ago
I was going into this video prepared to not understand anything. However, it's an extremely easy and clear proof.
I even think you were dragging it a bit on how the gap widens. It should be clear that approaching infinity gives you an approachingly infinite amount of zeros and as long as you raise it to the power of a real number (so not infinity), there will always be a time when the gap appears and becomes wider.

I would love to see the proof of this subset being 0 since it seems to me that there is a one to one correlation to all objects in either set.
Show less

Reply •

Danil Dmitriev 3 weeks ago
Hello, Mathologer! First of all, a very insightful and interesting video :) It was a bit hard for me to understand every bit of it on the first attempt, but the consequent review fixed everything. I will try to do the subtitles until tomorrow evening. Meanwhile, I have a few suggestions for how this video could be done even more understandable.

At 7:00 you're saying that all digits of the approximation to a polynomial will be correct, starting from some truncation. Some people have been noting already that they are correct only up to the end of non-zero digits of this approximation. I think that it would be better to add "will be correct up to the end of the approximation", or to give an alternative phrase like "all non-zero digits of our approximation will be correct". It would probably make this point of the video clearer.

The claim at 14:23 about the correctness of polynomial approximations starting from some truncations, was not clear to me on the first watch. I did not grasp the fact that the non-zero digits of our polynomial approximation will end on the n-th place, where n is the largest (fartherst to the right) place of the red-marked digits in powers of L in the expression.
I think that it would be beneficial to elaborate it a bit more; for instance, by saying that the value of our polynomial approximation will "end" on the red digit which is fartherst to the right across all powers of L in the polynomial (in case of the example on the screen, up to the 12th digit after the dot). And this is why, if we have at least some gap between the red and the green digits in all powers of L in the polynomial for some truncation, our approximation will be correct in all its non-zero digits for this truncation.
I am pretty sure that my explanation is far from ideal, but I hope that it will help you to make this point better in future.

Finally, I am really buffled by the fact that the "transcendental clone" set is (kind of?) of the same size as R, but at the same time has measure 0... You completely blowed my mind at this point of the video :)

Isn't this fact a contradiction, though? If two sets are connected by a one-to-one correspondence, then they must have the same cardinality. (Or at least so I thought until now :D) And continuum sets do not have zero measure. This seems to suggest that either R and the "transcendental clones" set do not have a one-to-one correspondence between them, or that the set of clones should also be of a non-zero measure... Since the latter seems to be wrong according to you, and the former also looks unlikely, I feel a bit lost. And thrilled to see the continuation video :)
Show less

Reply •

Danil Dmitriev 3 weeks ago
And one last question, to reflect on the fact that I am lost :) If transcendental clones do correspond one-to-one with R and they do have measure 0, does this say something about the existence of a set which has a larger cardinality than natural numbers, but smaller cardinality than R?

Reply •

Skip navigation
(NL) 3 weeks ago (edited)
We see it's true because (k+1)! + 1 > 2k! <-> k + 1 + 1/k! > 2, which is definitely true for k >= 1.

Reply •

paper (after just watching video Im sure I wouldnt be able to repeat the proof).

This all leads me to a question - we are not stuck with the n! space between digits, right? If we will call this video "the first method of creating a clone to every trans numder", then we can come up with "the second method" by making the space between numbers, say. 2*n! And then 3*n! and so on, so we also got infinitly many trans clones sets. Im perfectly sure we can come up with a method that will produce uncountably many methods of creating such sets, since we are not limited in zeros.
Right?

Dont know why anybody would need an uncountably infinite number of sets of uncountably infinite clones in one's everyday life, but you seem genuinly excited about it)

Cool tshirt
Show less

Reply •

Kumar Suyash Rituraj 3 weeks ago
i want that t shirt !!
Reply •

Darkstar 3 weeks ago
I wonder why Liouville chose this rather complicated number with digits at factorial(n). I mean wouldn't the same argument work if the ones were (for example) placed at 2^n or even n^2? If so, why the more complicated formula? If not, at what spacing does it break and why? For example, would it work if the runs of zeroes are of length floor(ln(n)) or
Reply •

MrCheeze 3 weeks ago (edited)
Darkstar I believe the only property that is necessary here is that n! grows faster than n^k asymptotically, for all k. Which means that 2^n would have worked just as well (but not n^2). I may be forgetting something here, though.
Reply • 1

Darkstar 3 weeks ago
But even in the case of the gaps growing slower than n^k, you'd just have to move "further out" for the argument to work again. It probably makes the proof less straightforward, but I think it would still work... Or maybe I'm missing something too :)
Reply •

Franz W 3 weeks ago
Are there practical uses for Liouville's number? If I switch all 0's with 1's and the other way round, is that number still transcendental?
Reply •

vote for no. 6 3 weeks ago (edited)
Franz W Yes, because if it was an algebraic number, say a, then L = 1/9 - a would also be algebraic.
Reply •

Enzo Giannotta 3 weeks ago
Thank you for the video! I wonderfully enjoyed it all.
Reply •

Polariod Fury 3 weeks ago
This makes sense, except when you are taking the L^2 term and adding L, I don't know what you're trying to show there.
Reply •

Mathologer 3 weeks ago
Well up to that point we've only shown that things work as claimed for all powers. But that is not good enough. We need to show that it also works for all polynomials with integer coefficients. L+L^2 is an example for such a polynomial and I use it to illustrate how what we've learned about the powers extends to polynomials :)
Reply •

Polariod Fury 3 weeks ago
Skip navigation
ah, ok.
Reply •

schadenfreudebuddha 3 weeks ago
But...the L1 to the 5 truncation isn't correct. It should round up to .00002, right?

DeadFish37 3 weeks ago
I seem to be missing the basic proof. I see no reason why a truncation of L should be a root, just because L is a root. L still has digits that its truncation doesn't, even if the truncation is correct up to its final non-zero digit
Reply •

Mathologer 3 weeks ago
Because all the digits of the polynomial evaluated at the truncation coincide with that of the polynomial evaluated at L (which we assumed to be all zeros if it is to be a solution to the respective polynomial equation :)
Reply •

František Mrkus 3 weeks ago
What is the transcedent number definition?
Reply •

Mathologer 3 weeks ago
I give the definition at the beginning of the video. Well, sort of. I am really focussing on the real transcendental numbers in this and the previous video. Anyway, in general a transcendental number is defined to be a number (real or complex) that is not a solution to any polynomial with integer coefficients :)
Reply •

František Mrkus 3 weeks ago
+Mathologer Okay, so the only way to write it is this equation?
1+1/10^1!+1/10^2!+1/10^3! etc... ?
Reply •

h4ck314 3 weeks ago
Haha seems that your argument is still more elaborate than the usual cardinality argument (namely that the set of algebraic numbers is countable).
Reply •

Hide replies

Mathologer 3 weeks ago
That's what the last video was about :)
Reply •

vote for no. 6 3 weeks ago
h4ck314 But this is a constructive proof. The other isn't.
Reply •

Steve's Mathy Stuff 3 weeks ago
You can construct a transcendental number using diagonalization on a list of the algebraic reals. Prof. Polster outlined a method for doing so in his last video and found the first 6 digits or so of a transcendental number. But, unless you have better intuition than I have, you probably won't know going in the first time what number will be produced. The order of the digits in the transcendental number depends on the order in which you list the algebraic numbers.

Liouville started with a specific number and showed that it was transcendental.
Show less
Reply •

Alex Tritt 3 weeks ago (edited)
<Trying to prove that adding an integer to a transcendental number gives a transcendental number, so spoilers if you want to figure it out for yourself>

Skip navigation

Let's say you have polynomial equation Σn a_n*x^n = 0 (eq A) for n between 0 and some finite N, a_n are integers. Let y = x + k, k integer. Then a_n*x^n = a_n*(y - k)^n, which is a polynomial in y with integer coefficients (raising y - k to a natural power results in some natural number (from Pascal's triangle) times a power of k (which is an integer) times a power of y (which makes it a polynomial in y)). Then the polynomial equation can be written as Σn b_n*y^n = 0 (eq B) for n between 0 and N, b_n being integers. So if x solves eq A then y = x + k solves eq B. So if x is algebraic then x + k is algebraic since it is also a solution to a polynomial equation with integer coefficients.
Let's say a number u is transcendental but v = u + (-k) is algebraic. Then by what we've

transcendental.
Show less
Reply •

Chris Bullers 3 weeks ago (edited)
I'm wondering why the definition of transcendental numbers is defined by not being the solution to polynomial equations with INTEGER coefficients, which implies that transcendental numbers MIGHT be the solutions to polynomials with non-integer coefficients. One can always multiply through an equation that is equal to zero by the product of the denominator of all of N rational coefficients to have an equation that is still equal to zero with integer coefficients,, so surely its trivially true to say that transcendental numbers are not the solutions to polynomial equations with RATIONAL coefficients. I would be interested to know then if its a known proof if you can not get transcendental numbers from polynomial equations with IRRATIONAL (non-transcendental) coefficients, as this seems like the next logical step. Intuitively it would seem not. In which case, the definition using integers seems even more unnecessarily restrictive.
Show less
Reply •

Hide replies

Mathologer 3 weeks ago
It is also true that every solution of a polynomial equation with algebraic coefficients is algebraic :)
Reply • 1

Chris Bullers 3 weeks ago
Thought so. I guess it IS a bit circular to define transcendental numbers as those which cannot be obtained from polynomials with coefficients of the set of reals minus the transcendental numbers!
Reply •

Treufuß 3 weeks ago
You are right, it **is** the same, since you can always multiply an equation such that you get integer coefficients. That's why it is easier to restrict yourself to integers and not need to think extra about rational numbers.

Reply • 1

Chris Bullers 3 weeks ago (edited)
Yeah I get that, but since its also true for non-rational algebraic numbers, I guess I always prefer the most generalised definitions. Trivial of course, but maybe more interesting that trans coeffs can give algebraic solutions (ex-e=0) which means you can map from one set to the other, but only in one direction... Rationals/Algebraics
Reply •

Cowslayer6789 3 weeks ago
That number is a little base 10y
Reply •

Hide replies

Mathologer 3 weeks ago
Well spotted :)
Reply •

Cowslayer6789 3 weeks ago
Mathologer would t be possible it prove that not only

10^-1!+10^-2!+10^-3!+...
But
X^-1!+X^-2!+X^-3!+...
Show less
Reply •

vote for no. 6 3 weeks ago
Cowslayer6789 The number whose base b expansion is the same is also transcendental, for any b >= 2.
Reply •

Kasper Meerts 3 weeks ago
Congratulations on 200,000 subscribers!
Skip navigation
Reply •

Mathologer 3 weeks ago
Thank you very much. Yes, it's party time this weekend :)
Reply • 2

Reply •

Art Schell 3 weeks ago (edited)

For any power L^X, at approximation A, Y = XA! and G = (X-1) + (A+1)A!. For arbitrarily large A, A+1 + X/(A+1)> X and thus G>Y.

Reply •

betabenja 3 weeks ago

uh. if f(x) represents the mapping from a real number x to the Liouville clone and you put in both the versions of the real number to get a transcendental number and not a transcendental number, then that means f(x) can distinguish between the two versions of the real number and they are therefore not equal.... where is the flaw?

Reply •

Hide replies

Steve's Mathy Stuff 3 weeks ago

Your premise is incorrect. For any x, f(x) cannot be both a transcendental number and an algebraic number. f is a function from R to R so it assigns to each real number a unique real number. As demonstrated in the video, f(1.23) = 1.220009000000000000000009000 ..., a transcendental number. f does not assign any other number to 1.23.

What might be confusing is that the function f is defined differently for real numbers with finitely many non-zero digits and real numbers with infinitely many non-zero digits. Using the wrong definition to find f(1.23) would lead one to believe that f(1.23) = 1.23000000000000000 ...

Show less

Reply •

betabenja 3 weeks ago

hm. It seems a bit unsatisfactory, but probably correct. Essentially you are saying that if f(x) is defined such that it always gives a Liouville number the choice of which representation of R to use must be included within f(x).

Otherwise, f(x) is just a function which sometimes give transcendental numbers and sometimes does not. Still, in this second version lets call it g(x) where you just blindly take the next digit and place it n! further down the decimals I find it difficult to believe you might arrive at drastically different answers. If the two representations of x are equal, then g(x) is not a function that can be written algebraically, in which case, neither is f(x).... Thats probably where I'm going wrong, treating f(x) as a function rather than a mapping.

Show less

Reply •

Steve's Mathy Stuff 3 weeks ago

Yes. f is defined so f(x) is a Liouville number.

A mapping is a function. Different branches of mathematics add extra baggage to the term, but a mapping is always a function.

"in this second version lets call it g(x) where you just blindly take the next digit and place it n! further down the decimals I find it difficult to believe you might arrive at drastically different answers."

Huh? Unless I am completely misunderstanding the definition of g(x), you can see that g(x) is drastically different from f(x) for numbers with only finitely many non-zero digits.

" If the two representations of x are equal, then g(x) is not a function that can be written algebraically, in which case, neither is f(x)"

The natures of g and f are not dependent on 1.23 being equal to 1.22999 ...But if you mean g and f are not polynomials, that is correct.

Show less

Reply •

marioalonsojr120 3 weeks ago

this is a wonderful video as a birthday present.

Reply •

Acrom McLain 3 weeks ago

Skip navigation sum from 1 to n of n! < n*n! < (n+1)! +1

Reply •

vote for no. 6 3 weeks ago

Acrom McLain Do you mean sum k = 1 to n of k! < (n+1)! + 1?

HYEOL 3 weeks ago
Guess I just have to watch it more than once
thumb up
Reply •

L'ascolto Del Venerdì 3 weeks ago
666: math and illuminati... AGAIN!
Reply •

Psyq Watts 3 weeks ago
Bravo! Great proof!
Reply •

John Jönsson 3 weeks ago
nth
Reply •

Pyry Kontio 3 weeks ago
Yeah, another Mathologer video! Gonna start watching it now!
Reply •

bridogg154 3 weeks ago
I love u
Reply •

Ovidiu Hancu 3 weeks ago
Uuuuu maths
Reply •