MathSnacks
Prime Power

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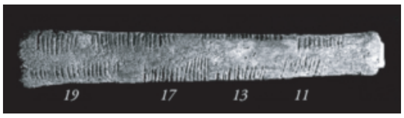
Prime Cuts



The 20 000 year old *Ishango Bone* was discovered in Congo, Africa in 1961. It contains various groups of notches, including in order the groupings 11, 13, 17 and 19. Is this coincidence, or was this the calculation of a Prehistoric Pythagoras? *

Some 20 000 years later, the real Pythagoras knew about even and odd numbers, but seemingly little else.

Natural Primes

In the movie "Contact", the aliens send their radio pulses in prime number groups, to indicate their intelligent origin. But it is possible that prime numbers also occur in nature. Periodical cicadas have synchronized births, with mass hatchings once every 13 or 17 years. These prime number life cycles may ensure that cicadas do not repeatedly hatch into the waiting mouths of their predators; any predator with a life cycle of less than 13 years will not be prepared for two consecutive hatchings.

Producing Primes

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70

The first prime number is 2. Circling all proper multiples of 2, the smallest integer that is not circled is 3, the second prime number. Next, circling all proper multiples of 3, the smallest integer that is not circled is 5, the third prime number. Circle the proper multiples of 5, and so on. This method of distinguishing the primes from the composite numbers is the famous *Sieve of Eratosthenes* (*c.* 250 B.C.)

Euclid (*c.* 300 B.C.) proved that there are *infinitely* many primes by showing that from any collection $p_1, p_2, \ldots, p_n$, of *finitely* many primes, we can always create a new one. To do this, set

$$q = p_1 \cdot p_2 \cdot p_3 \cdots p_n + 1$$

When q is divided by any of p_1 to p_n, the remainder is 1, so *none* of these prime numbers are factors of q. But q *does* have prime factors: we can choose any of them to be our new prime!

Prime Patterns

$2^{24,036,583}-1$

It is difficult to see any pattern in the sequence of prime numbers, to guess where the next will occur. Write $\pi(n)$ for the number of primes no bigger than n. A careful use of Erastothenes's approach shows that $\pi(n)/n$ is very small if n is large; what this means is that the *probability* of a large number n being prime is very small.

You can win a million dollars for proving Bernhard Riemann's (1826 - 1866) *Hypothesis* on the distribution of prime numbers. More easily, join the hunt for huge prime numbers. The largest known prime, with almost a million digits, was discovered in 2004 on a personal computer. *

Ripper References*

http://www.mersenne.org
A. Marshack, The Roots of Civilisation, Moyer Bell, 1991