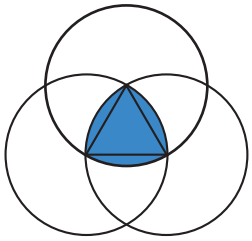
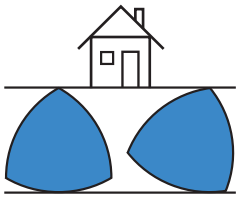
MathSnacks
"Semi" Circle

by Marty Ross, Burkard Polster, and QED (the cat)

Rolling Reuleaux

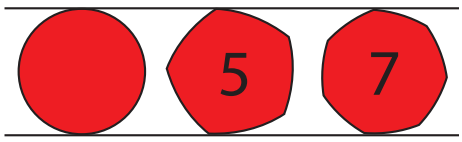




Start with an equilateral triangle, and use it to draw three circles: each circle is centered at one vertex and passes through the other two vertices. The region captured by the three circles is known as Reuleaux's triangle.

Reuleaux's triangle has *constant width*, equal to the radii of the circles. That is, no matter which way a Reuleaux triangle is squeezed between two parallel lines, the distance between the lines stays the same. So, if you make huge rollers in the shape of Reuleaux triangles, you can transport your house smoothly along them.

Repeating Reuleaux



There are many other shapes of constant width (SCW's). The circle is an SCW, obviously! More interstingly, we can take a regular polygon with an odd number of sides and draw the circles as for Reuleaux: the centre of each circle is at a vertex, and the circle passes through the two vertices furthest away. The result is an SCW.

Ripper Reference

The Enjoyment of Mathematics, H. Rademacher, O. Toeplitz, Princeton, 1966.

Dramatic Drill

A Reuleux triangle will roll smoothly inside a square of just the right size. It touches all four sides at all times, and all of the square gets touched except for small rounded corners. Based on this, there is an ingenious machine which can drill (almost) square holes.

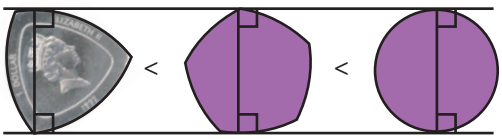
The famous *Wankel engine* is similarly based on the constant width property of Reuleux's triangle.

Clever Coins

When a circular coin is inserted in a vending machine, it is identified by measuring its width. This means that an SCW-shaped coin will be just as easy to recognize. The British have issued some beautiful coins of this type.

The Australian 50 cent coin is not an SCW. But it is a regular 12-gon, and the large number of sides means that it is very close to being circular.

Altered Area



Every SCW has a number of "circle properties". First, any "diameter" of an SCW meets the encasing parallel lines at right angles. This can be used to show that an SCW manhole cover cannot fall through the hole. This is why manhole covers are round!

Secondly, Any SCW has the same perimeter as the circle of the same width. It then follows that the circle is the SCW of *greatest* area amongst all SCW's of that width.

It is also known that Reuleaux's triangle is the SCW of *smallest* area for a given width. So, we should make Reuleaux coins, and save that precious metal!