Repeating Rectangle

A rectangle is golden if removing a square leaves a smaller rectangle of the same proportions. So, if $L$ and $W$ are the dimensions of such a rectangle, then

$$\frac{L-W}{W} = \frac{W}{L}.$$  

Golden rectangles are famous for their role in aesthetics, though this is as much numerology as mathematics.*

MathSnacks
the Golden Ratio
by Marty Ross,
Burkard Polster,
and QED (the cat)

Ideal Irrational

$\phi$ is the number most easily proved to be irrational. For, supposing $\phi$ is rational, we could make a golden rectangle with sides $L$ and $W$ integers. And so, the first square pictured also has integer sides, of length $L-W$. Then, the smaller square still has integer sides, and so does the next smaller one, and the next one, and so on. But this infinite diagram, with all sides positive integers, is clearly impossible: so, the original golden rectangle could not have had integer sides, and thus $\phi$ is irrational!

Incredible Icosahedron

The mathematics of golden rectangles contains genuine beauty. Defining the golden ratio $\phi$ (Phi) to be $L/W$, we see

$$\phi - 1 = \frac{1}{\phi}.$$  

This equation, together with the Pythagorean Theorem, shows how to physically make an icosahedron, by slotting three golden rectangles together, as pictured.*

Ingenious Infinitum

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \ldots}}}.$$  

But what is $\phi$? We can multiply the equation $\phi = 1+1/\phi$ by $\phi$, and then solve the resulting quadratic equation to give

$$\phi = \frac{1 + \sqrt{5}}{2}.$$  

Or, since $\phi = 1+1/\phi$, we can substitute $1+1/\phi$ for $\phi$, giving

$$\phi = 1 + \frac{1}{1 + \frac{1}{\phi}}.$$  

Substituting again and again and again....

Fibonacci Formula

$$\frac{\phi^n + (1 - \phi)^n}{\sqrt{5}}$$  

The sequence 1, 1, 2, 3, 5, 8, ... is the famous Fibonacci sequence. The next Fibonacci number is $5+8=13$, and so on. What if we want the 1000th Fibonacci number, or in general the $n$th one? We can churn them out, one by one, or we can use the magical formula above.*


Brilliant Books*

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