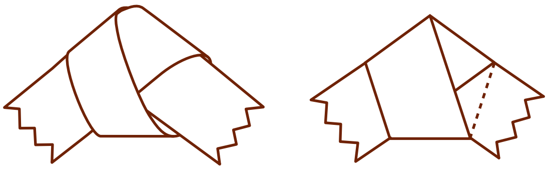
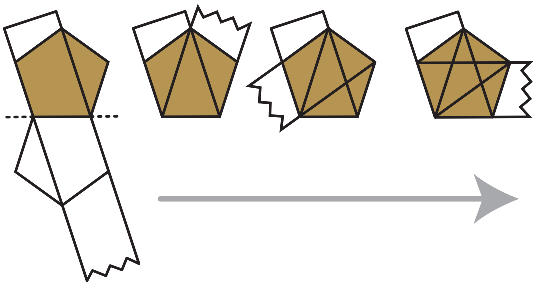
# MathSnacks
## Going, Going, Gon

by Marty Ross, Burkard Polster, and QED (the cat)

### Perfect Pentagon



Tie a paper strip into a knot and pull on the ends until the knot is completely flat. Cut off the excess bits of paper on both sides and you are left with a regular pentagon. Why does this works?



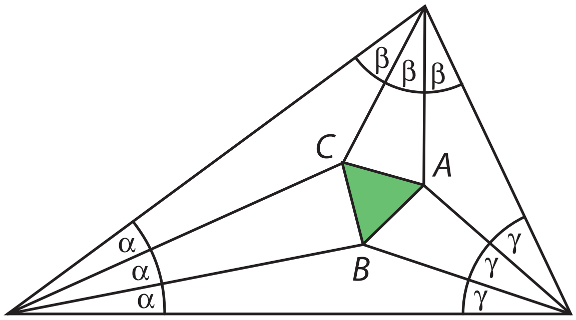
Consider two regular pentagons with a common side, plus a paper strip running through both of them. If we fold the bottom pentagon onto the top along the common side, the paper strip will neatly align itself along one of the sides of the top pentagon. Continuing, if we keep folding the paper strip around this top pentagon, we will successively define all its sides. We unwrap the now creased paper strip and discard the pentagons. Finally, we can tie the paper strip into a knot and flatten it such that no new creases appear.

The regular polygons with more than five sides can also be knotted using one or two paper strips. However, the practical execution of most of these constructions is very awkward.

### Ripper Reference*

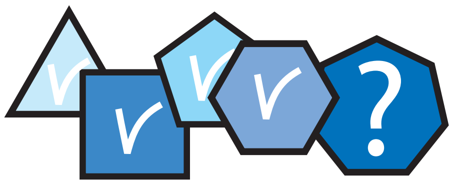
www.cut-the-knot.org/triangle/Morley

### Morley's Miracle




In 1899, the mathematician Frank Morley discovered an amazing theorem about triangles, now commonly referred to as *Morley's Miracle*. Basically, it says that in any triangle there is hidden an *equilateral* triangle. Here is how you find it: start with any triangle and trisect each of its angles. Then three of the points of intersection *A*, *B*, and *C*, are the vertices of an equilateral triangle. Amazing, and also amazing that the ancient Greeks missed out on this one!

### Harried Heptagon




One of the famous geometry problems of antiquity consisted in finding ways to construct regular $n$-gons using only a compass and an unmarked ruler. It is easy to construct equilateral triangles, squares and regular pentagons. Then, because the bisection of angles is also easy, we can construct all regular $n$-gons where $n$ is of the form $2^k$, $2^k3$, or $2^k5$: thus the list begins 3, 4, 5, 6, 8, 10, 12, 16, 20, 24, 32, 40,...

For centuries this is all that people knew. Finally, in 1796 Carl Friedrich Gauss discovered a wonderful connection between this problem and the *Fermat primes*. These are prime numbers of the form $2^{2^n}+1$, the first five of which are 3, 5, 17, 257, and 65537 (and we don't know of any more!). Gauss proved that a regular $n$-gon can be constructed with ruler and compass if $n$ can be written in the form $2^k m$ where $m$ is 1 or the product of different Fermat primes. For example, a regular 7-gon cannot be constructed with ruler and compass, but regular 15-gons and 17-gons can.