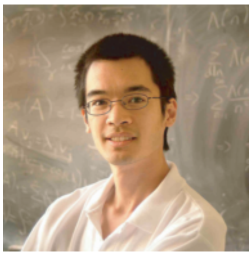
MathSnacks

Proofs Without (too Many) Words

by Marty Ross, Burkard Polster, and QED (the cat)

First Fields



In 2006, Terry Tao became the first Australian to be awarded the Fields Medal, the highest prize in mathematics. The official announcement described Terry as "a supreme problem solver whose spectacular work has had an impact across several mathematical areas ... He combines sheer technical power ... and a startlingly natural point of view that leaves other mathematicians wondering 'Why didn't anyone see that before?'

Prime Progression

7 157 307 457 607 757 907

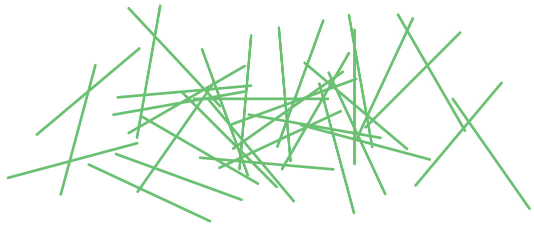
In an arithmetic sequence the difference between any two consecutive terms is the same. The above is an arithmetic sequence of length 7 of prime numbers. It is extremely difficult to find such sequences; currently, the longest known arithmetic sequence of primes is of length 24. In 2004, Terry Tao and Ben Green proved that there must be arithmetic sequences of primes of arbitrarily long length. Their proof is non-constructive: they give no hints on how to explicitly find such sequences.

Ripper References*

Terry Tao's Webpage: www.math.ucla.edu/~tao

Fields Medal Announcement: www.icm2006.org/dailynews/fields_tao_info_en.pdf

Krazy Kakeya



The *Kakeya Problem* asks what is the set of *smallest area* needed to rotate a needle of unit length. In 1928, Abram Besicovitch proved the counter-intuitive result that no matter how little area A we are permitted, a planar region of area A can be found within which the needle can be rotated!

A *Kakeya Set* is a region which contains an interval of unit length in every direction. So, any candidate for the Kakeya Problem is a Kakeya Set, but not necessarily vice versa. In 1971, Roy Davies proved that a Kakeya Set in the plane must be 2-dimensional; no one knows the dimension of Kakeya Sets in three- and higher-dimensional spaces; in 2001, Terry Tao and Nets Katz proved that the dimension of a Kakeya Set in n-dimensional space must be at least

$$(2-\sqrt{2})(n-4)+3$$

Horn's Hermitian

$$\begin{pmatrix} 4 & 1-i \\ 1+i & 5 \end{pmatrix}$$

The conjugate of a complex number a+b*i* is a-b*i*. Take a square matrix M of complex numbers and transform it into a second matrix by first flipping it across its upper left to lower right diagonal and then turning all its entries into their conjugates. If the resulting matrix is the same as M then it is called *Hermitian*. *Horn's Conjecture* asks about the so-called eigenvalues of the sum of two Hermitian matrices, an important question in the field of numerical analysis. In 1999, Terry Tao and Allen Knutson proved Horn's Conjecture. The Fields Medal announcement referred to Tao (and Knutsen) having proved this as "... akin to an English-language novelist suddenly producing the definitive Russian novel."