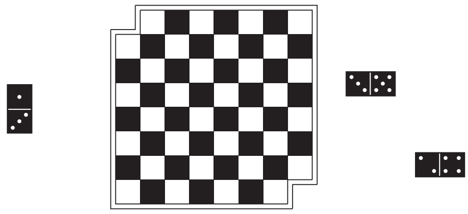
MathSnacks
Perfect Puzzles

by Marty Ross, Burkard Polster, and QED (the cat)

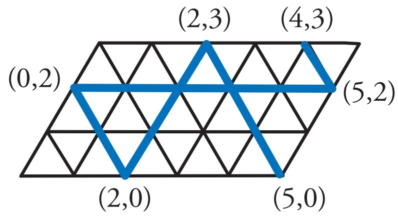
Chessboard Challenge



It is easy to see that a chess board can be tiled with dominoes, each of which covers two squares. Suppose we remove two diagonally opposite corner squares. Is it it still possible to tile what's left?

Answer: No! In any tiling, each domino covers one white square and one black square. Hence any tiling covers the same number of white and black squares. Since the indented board has two more white squares than black squares, it cannot be tiled.

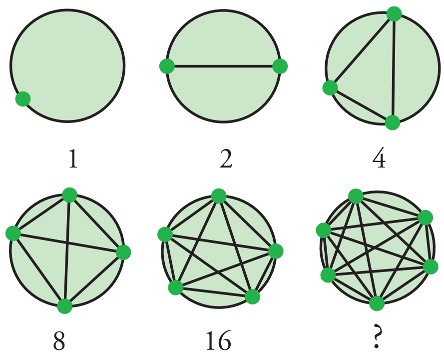
Juggling Jugs



In the movie *Die Hard: With a Vengeance,* John (Bruce Willis) and Zeus (Samuel Jackson) are given a 5 gallon jug and a 3 gallon jug; then (with 30 seconds to think before they are blown to smithereens), they are ordered to use a fountain to fill the larger jug with exactly 4 gallons of water, using nothing but the jugs.

Solution: The points in the grid correspond to the different ways in which the two jugs can be filled with water. The horizontal lines represent the filling or emptying of the 5 gallon jug; the lines of positive slope represent the filling or emptying of the 3 gallon jug; and the lines of negative slope represent the transferring of water between the jugs. Then, one solution to our problem is traced by the blue path: (5,0)-(2,3)-(2,0)-(0,2)-(5,2)-(4,3). So, you start by filling the 5 gallon jug from the fountain, then you fill the 3 gallon jug from the 5 gallon jug (leaving 2 gallons in the 5 gallon jug), then you empty the 3 gallon jug, etc.

Sneaky Circles*



What's next in the sequence: 1, 2, 4, 8, 16,...? Select n points on the edge of a circle, and join all the points to each other by straight lines. Into how many regions does this cut the circle?

Answer: The first five cases suggest that the answer is 2^{n-1}. However, for $n = 6$ the answer depends upon where we place the points, and the *maximum* number of regions is 31, not 32! In general, the maximum number of regions is

$$\frac{1}{24}(n^4 - 6n^3 + 23n^2 - 18n + 24).$$

Coin Confusion*

If a coin rolls without slipping around another coin of the same size, how many times will it rotate while making one revolution? Just once, or twice? What if the fixed coin is twice the diameter of the rotating coin? How many revolutions will the coin make if it rolls around 2 coins (all coins being of the same size) that are place side by side? Or n coins?

Answers: In the next issue of *Vinculum*!

Ripper References*

David Acheson, 1089 and All That, Oxford, 2002
Martin Gardner, Mathema-