MathSnacks
The Famous Five
by Marty Ross, Burkard Polster, and QED (the cat)

Plato's Penta

tetra (4) octa (8) icosa (20) hexa (6) dodeca (12)
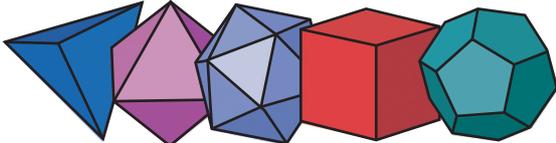
hedron (= faces)

There are infinitely many flat regular polygons– the equilateral triangle, the square, the regular pentagon, and so on. By comparison, there are only five regular polyhedra in three dimensions. Plato associated the tetrahedron with fire, the octahedron with air, the icosahedron with water, the cube with earth and the dodecahedron with the basic building block for the heavens. The Greek numbers in the names refer to the numbers of faces of these shapes.

Fixedly Five

A regular polyhedron is one having identical regular polygons as faces, the same number of such faces around each vertex, and with no concavities. Why should there be exactly five of them?

For several copies of a regular polygon to combine into a corner, they have to leave a gap when arranged around a point in the plane. This is only possible with three or four or five equilateral triangles, three squares, and three pentagons.

This gives a total of five possible corners, every one of which corresponds to one of the Platonic solids.

Cunning Cuts

When you slice the Platonic solids in just the right way you can expose unexpected cuts. For example, you can get equilateral triangles and regular hexagons from the cube and squares from the tetrahedron. In the pictures above, the vertices of these polygons are all midpoints of edges of the cube or tetrahedron.

Corpus Cubus
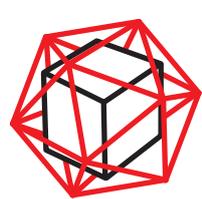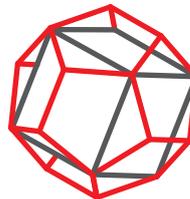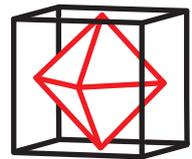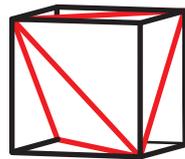
All Platonic solids can be found in and around the cube. Take every second vertex of the cube and you get the vertices of a tetrahedron. The six midpoints of a cube's faces are the vertices of an octahedron. Placing suitable identical tents on its faces gives the dodecahedron. And, even the icosahedron can be circumscribed around the cube in a symmetric fashion. And the cube is not special in this respect: take any of the other solids and the other four can be inscribed or circumscribed just as nicely.

Ripper Reference
Daud Sutton, Platonic & Archimedean Solids, Walker, 2003