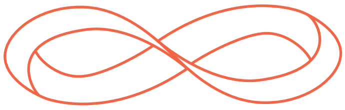
VINCULUM VOLUME 42 NUMBER 1 MARCH 2005

MathSnacks
Inconceivable Infinity
"Infinity is where things happen that don't."

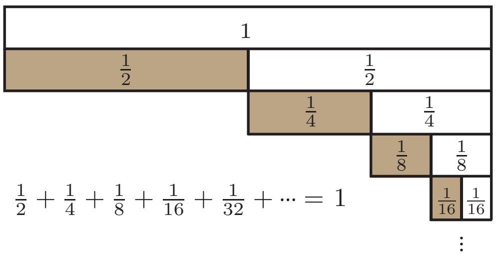
by Marty Ross, Burkard Polster, and QED (the cat)

Suspicious Symbol



The common symbol for infinity was introduced by John Wallis in 1655. It is often combined with the mystery of the Möbius band. The symbol stands for many things and explains nothing.

Romantic Zeno



$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\cdots=1$$

In the movie *I.Q.*, Meg Ryan walks half the distance to Tim Robbins, and then half the remaining distance, then half the remaining distance, ... Does she ever reach him? The *potentially* infinite process means that Meg can get as close to Tim as she desires. The *completed* infinity view is to sum *all* the fractions (the brown boxes fill the top box) and to claim she actually gets there!

Disappearing Decimal

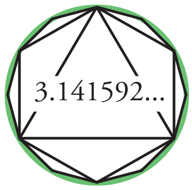
$$0.99999... = 1$$

Why is that?* Well, whatever x = 0.99999.... is, multiplying by 10 moves the decimal point, to give

$$10\,x = 9.99999...$$

Subtracting x gives $9x = 9$, and so $x = 1$.

Dodgy Dots



The dots suggest "and so on, forever", but what do they *really* mean. With 0.9999... it is clear that each dot is a 9. But the decimal expansion of π is mysterious, and here the dots merely cloak the mystery. The number π *can* be simply seen as an infinite object, as the area of an infinite-sided polygon. Does that make any sense? Yes! As a completed infinity, the process of considering the area polygons of radius 1 with more and more sides.

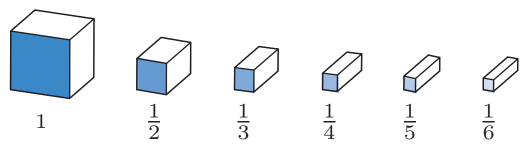
Stunning Sums

$$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots=\frac{\pi^2}{6}$$

This beautiful sum was found by Leonhard Euler in 1735. No one knows a simple way to express the sum of reciprocal cubes. On the other hand, if we remove the powers, we find

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots=\infty$$

Paradoxical Playtoy



Imagine an infinite set of square blocks, all of thickness 1, and of side lengths 1, 1/2, 1/3, ... The total *volume* of the blocks is $\pi^2/6$, but the *surface area* (just consider the tops of the blocks) is infinite. So, it would take a finite amount of wood to build the blocks, but an infinite amount of paint to paint them!

Ripper References*

B. Polster, *Q.E.D.*, Walker, 2004
E. Maor, *To infinity and Beyond*, Princeton, 1991