The following proof is from the book Lectures on elliptic curves by J.W.S. Cassels (pages 55 and 56).

We conclude this section by giving one of Fermat's own descent arguments.

He wished to show that there are no integer solutions of

$$X^4 + Y^4 = Z^4 \qquad X \neq 0, Y \neq 0.$$

This is a curve of genus 3 (not that Fermat knew about the genus), but he remarked that it is enough to disprove

$$X^4 + Y^4 = Z^2 \qquad X \neq 0, Y \neq 0 \qquad (*)$$

On writing (*) in the shape

$$(Z/Y^2)^2 = 1 + (X/Y)^4$$

one sees that we have an elliptic curve, though not given in canonical form. However, following Fermat, we consider integer solutions of (*).

If (*) has an integral solution, we take one $(\mathbf{x}, \mathbf{y})$ for which

$$\max(|x|, |y|)$$

is $> 0$ and as small as possible. (|| is the absolute value). Then $x$, $y$, $z$ have no common factor, and indeed are coprime in pairs. Since $x^4 \equiv 1$ mod 4 if $x$ is odd, one of $x$, $y$ must be odd and the other even. We suppose that

$$2 \mid x, \qquad 2 \not| \, y, \qquad 2 \not| \, z.$$

Write (*) in the shape

$$(z + y^2)(z - y^2) = x^4.$$

Since $z$, $y$ are both odd, the two factors on the left are divisible by 2 but only one is divisible by 4. Hence (taking $z > 0$) we have two possibilities, where $u, v \in \mathbb{Z}$:

| | First Case | Second Case |
|---|---|---|
| $z + y^2 =$ | $8u^4$ | $2u^4$ |
| $z - y^2 =$ | $2v^4$ | $8v^4$ |

The first case gives

$$y^2 = 4u^4 - v^4,$$

which is impossible mod 4. Hence we have the second case:

$$y^2 = u^4 - 4u^4.$$

Now

$$(u^2 + y)(u^2 - y) = 4v^4,$$

and so

$$u^2 + y = 2v^4$$
$$u^2 - y = 2s^4$$

for some $r$, $s \in \mathbb{Z}$. Hence

$$v^4 + s^4 = u^2.$$

This is another solution of (*). Further,

$$x^4 = 16u^4v^4 = 16u^4r^4s^4.$$

This contradicts the assumed minimality of the original solution, and so we have a contradiction.