WE, THE LIVING

Michael A.B. Deakin, Monash University

I went to school in Tasmania and there, when I was in 5th grade, a religion teacher asked us how we knew that we would die. Eventually, one kid stuck his hand up and said, when called on to speak: "Well, er, most people have so far". This answer was seen by the teacher as a piece of typical Tasmanian understatement and the reply was accepted as correct.

But is it?

The world's population is exploding – i.e. growing at an alarmingly rapid rate. I have seen it written that "Of all the people who have ever lived, half are alive at this moment". This statement (call it Statement S), if true, denies the Tasmanian boy's reasoning.

I set out to see how true Statement S actually is. In what follows, I develop a simple mathematical model. It does accept that we will all die – we know this on grounds other than the Tasmanian boy's answer. There will be a lot of simplifying assumptions in the model but they will not be so wild as to invalidate the general conclusions to be drawn from it.

It will be convenient to measure *time* in 5-year periods. Imagine that every five years we take a census of the total human population of planet Earth. To make matters clear, suppose we are about to count all the people alive as we reach 0:00 hours Greenwich Mean Time on January 1, 1990. Call this number $X(0)$.

Now suppose that a similar exercise had been carried out precisely five years previously on January 1, 1985. Call the count then $X(1)$. And for the similar count on January 1, 1980, write $X(2)$. And so on.

As a (very good) approximation, we will suppose that the census counts

$$X(0),\ X(1),\ X(2),\ \ldots$$

form a decreasing geometric sequence. (This is known as Malthus's Law, after the Reverend Thomas Malthus, who first formulated it; see *Function Vol. 2, Part 3*, p.21.) We thus write

$$\begin{aligned} X(1) &= rX(0) \\ X(2) &= rX(1) \\ X(3) &= rX(2) \end{aligned}$$

etc., and in general

$$X(n+1) = rX(n). \tag{1}$$

To relate our model to reality, we first need an estimate of the value of r. Malthus, back in 1798, took the doubling time – the period that it took for a population to double its size – as 27 years. More recently it has become somewhat less. A quite reasonable estimate is 25 years, that is to say, 5 census periods.

From this, we can conclude that

$$X(5) = r^5X(0) = \tfrac{1}{2}X(0), \tag{2}$$

and so

$$r^5 = \tfrac{1}{2}$$

or

$$r = 2^{-1/5} \simeq 0.87, \tag{3}$$

to a good approximation.

At first it might be thought that all we need do now to estimate the total number of people who have ever lived is to add up the series

$$X(0) + X(1) + X(2) + \dots .$$

This, however, would give an overestimate, as many people will appear in more than one census. Some could end up being counted 20 times or more.

To overcome this problem we need to look at the Malthusian model in more detail.

At the next census there will be $X(0)$ people. Of these, there will be the $X(1)$ people from the last census, along with those born since, but less those who have died in the meantime. Let $BX(1)$ be the number born and $DX(1)$ the number who have died. B, D are referred to respectively as the *birth* and the *death rates*. We will treat them as constant: a reasonably good approximation.

We thus have

$$X(0) = X(1) + BX(1) - DX(1),$$

and similarly

$$X(1) = X(2) + BX(2) - DX(2),$$

and, in general,

$$X(n) = X(n+1) + BX(n+1) - DX(n+1).$$

I.e.

$$X(n) = (1+B-D)X(n+1). \tag{4}$$

Compare Equation (4) with Equation (1). This gives

$$1 + B - D = \frac{1}{r}. \tag{5}$$

Then at the next census $BX(1)$ previously uncounted people will be included. At the last $BX(2)$ were counted for the first time. And so on. The total number (T, say) of people who have ever lived is thus

$$BX(1) + BX(2) + BX(3) + \dots .$$

Thus

$$\begin{aligned} T &= B[X(1) + X(2) + X(3) + \dots] \\ &= B[rX(0) + r^2X(0) + r^3X(0) + \dots] \\ &= BrX(0)[1 + r + r^2 + \dots]. \end{aligned}$$

So

$$T = \frac{BrX(0)}{1-r}. \tag{6}$$

To proceed further we need to know a value for B. The simplest way to estimate this is indirectly, via Equation (5).

Let us suppose that the average life-span is 40 years. Indeed, to simplify the calculation, suppose everybody lives *exactly* 40 years. This is unrealistic, of course; we'll see later what might happen if we look at matters more realistically. But if we take the simple calculation first we find those dying between the 1985 and the 1990 census will be precisely those born between the 1945 and the 1960 censuses. The first figure is $DX(1)$, the second $BX(9)$. Thus

$$DX(1) = BX(9)$$

or

$$D = Br^8 \simeq 0.33B. \tag{7}$$

Now combine Equation (7) with Equation (5) to get

$$1 + 0{\cdot}67B = 1{\cdot}15,$$

i.e.

$$B \simeq 0.22. \tag{8}$$

We are now in a position to check Statement S. The fraction of all people who are alive of those who have ever lived is

$$\frac{X(0)}{T} = \frac{X(0)(1-r)}{BrX(0)} = \frac{1-r}{Br}. \tag{9}$$

Substitute in the values for r, B from Equations (3), (8) to find

$$X(0) = 0{\cdot}68T. \tag{10}$$

In other words, about 68% of all humans are still alive! The Tasmanian boy's answer was incorrect.

How much faith can we really put in this figure?

First let us see the effect of varying the assumed life-span. We supposed this to be 8 census periods. Now consider instead N census periods. Equation (7) is now altered, and when we combine it with Equation (5), we find that

$$B = \frac{1-r}{r(1-r^N)}, \tag{11}$$

so that

$$\frac{X(0)}{T} = 1 - r^N. \tag{12}$$

Now if $N = 1$ (a value typical of a very poor third-world country), this gives 0.13 and Statement S is clearly false. However, if $N = 15$ (a value about right for an affluent Western country), we find 0·87 and Statement S is more than fulfilled.

Statement S is most nearly true (i.e. $X(0) = \frac{1}{2}T$) if $N = 5$. So life expectancies (at birth) of less than 25 years do not yield the truth of Statement S. Those above 25 years do. This, of course, is precisely because we so chose r as to yield a doubling time of 25 years.

It is hard to find good estimates of N. I looked up some U.N. figures and was surprised to find that even in India and Bangladesh life expectancies at birth are claimed to exceed 50. I.e. $N > 10$. However, very poor countries simply do not have the relevant statistics. $N = 8$ is probably about right – as close as we can tell.

Another way to check our value of N is to look at U.N. figures for B. Our value corresponds to an annual crude birth-rate of 40·6 per thousand. (That is to say, $1{\cdot}0406^5 = 1{\cdot}22$, 0·22 being the value of B given by Equation (8).) This is high, though not impossibly so; you will find it in many third-world countries (again, however, these being the ones for which statistics are not particularly reliable). Probably it indicates that we have, if anything, underestimated N : that is to say, underestimated $X(0)/T$.

Our assumption that everyone died at the same age was, of course, quite unrealistic. Nevertheless, it probably doesn't affect the outcome of the calculation very much. There will be groups in the population for which N is small and these will be balanced by groups for which N is large. The different values of $X(0)/T$ would need, in a more realistic model, to be averaged. This might produce some discrepancy from the value calculated above, but it could hardly alter it too drastically. So Statement S seems safe.

Do I then believe Statement S?

Well, perhaps.

There is a real problem with Equation (1), the Malthusian model. The key to the doubt lies in Equation (6), where I summed an infinite series – although, of course, we have not been around for infinite time. This does not *directly* affect very much the accuracy of Equation (6) and the calculations based on it; but it does throw some doubt on Equation (1).

Right now, there are about 5.2 billion people on earth (a billion being 1,000,000,000). So, one census ago there were 4.5 billion or so; one census period before that there were 3.9 billion. Etc. And 156 census periods ago,on this logic, there would have been just two people. Now this is less than 800 years ago – and even the notorious Bishop Usher† dated Adam and Eve before that!

The Malthusian model is, in fact, a very good description of recent human history. But humankind has been around a lot longer than the few centuries over which it has been valid. For most of history, it is believed, the human population lived a hunter-gatherer existence, with the population essentially constant, $B = D$ and $r = 1$.

We don't know very accurately how long this period lasted. After consulting a number of authoritative texts, I found a (very rough) consensus that the origin of our species, *Homo sapiens*, was about 100,000 years ago.

Even more uncertain than this figure is the estimate of how many people there have been during this long period. I looked and looked but failed to find any reliable estimates. So eventually I pulled a figure out of the air: 1 million.

The other thing we need to know is how long people lived back then. Lacking information, I simply assumed 40 years – as in the previous analysis.

So we "guesstimate" that our value of T has left out something like

$$100{,}000 \times 1{,}000{,}000/40$$

people, i.e. 2.5 billion. Now our previous estimate of T was, from Equation (10), 6·75 billion. If we add to this the 2·5 billion we've just estimated, we get 9·25 billion: a little less than twice the current population. So Statement S seems true, but just.

But there are so many uncertainties. Statement S may well be true. Equally, it may well be false. Of course, if we wait another doubling period, it will almost certainly be true if population goes on increasing as it is now.

But, of course, supposing even it were false, it does not support the Tasmanian boy's logic. Say even 90% of all people had in fact died, would you accept an experiment that gave only 90% support for a supposed universal truth?

* * * * *

† Bishop Usher attempted to date the creation by means of a very detailed study of Old Testament chronologies, which he took to be literally true in every slightest particular. He assigned a date of 4004 BC to the creation. Very few, if any, reputable biblical scholars would today agree with this methodology.