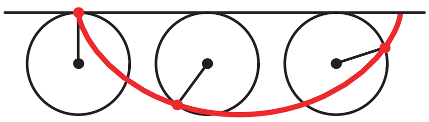
MathSnacks
Rolling, Rolling, Rolling

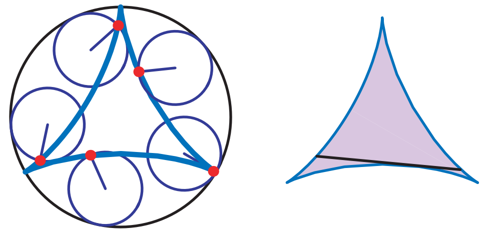
by Marty Ross, Burkard Polster, and QED (the cat)

Dashing Descent



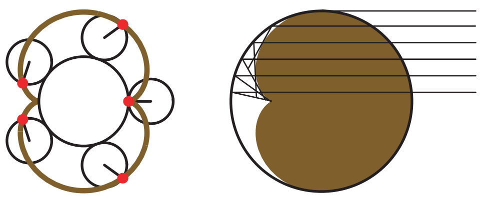
The curve traced by a point on the rim of a circle as it rolls along a straight line is called a *cycloid*. Here are two of its amazing properties. First, the time it takes for a ball to roll down the cycloid to the bottom point is always the same, no matter the point (excluding the bottom) from which the ball is released. Second, imagine any other curve connecting a top point of the cycloid to any other point on the cycloid. Then, no matter the choice of this second curve, it takes less time for a ball to roll down the cycloid.

Tricky Turnaround



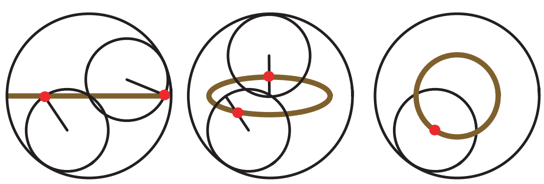
Rolling a circle inside another circle of triple the diameter produces the *deltoid* (*above, left*). As illustrated in the example shown on the right all tangent segments of this curve have the same length. Let's say this length is one. This then means that *we can rotate a line segment of length one a full turn while remaining inside the curve.* For a long time, mathematicians believed that the curve with this property that contained the smallest area was the deltoid. It came as a huge surprise when it was discovered that, choosing *any* small positive area A, it is possible to construct a (very strange) curve containing area A within which the line segment can be rotated.

Coffee Cup Curve

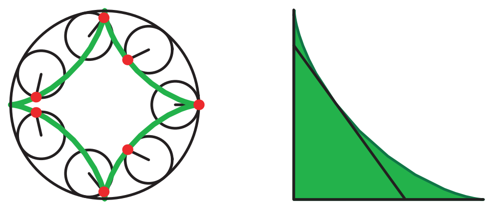


Rolling a circle around the outside of another circle with double the diameter produces the *nephroid* (*above, left*). You can see half of this curve every time you have a cup of coffee outside on a sunny day. After being reflected on the inside of your cup the rays of the sun envelope this curve (*above, right*)

If you roll the smaller circle on the inside of the larger one, a point on the circumference of the inner circle traces out a diameter of the larger circle (*below, left*). The curve traced by an inner point of the smaller circle is an ellipse (*below, middle*). This ellipse is a circle exactly when the chosen inner point is the center of the smaller circle (*below, right*).



Ladder Curve



Rolling a circle inside another circle of four times the diameter produces the *astroid* (*above, left*). Imagine a ladder leaning against a wall, with the wall and the ground perpendicular. If the ladder slides to the ground, the region bounded by one quarter of the astroid consists of exactly those points that at some instant during the sliding action coincide with the ladder.

The ratios of the radii of the circles used for producing the curves on this page are: $1/\infty$ - quickest descent; 1/2 - coffee cup/ellipse; 1/3 - smallest turnaround; and 1/4 - ladder. The missing ratio 1/1 corresponds to the solution of the coin puzzle discussed earlier on in this issue.

Ripper References*

We used the free *Mathematica* notebooks dealing with curves at http://mathworld. wolfram.com to produce the diagrams on this page