\( \phi \) from Fibonacci?

\[ a, b, a+b, a + 2b, 2a + 3b, 3a + 5b, \ldots \]

D.B. claims that "phi [the golden ratio] was derived from the Fibonacci sequence." In truth, Euclid thought of \( \phi \) around 300 BC, about 1500 years before Fibonacci came up with his numbers. It is true that you can get \( \phi \) from the Fibonacci sequence; however, you can get \( \phi \) from many sequences. Start with any positive numbers \( a \) and \( b \), and form a sequence by adding, as shown. (If \( a = b = 1 \), then this is exactly the Fibonacci sequence). Then, whatever \( a \) and \( b \), the ratio of adjacent terms in this sequence approaches

\[ \phi = 1.618... \]

The beautiful chambered nautilus forms (approximately) a logarithmic spiral: that is, the ratio of two successive radii does not depend upon the direction. However, this constant ratio is about 2.9: despite what D.B. says, it has nothing to do with \( \phi \! \)!

Fibbing Bees

What proportion of bees in a hive are female? D.B. claims that the ratio of females to males is \( \phi \). In reality, the ratio is anywhere from 50:1 to 100:1.

What is true is that the number of ancestors of a given bee are all Fibonacci numbers. Suppose \( F_n \) and \( M_n \) are the number of female and male ancestors in the \( n \)th generation. Then \( F_{n+1} = F_n + F_{n-1} \). But, \( M_n = F_{n-1} \) since male bees have no father. Combining these equations, we find

\[ F_{n+1} = F_n + F_{n-1}. \]