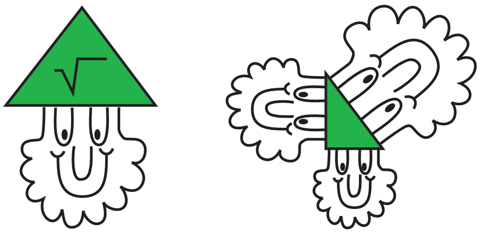
MathSnacks Pythagoras & Co.

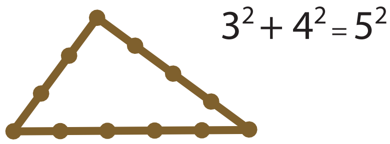
by Marty Ross, Burkard Polster, and QED (the cat)

Scary Scarecrow



In *The Wizard of Oz*, the Scarecrow is granted his Doctor of Thinkology and promptly recites: *The sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side.* What are the three mistakes? Are there sides of *any* isosceles triangle which satisfy the Scarecrow's equation? Danny Kaye sings a correct version of the Pythagorean Theorem in *Merry Andrew*.

Tantalising Triples



A *Pythagorean triple* consists of three natural numbers that satisfy the Pythagorean equation. This leads to an ancient method of constructing a right angle: taking a loop of rope with 3+4+5=12 equally spaced knots, then the *converse* of the Pythagorean Theorem guarantees that the triangle is right-angled.

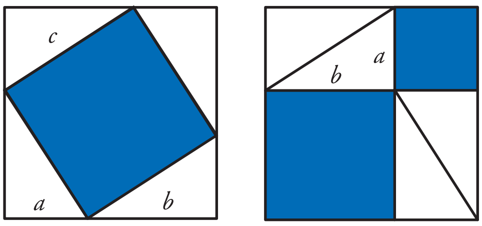
A simple way to construct some (but not all) Pythagorean triples is based on the equality

$$(n+1)^2 = n^2 + (2n+1).$$

Just choose $2n+1$ to be an odd square! For example, $2n+1=25$ corresponds to the triple 5:12:13. There is also an elegant formula which gives rise to *all* Pythagorean triples.*

The presence of Pythagorean triples on the Babylonian clay tablet Plimpton 322 demonstrates that people knew of the Pythagorean Theorem long before Pythagoras.

Perfect Proof



Of course, the Pythaogorean Theorem says that for a right-angled triangle the area of the large square is the sum of the area of the two smaller squares. In algebraic terms, this is the famous equation

$$a^2 + b^2 = c^2.$$

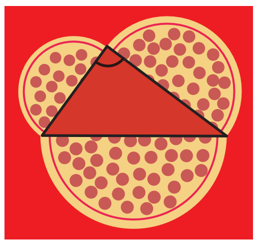
Proof: Fit four copies of the triangle in the corners of a square of size $a+b$ *(above, left)*. This leaves a tilted square of sidelength c in the middle. Summing the areas, we have

$$c^2 + 4 \cdot 1/2 \cdot a \cdot b = (a+b)^2,$$

and thus $c^2 = a^2 + b^2$.

We can dispense with the algebra by reshuffling the triangles *(above, right)*. We then simply observe that the blue areas in both diagrams are equal: c^2 on the left and $a^2 + b^2$ on the right. There are many other lovely proofs of the Pythagorean Theorem.*

Pizza Puzzle



A large pizza costs the same as a medium and a small pizza together. Which should we buy? To answer this, cut the pizzas in half, and make a triangle out of the diameters. If the angle opposite the large pizza is a right angle, then the area of the large pizza is exactly the sum of the areas of the two smaller pizzas: that's Pythagoras in action! If the angle is greater than a right angle, then it's better to buy the large pizza, and otherwise we should buy the two smaller pizzas.

Brilliant Books*

R. Nelson, Proofs Without Words, I & II, MAA, 1993, 2000.
J. Stillwell, Numbers and Geometry, Springer, 1997.