

$$f(1)+f(2)+f(3)+\dots+f(n) =$$

$$\begin{aligned}
 & \int_0^n f(t) dt \\
 & + \frac{f(n)-f(0)}{2} \\
 & + \frac{B}{2!} (f'(n)-f'(0)) \\
 & + \frac{B}{4!} (f'''(n)-f'''(0)) \\
 & \dots
 \end{aligned}$$

There is a slight asymmetry in the formula in that all the infinitely many terms on the right side of the equation are “bounded” (gesture inverted comma) by an n (click) and a 0 (click) whereas the $f(1) + f(2) + \dots$ sum on the left side of the equation is bounded by an n (click) and a 1

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 & + \frac{f(n) - f(0)}{2} \\
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 & + \frac{B^4}{4!} (f'''(n) - f'''(0)) \\
 & \dots
 \end{aligned}$$

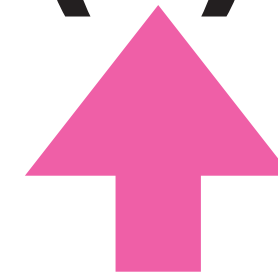
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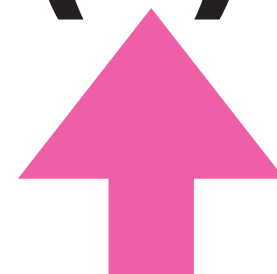
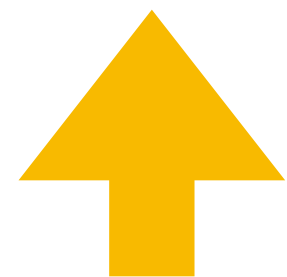
$$+ \frac{B^2}{2!} (f'(n) - f'(0))$$

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$$f(1) + f(2) + f(3) + \dots + f(n) =$$



$$\int_0^n f(t) dt$$

0

$$f(n) - f(0)$$

2

+

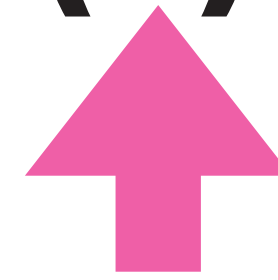
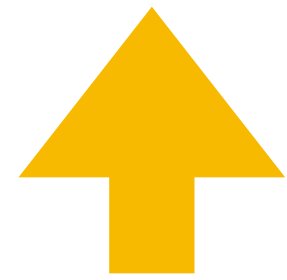
$$+ \frac{B_2}{2!} (f'(n) - f'(0))$$

$$+ \frac{B_4}{4!} (f'''(n) - f'''(0))$$

...

There is a slight asymmetry in the formula in that all the infinitely many terms on the right side of the equation are “bounded” by an n and a 0 whereas the $f(1) + f(2) + \dots$ sum on the left side of the equation is bounded by an n and a 1

$$f(\mathbf{1}) + f(2) + f(3) + \dots + f(\mathbf{n}) =$$



$$\int_{\mathbf{0}}^{\mathbf{n}} f(t) dt$$

$$+ \frac{f(\mathbf{n}) - f(\mathbf{0})}{2}$$

$$+ \frac{\mathbf{B}_2}{2!} (f'(\mathbf{n}) - f'(\mathbf{0}))$$

$$+ \frac{\mathbf{B}_4}{4!} (f'''(\mathbf{n}) - f'''(\mathbf{0}))$$

...

Obviously we won't be able to sleep tonight unless we balance things out :) Luckily this is easy to do. Just add $f(0)$ on both sides like this.

$$f(0) + f(1) + f(2) + f(3) + \dots + f(n) =$$

$$+ \int_0^n f(t) dt + f(0) + \frac{f(n) - f(0)}{2}$$

$$+ \frac{B^2}{2!} (f'(n) - f'(0))$$

$$+ \frac{B^4}{4!} (f'''(n) - f'''(0))$$

...

$$f(0) + f(1) + f(2) + f(3) + \dots + f(n) =$$

$$\begin{aligned}
 & \int_0^n f(t) dt \\
 & + \frac{f(n) - f(0)}{2} + f(0) \\
 & + \frac{B^2}{2!} (f'(n) - f'(0)) \\
 & + \frac{B^4}{4!} (f'''(n) - f'''(0)) \\
 & \dots
 \end{aligned}$$

$$f(0)+f(1)+f(2)+f(3)+\cdots+f(n)=$$

$$\int_0^n f(t)dt$$

$$+ \frac{f(n)-f(0)}{2} + f(0)$$

$$+ \frac{B}{2!} (f'(n)-f'(0))$$

$$+ \frac{B}{4!} (f'''(n)-f'''(0))$$

...

$$f(0)+f(1)+f(2)+f(3)+\cdots+f(n)=$$

$$\begin{aligned}
 & \int_0^n f(t) dt \\
 & + \frac{f(n)+f(0)}{2} \\
 & + \frac{B}{2!} (f'(n)-f'(0)) \\
 & + \frac{B}{4!} (f'''(n)-f'''(0)) \\
 & \dots
 \end{aligned}$$

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 & \dots
 \end{aligned}$$

Okay, so at this stage the lower bound is 0 everywhere in sight and we'll definitely be able to sleep tonight in this respect.

$$f(0) + f(1) + f(2) + f(3) + \dots + f(n) =$$

$$\begin{aligned}
 & \int_0^n f(t) dt \\
 & + \frac{f(n) + f(0)}{2} \\
 & + \frac{B^2}{2!} (f'(n) - f'(0)) \\
 & + \frac{B^4}{4!} (f'''(n) - f'''(0)) \\
 & \dots
 \end{aligned}$$

Having said that there is nothing special about 0 and we can in fact replace 0 by any integer less than the upper bound n to arrive at the completely general formula. In fact, most books start with 1 as the lower bound instead of 0. So let's also perform this final mathematical cosmetic surgery.

$$f(\mathbf{1}) + f(2) + f(3) + \dots + f(n) =$$

$$\int_1^n f(t) dt$$

$$+ \frac{f(n) + f(\mathbf{1})}{2}$$

$$+ \frac{\mathbf{B}_2}{2!} (f'(n) - f'(\mathbf{1}))$$

$$+ \frac{\mathbf{B}_4}{4!} (f'''(n) - f'''(\mathbf{1}))$$

...