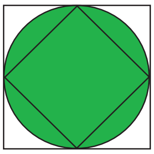
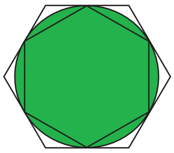
MathSnacks
how i wish i could calculate pi

by Marty Ross, Burkard Polster, and QED (the cat)

Piblematic



$\pi = 3$

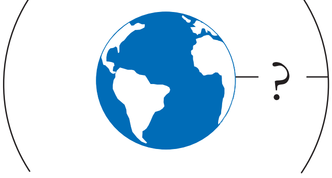


Well, so implies the Bible: *And he made a molten sea (vessel), ten cubits from the one brim to the other: it was round ... and a line of thirty cubits did compass it round about.* (I Kings 7:23). Other ancient approximations for π are 3 1/8 (Mesopotamia, *c.* 1800 B.C.) and $(16/9)^2$ (Egypt, *c.* 1650 B.C.).

What π really *is* is the ratio of the *circumference* of any circle to its *diameter*. Archimedes (*c.* 250 B.C.) was the first to use this definition to properly estimate π. By inscribing and circumscribing a circle with rgular polygons, he proved that π lies between 3 10/71 and 3 1/7. (What are the approximations given by the squares and hexagons above?) More importantly, Archimedes' method can be used to obtain π to any desired accuracy.

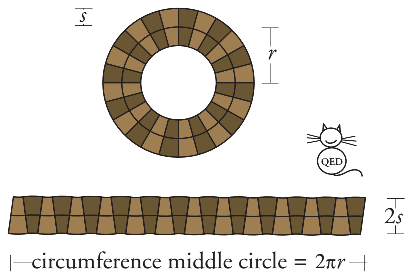
Of course, all of the above rational values for π (*including the decimal number hidden in our title*) can only be approximations, since π is irrational (Joseph Lambert, 1766).*

Pi Puzzle



A simple formula can give unexpected answers! Suppose there is a loop of string that is tightly wrapped around the Equator. After lengthening the loop by one meter, it will lie a bit above the surface of the Earth: how far? Compare your result to the radius of a circle whose circumference is one meter. What happens if you replace the Earth by a sphere of different radius, but leave the rest of the thought experiment unchanged.

Pi Pies



From the formula for the circumference of a circle, a simple idea allows us to derive the area formulae for rings and circles. Take a circle of radius r and thicken it to make a ring of width $2s$. Dissect the ring and then arrange the resulting slices into a wobbly rectangle. As we use more and more slices, we obtain *exactly* a rectangle. Hence

$$\textit{area ring} = \textit{area rectangle} = 4\pi rs.$$

By choosing $r = s$, the ring turns into a circle of radius $R = 2r$. This gives the area formula for a circle:

$$\textit{area circle} = \pi R^2.$$

Now, think of the first diagram as the view from above of a do-nut, a *torus*. Dissecting and rearranging as before, eventually gives a cylinder, (which from the top looks like a rectangle). Hence

$$\textit{surface area torus} = \textit{surface area cylinder} = 2\pi r \cdot 2\pi s,$$
$$\textit{volume torus} = \textit{volume cylinder} = 2\pi r \cdot \pi s^2.$$

Precisely Pi

$$\frac{8}{1\cdot 3} + \frac{8}{5\cdot 7} + \frac{8}{9\cdot 11} + \cdots$$

This infinite sum is equivalent to the oldest explicit expression for π (Madhava, *c.* 1400). There are many, many others, the flipside of which is that π make surprising appearances in the solutions to many problems. For example, pick two natural numbers m and n at random. What is the probability that m and n have no common factors? The answer is $6/\pi^2$!

Ripper References*

P. Beckmann, A History of Pi, St.Martin's Press, 1971.
Pi (the movie, 1998), available on VHS and DVD.