How Much is a $5 Betting Coupon Worth?

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When it first opened, the Crown Casino in Melbourne, Australia charged for admission. In return, the patron was given a $5 betting coupon:

The $5 coupon can be used to make any standard casino bet. If the bet succeeds, the patron receives their normal winnings; win or lose, the Casino collects the coupon.

What is the value of such a coupon? One quickly realizes that it is worth less than $5, even discounting the Casino's edge. For example, making a 50-50 bet—say, betting red or black on roulette, but ignoring the zero—the coupon is worth $2.50: half the time we win $5 (and the coupon has disappeared), and half the time we win nothing. Allowing for the single zero on an Australian roulette wheel, the average win is $18/37 \times 5 = \$2.43$.

Surprisingly, we can do better. (This was pointed out to me by my gambling colleague, Gary Watt.) Suppose we take 37 coupons, betting a coupon on each single number in roulette. Then, whatever number comes up, we win $35 \times \$5 = \$175$. Thus the value of each coupon when played on such a bet is $1/37 \times \$175 = \$4.73$. In fact, this was the best option available at the Crown Casino.

In general, suppose we make a bet with a probability of $1/n$ of success, and suppose that the expected return on this bet is $R \in [0,1]$. For example, the expected return on any roulette bet is $R = 36/37$ (e.g., after spending a dollar to play the game, you can win $36 with probability 1/37 or you can win $2 with probability 18/37). Then a normal successful bet of $5 will win $(Rn-1) \times \$5$, plus the original $5 staked, giving a total return of $Rn \times \$5$. Placing the coupon on such a bet, we don't have any $5 stake returned, and thus the value of the coupon here is

$$V = 5\frac{(Rn-1)}{n} = 5\left(R - \frac{1}{n}\right).$$

The above calculation is very simple but it doesn't quite dispel the paradoxical flavor, the fact that the value of the coupon depends upon the probability of the bet succeeding. Perhaps the clearest way to think about it is to realise that, *if your bet succeeds then the coupon has been of no value to you*: the coupon has disappeared anyway, and the $5 stake you otherwise would have had to risk has gone back into your pocket. Thus, expected returns being equal, you want to make the bet with the highest probability of losing.