$$f(1)+f(2)+f(3)+\cdots+f(n) = \int_0^n f(t)\,dt + \frac{f(n)-f(0)}{2} + \frac{B_2}{2!}\left(f'(n)-f'(0)\right) + \frac{B_4}{4!}\left(f'''(n)-f'''(0)\right) \cdots$$

There is a slight asymetry in the formula in that all the infinitely many terms on the right side of the equation are “bounded” (gesture inverted comma) by an n (click) and a 0 (click) whereas the f(1) + f(2)+etc. sum on the left side of the equation is bounded by an n (click) and a 1

$$f(1)+f(2)+f(3)+\cdots+f(n) =$$

$$\int_0^n f(t)\,dt$$

$$+\ \frac{f(n)-f(0)}{2}$$

$$+\ \frac{B_2}{2!}\left(f'(n)-f'(0)\right)$$

$$+\ \frac{B_4}{4!}\left(f'''(n)-f'''(0)\right)$$

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Obviously we won't be able to sleep tonight unless we balance things out :) Luckily this is easy to do. Just add f(0) on both sides like this.

$$f(0)+f(1)+f(2)+f(3)+\cdots+f(n)=$$

$$\int_0^n f(t)\,dt$$

$$+\ \frac{f(n)-f(0)}{2}\ +\ f(0)$$

$$+\ \frac{B_2}{2!}\left(f'(n)-f'(0)\right)$$

$$+\ \frac{B_4}{4!}\left(f'''(n)-f'''(0)\right)$$

$$\cdots$$

$$f(0)+f(1)+f(2)+f(3)+\cdots+f(n)=$$

$$\int_0^n f(t)\,dt$$

$$+\ \frac{f(n)-f(0)}{2}+f(0)$$

$$+\ \frac{B_2}{2!}\left(f'(n)-f'(0)\right)$$

$$+\ \frac{B_4}{4!}\left(f'''(n)-f'''(0)\right)$$

$$\cdots$$

$$f(0)+f(1)+f(2)+f(3)+\cdots+f(n)=$$

$$\int_0^n f(t)\,dt$$

$$+\quad \frac{f(n)-f(0)}{2} + f(0)$$

$$+\frac{B_2}{2!}\left(f'(n)-f'(0)\right)$$

$$+\frac{B_4}{4!}\left(f'''(n)-f'''(0)\right)$$

$$\cdots$$

$$f(0)+f(1)+f(2)+f(3)+\cdots+f(n)=$$

$$\int_0^n f(t)\,dt$$

$$+\frac{f(n)+f(0)}{2}$$

$$+\frac{B_2}{2!}\left(f'(n)-f'(0)\right)$$

$$+\frac{B_4}{4!}\left(f'''(n)-f'''(0)\right)$$

$$\cdots$$

$$f(0)+f(1)+f(2)+f(3)+\cdots+f(n)=$$

$$\int_0^n f(t)\,dt$$

$$+\ \frac{f(n)+f(0)}{2}$$

$$+\ \frac{B_2}{2!}\left(f'(n)-f'(0)\right)$$

$$+\ \frac{B_4}{4!}\left(f'''(n)-f'''(0)\right)$$

$$\cdots$$

Okay, so at this stage the lower bound is 0 everywhere in sight and we’ll definitely be able to sleep tonight in this respect.

$$f(0)+f(1)+f(2)+f(3)+\cdots+f(n)=$$

$$\int_0^n f(t)\,dt$$

$$+\frac{f(n)+f(0)}{2}$$

$$+\frac{B_2}{2!}\left(f'(n)-f'(0)\right)$$

$$+\frac{B_4}{4!}\left(f'''(n)-f'''(0)\right)$$

$$\cdots$$

Having said that there is nothing special about 0 and we can in fact replace 0 by any integer less than the upper bound n to arrive at the completely general formula. In fact, most books start with 1 as the lower bound instead of 0. So let's also perform this final mathematical cosmetic surgery.

$$f(1)+f(2)+f(3)+\cdots+f(n)=$$

$$\int_1^n f(t)\,dt$$

$$+\ \frac{f(n)+f(1)}{2}$$

$$+\ \frac{B_2}{2!}\left(f'(n)-f'(1)\right)$$

$$+\ \frac{B_4}{4!}\left(f'''(n)-f'''(1)\right)$$

$$\cdots$$