Function

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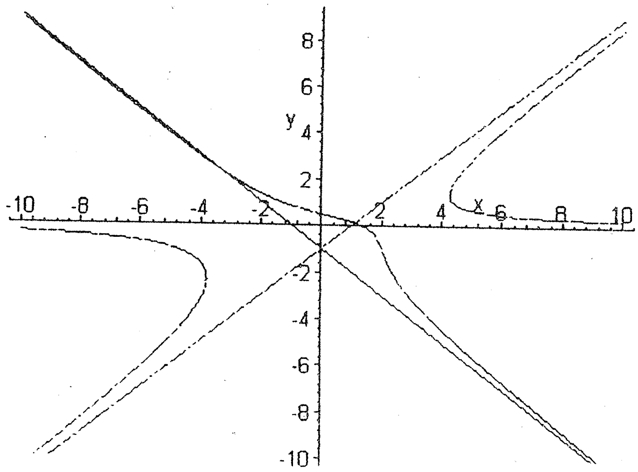

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The Front Cover: Linear Asymptotes

Julius Guest, Alexander St, East Bentleigh

1. Introduction

Graph *sketching*, as opposed to accurate plotting, is a very useful tool for examining the behaviour of a curve in a plane. The aim of graph sketching is to display the main, most interesting features of the curve, but to find these without the labour of plotting the curve. Plotting entails finding many details that are of no particular interest, and so can be inefficient. The details we concentrate on when *sketching* a curve are such features as maxima, minima, points of inflection, zeroes, etc. Among these features are *asymptotes*.

Asymptotes are curves or straight lines which represent some limiting behaviour of the curve under discussion, for example when one or both of its variables approach some special value. Here I shall concentrate on *linear asymptotes*, which are straight lines that represent the limiting behaviour. By no means do all curves possess such asymptotes. A curve has a linear asymptote if its slope tends to become constant under some limiting condition, but some curves (e.g. $y = x^2$) do not have this property.

I will discuss how linear asymptotes arise in the case of Cartesian co-ordinates.

2. Cartesian Co-ordinates: First Illustration

Begin with the case in which the curve is given by a formula of the type

$$F(x, y) = 0$$

and take as an example the curve represented by

$$y^3 - x^2y + 2y^2 + 7y + 3x - 4 = 0. \qquad (1)$$

We can look for linear asymptotes by substituting into Equation (1) the formula for a straight line $y = mx + c$, which yields

$$(mx+c)^3 - x^2(mx+c) + 2(mx+c)^2 + 7(mx+c) + 3x - 4 = 0. \qquad (2)$$

If we now equate the coefficient of x^3 to zero, we find

$$m(m^2 - 1) = 0 \qquad (3)$$

Next equate the coefficient of x^2 to zero, which gives

$$c(3m^2 - 1) + 2m^2 = 0 \qquad (4)$$

There are three possible values of m, given by the three solutions of Equation (3):

$$m_1 = -1, \qquad m_2 = 0, \qquad m_3 = +1. \qquad (5)$$

Each of these has a corresponding value of c given by Equation (4)

$$c_1 = -1, \qquad c_2 = 0, \qquad c_3 = -1. \qquad (6)$$

So the curve represented by Equation (1) has three linear asymptotes:

$$y = -x - 1, \qquad y = 0, \qquad y = x - 1.$$

These are found by substituting the values of m and c from Equations (5) and (6) into the standard equation for a straight line.

The graph on the Front Cover illustrates the relationship of the asymptotes to the curve representing the functional relationship indicated by Equation (1). The first and the third of these asymptotes are shown explicitly. The second coincides with the horizontal axis.

3. Cartesian Co-ordinates: Second Illustration

Now take a second example. Consider

$$y = \frac{2x^3 + x^2 - 11x + 13}{x^2 + 2x - 3}.$$

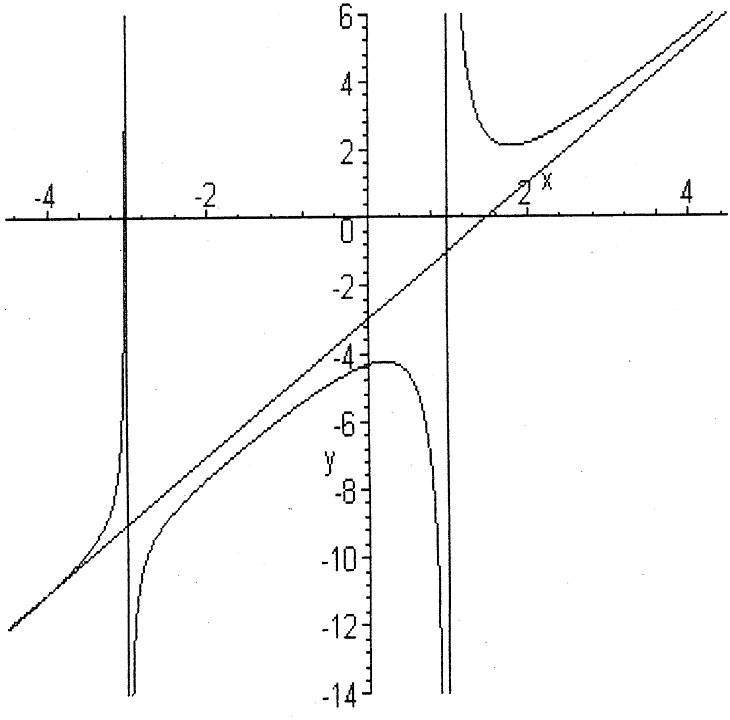
We note first that the denominator factorises to $(x+3)(x-1)$, so, as either $x \to 1$ or $x \to -3$, then y must tend to infinity, and we find two asymptotes, parallel to the y-axis: $x=1$ and $x=-3$.

If we divide the cubic in the numerator by the quadratic in the denominator, we find

$$y = 2x - 3 + \frac{x+4}{x^2 + 2x - 3}$$

and as $x \to \infty$, we obtain the oblique asymptote $y = 2x-3$.

The figure below shows the behaviour of the function and its three asymptotes.



Mathematics at Homebush and Flemington

Michael A B Deakin, Monash University

Let us start with a simple, but nonetheless surprising, result. I first saw it in 1950 in a book of puzzles for school students. I will begin with the question posed there, but will present it in a slightly more modern form.

Suppose a wire were stretched all around the circle of the earth's equator (duly supported over oceans, etc), and that once it was in place a length of one metre was added to it, and the wire once more stretched into a circular form. How high off the ground would the wire now be?

The answer ("just under 16cm") comes as a surprise to most people encountering the problem for the first time. But we may readily deduce this result. Take the radius of the earth to be R. Then the circumference of the earth (and so also of the wire) will be $2\pi R$. If this circumference is extended by a length l, then the new circumference will be $2\pi R + l$, which is exactly the circumference of a circle of radius $R + \frac{l}{2\pi}$. In the case just given, l is 1m and so the extra term (the addition to the radius) comes to 15.915.... cm.

But now notice two things about this calculation. The first is that the radius R of the earth plays no part in the final answer. That answer holds good *whatever* radius we begin with. So if in the question we replaced the earth with a tennis-ball (say), the same answer would result.

The second point to note is that we may turn the calculation around and suppose that the radius R has been increased, and so seek the increase in the circumference. Suppose R is increased to $R + r$. Then the new circumference is $2\pi(R + r)$, and the increase is $2\pi r$, which is the l of the previous calculation.

What brought this back to mind was watching the track events at the recent Olympics. Athletes competing in sprint races run in lanes and each lane is a fixed distance out from its inside neighbour. Typically eight lanes are spaced across the track, and each lane is all or part of a composite curve. This curve is made up of two semi-circular "ends" with straight connecting sections in between. The distance right around one of these lanes will be $2\pi R + 2L$, where R is the radius of the semi-circular piece and L is the length of the straight. For each lane, L is the same, but as we go out from the inside lane to the outside one, R increases. Take the distance from one lane to the next to be r (say).

The races are so organised that each competitor runs the same distance as the others. Consider the simplest case in which the race consists of one circuit of the track (400m). One lane will be exactly 400m in total length. Suppose for argument's sake that this is the innermost lane. Then the next lane out will be $2\pi r$ too long, and the next one beyond that will be even longer by a further $2\pi r$. Etc. This follows exactly as before; the extra distance L does not affect this figure, as it is the same for all competitors.

In order to even things up and to make the contest fair, the competitors in the outer lanes are each moved forwards from the starting position of the innermost athlete, and because the difference in the radius is the same from each lane to the next, so also is the apparent starting advantage given as we move out from lane to lane. The amount of the "stagger" is $2\pi r$ in each case. As with the problem with which we began, the radius of the innermost track is irrelevant.

This same principle applies to other distances as well. For example, the 200m race comprises half a circuit for the inner competitor, and each of the other athletes is advanced a distance πr per lane as we go from the inner lane to the outer one.

(Before we move on, it may be as well to remark that the fact that the distances have been made equal does not make the other conditions the same. As was several times pointed out by Games' commentators, the runners in the inside lanes have to cope with the tighter curvature experienced there, but they have the advantage of keeping their rivals in view.)

The shape we have been considering is a simple example of a "convex closed curve". The word "closed" refers to the fact that its ends join up. By "convex" is meant that it "bulges outward" everywhere. Or rather it is more exact to say that it nowhere "bulges inward". (Recall that on the straight stretches it doesn't "bulge" at all.) Any point on any chord of a closed convex curve lies either on the curve or else within its interior. Another way to put it is that a convex curve shape, if made into a sculpture and tied about with string, stays everywhere in contact with that string.

We can use the insights already gained to build more general results onto the simple calculations we have been doing. Think first of a figure made up of parts of two circles as in Figure 1. This shows a closed convex curve whose lower half is the semi-circle $y=-\sqrt{R^2-x^2}$ and whose upper half is the circular arc $y=R-\sqrt{2R^2-x^2}$. (In the illustration, $R=1$.) Imagine expanding the size of the curve by an amount *r*.

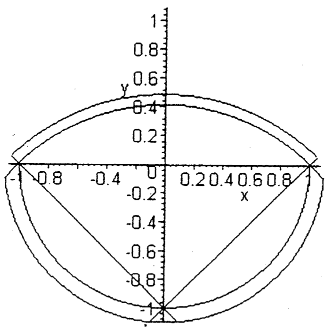


Figure 1

Now we need to be quite precise about what we mean by this expansion. It is *not* a mere enlargement of the figure (although in the case of the simple circle, this comes to the same thing). Rather, we want the outer curve to be everywhere distant *r* from the inner one. Or to put it more precisely, the distance between a point on the outer curve and the nearest point on the inner one is always to be *r*.

So, in this case, we proceed as follows. On the lower semi-circle, we build another semi-circle with the same centre, but of radius $R+r$. This has an arclength of $\pi(R+r)$. We may also similarly build a circular arc on the upper portion of the curve. This upper portion has an arclength $R\alpha$, where α is the angle subtended at the centre of the circular arc: in this case $\alpha=\pi/2$. (Remember to measure in *radians* !) So when *this* figure expands, we find an arclength of $(R + r)\alpha$. But our new curve is no longer closed. We have a problem with the corners. To overcome this, swing a circular arc of radius r with each corner as centre, and so chosen as to join up the "loose ends". This ensures that the shortest distance from a point on the larger curve to one on the smaller is always r. Readers may care to check that these circular arcs each subtend an angle $\frac{\pi}{2}-\frac{\alpha}{2}$ at their respective corners, so that each has a length of $r\left(\frac{\pi}{2}-\frac{\alpha}{2}\right)$. We may now add up the lengths of all the pieces of the expanded curve. The result is

$$\pi(R+r)+(R+r)\alpha+r\left(\frac{\pi}{2}-\frac{\alpha}{2}\right)+r\left(\frac{\pi}{2}-\frac{\alpha}{2}\right)=(\pi+\alpha)R+2\pi r$$

which is the length of the original curve plus an extra $2\pi r$.

So our earlier results with circles and running tracks also hold more generally.

To see how much more generally, let's look at another example, in some ways a simpler one. Imagine a square of side R. This will have a total perimeter of $4R$. Now suppose each side to be pushed outwards by a distance r. This will produce a figure comprising four disconnected straight line segments each still of length R. To join them, use four circular arcs, each of radius r and centred on the vertices of the original square. We easily see that the distance between the two curves is always r. Because of the simple geometry of this case, *all* the extra length in the expanded figure comes from the circular arcs, and it is a very simple matter to check that the additional length comes to $2\pi r$.

This new example gives the key to the general result.

Any closed convex curve C may be approximated to arbitrary accuracy by means of a closed inscribed polygon. (This is in fact a general property of all "reasonable" closed curves, i.e. those not involving fractals, and fractals cannot be convex.) When we say "to arbitrary accuracy", we mean that the difference between the exact curve C and its approximating polygon may be made as small as we like, merely by taking enough sides in the polygon.

But however many sides we have in the polygon and however irregular these are, the argument given for the square will hold and we will have a number of circular arcs added to the expanded perimeter. The total of all the angles involved will be 2π (a result known to Euclid) and so the combined length of the circular arcs will be $2\pi r$. This same property must therefore apply to the limiting curve C.

So let us now summarise.

> *If C is any convex closed curve, and C' (let us call it) is another closed convex curve distant r "further out" from C (that is to say, the shortest distance from any point on C' to a point on C is always r), then the extra length of C' as compared to C is $2\pi r$.*

I once had occasion to use this result in a discussion with a racing trainer. He was interested in how much further a horse ran if it failed to get a place on the rails and so ran "one wide". If the race consists of exactly one circuit of a closed convex track, the answer is now easy. Suppose (to make the numbers easy) that the extra distance between the horse and the rail is 1m. Then in units of metres, the horse runs an extra 2π or about 6.28m. This may not sound much in a race of 1500m or so, but on the other hand 6.28m is about 3 or 4 lengths, and this is quite a convincing winning margin in such a race.

Notice that this is a different question from that considered earlier for human sprint-races. (However it can apply to longer human races such as the distance events where the competitors do not run in lanes.)

But this answer is still limited. We can only apply it to an integral number of laps of the racing circuit. What if, as often happens, the number of laps is not integral?

Well, we can extend the analysis to cover this case also. Think back again to the case of the square. Each of the four corners corresponded to an angle of $\pi/2$, and an addition to the perimeter of $\pi/2$. So if we change direction once, the distance increases by an amount $\pi/2$; if we change direction twice, the amount becomes $2\pi/2$. And so on.

More generally, if we have an irregular closed convex polygon, each corner will contribute an angle α (say) where the sum of all the values of α is 2π. After going round some number m (say) of the corners, the horse will have changed direction by an angle equal to the sum of the first m values of α. Call this total change in the horse's direction θ. Then the extra distance the horse runs will be $r\theta$.

This is now the more general result we seek. It applies for all values of θ, not just to integral multiples of 2π. Notice that it is given in terms of θ, the change in the *direction* in which the horse is travelling. This makes sense; when the horses line up at the barrier, they are all facing in the same direction, and when they finish, they all cross the finishing line running in the same direction. So, for example, if a horse completes 1.5 circuits of the track, ending up facing north (say), when at the start it faced south, then running one wide contributes 9.42m to the total distance run. *Notice that in this case the 0.5 refers to the direction, not to the length of the partial circuit.*

So we have simple formulae that apply to a wide variety of different cases, and all are closely related to a very simple geometric problem.

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Functions With No Zero Derivatives

A number of simple transcendental functions, most notably e^x, have the property that their derivatives never vanish, or never vanish at certain points, no matter how many times we take the derivative. Interestingly, it's possible to express some well-known problems in terms of functions that have no zero derivatives. For example, consider the simple function

$$f(x)=\left(\frac{a}{ax+1}\right)^2+\left(\frac{b}{bx+1}\right)^2-\left(\frac{c}{cx+1}\right)^2$$

where a, b, and c are integers. The non-vanishing of every derivative of this function at $x=0$ is equivalent to Fermat's Last Theorem, because the nth derivative is

$$f^{(n)}(x)=(-1)^n(n+1)!\left\{\frac{a^{n+2}}{(ax+1)^{n+2}}+\frac{b^{n+2}}{(bx+1)^{n+2}}-\frac{c^{n+2}}{(cx+1)^{n+2}}\right\}$$

At $x=0$ this reduces to $(-1)^n(n+1)!\{a^{n+2}+b^{n+2}+c^{n+2}\}$, which, by Fermat's Last Theorem (now proved!), cannot equal zero when n is any positive integer and a, b, c are any integers.

Adapted from http://www.seanet.com/~ksbrown/kmath160.htm

Letters to the Editor

The Part-Buried Pipe

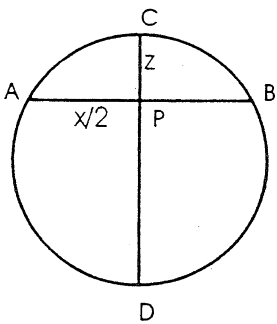
The April issue of *Function* arrived a few days ago, and I was particularly interested in your article on the part-buried pipe. I recall reading the two articles on the subject in the *AustMS Gazette*, and I remember noticing at the time that Professor Love's formula for the radius, r, in terms of x and z, could be obtained without recourse to Euclid's theorem of the two chords. Specifically, apply Pythagoras to the right triangle with vertices at O, A and P, where O is the centre of the circle, and A and P are as shown in Figure 1 of your article. The side-lengths are $r - z$, $x/2$ and r, so we obtain:

$$(r-z)^2 + (x/2)^2 = r^2$$

Solving for r yields Professor Love's formula.

Peter Grossman
Intelligent Irrigation Systems

[*For the reader's convenience, the diagram referred to is reproduced below. Eds.*]



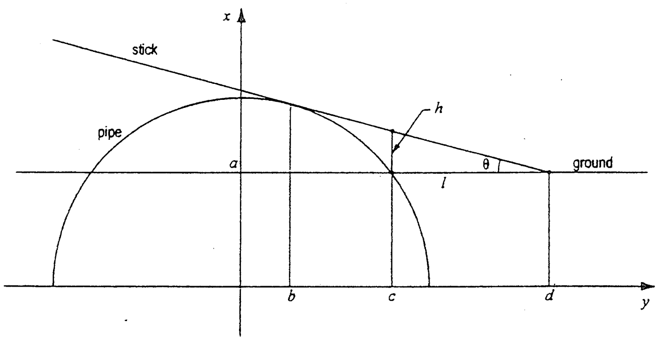
And More on Same

In response to your interesting article on determining the radius of a partially buried pipe, I am pleased to invite you to check out the practical feasibility of an alternative set of measurements which would also determine the radius.

You will need a tape measure and a perfectly straight stick of wood or plastic which is to be rested on the pipe perpendicular to its axis, so that it will be parallel to the cross-section of the pipe upon which it rests. The ground around the stick must be sufficiently level that the line $y = a$ will be an adequate model for the ground level in the diagram opposite.

The stick then represents a tangent line to the curve (of the pipe) at an unknown point where $x = b$. This line may be regarded as the graph of the function

$$y = -\frac{bx}{\sqrt{r^2 - b^2}} + \frac{r^2}{\sqrt{r^2 - b^2}} = T_b(x), \text{ say.}$$



The stick meets the ground when $x = d$. d satisfies the equation $T_b(x) = a$, and so we find

$$d = \frac{r^2 - a\sqrt{r^2 - b^2}}{b}$$

It seems feasible to accurately measure the distance $l = c - d$ between the end of the stick and the point where the pipe meets the ground. We also want to measure the *slope* of the stick, which we can conveniently treat as positive. This could be done in one of two ways:

(i) Measure the height h, so that s = slope = h/l

(ii) Use a protractor to measure the angle θ, so that s = slope = $\tan\theta$.

In terms of b, we have s = slope = $y'(b) = \frac{b}{\sqrt{r^2 - b^2}} = \frac{1}{\sqrt{(r/b)^2 - 1}}$.

Thus $\frac{r}{b} = \csc\theta$, and so r/b, along with l, may be regarded as known.

The formula for c now gives

$$l = \frac{r^2}{b} - a\sqrt{(r/b)^2 - 1} - d = \frac{r^2}{b} - a\frac{l}{h} - d = r\csc\theta - a\cot\theta - d ,$$

which may be looked at as a linear equation in the unknowns r, a.

We can, moreover, place the stick anywhere we like along the line $y = a$, producing as many pairs of values (l,θ) as we might want or need, say 3: $l_1 < l_2 < l_3$; $\theta_1, \theta_2, \theta_3$. The resulting expressions for, say, $l_2 - l_1$ and $l_3 - l_2$ cancel out the d and leave two equations in r and a.

Unless we are very unlucky, these equations should have a unique solution, and this can be checked against any further values of l and θ that we may care to obtain, thereby affirming the precision of our measurements and the accuracy of the answer.

J G Kupka
Monash University

Dr Fwls caught out?

This time (April 2000) our dear doctor was a little mischievous. As a matter of fact the solution to the equation tan $\theta = i$ can be found quite easily. It is known that

$$\tan\theta = \frac{\exp(i\theta) - \exp(-i\theta)}{i(\exp(i\theta) + \exp(-i\theta))}$$

Setting this equal to i results in the equation $\exp(i\theta) = 0$, and no finite complex number satisfies this. θ would have to be infinite and the laws of ordinary complex arithmetic no longer apply!

Julius Guest
Alexander St, East Bentleigh

A Hat of a Different Colour

Here is an intriguing problem that has become quite famous since it was described in *The New York Times* of April 10 this year. It was devised by Todd Ebert, a computer science instructor at the University of California at Irvine, who first produced it in his Ph D thesis. It now exists in many forms, but here we describe the very simplest of them. This gives the nub of the problem, and allows its essentially paradoxical character to be appreciated. However, the more advanced versions are much more complicated to analyse!

It has applications to coding theory, and so it is not merely a recreational curiosity. Its current popularity depends on its applicability as well as its novelty and the element of surprise in its answer.

Here then is the problem.

> Three people, Ada, Bet and Col, are to enter a room and compete for a prize of $3million. They are not in competition with each other but rather must combine to win the prize jointly, if they succeed in a task put to them. Before they enter the room they may hold a strategy meeting, but once they are inside, they are forbidden any communication with one another. Before they go in, they know the task they will have to perform. It is this. As they enter, they will each have a coloured (red or blue) hat placed on their head, and will be able to see the hats on the heads of the others, but not on their own. The hats will be red or blue as decided by the toss of an unbiassed coin, so the colours of the hats are completely random and independent. At a signal, each must at once give an answer to the question "What colour is the hat on your own head?". Three answers are allowed: "red", "blue" or "pass". The prize will be won if two conditions are satisfied: at least one player must guess correctly, and no player may guess incorrectly.
>
> What strategy should they adopt to maximise their chance of winning the prize?

Our naïve expectation would be that, because the colours of the hats are completely random, then the trio have no better than a 50:50 chance of the prize, or perhaps even less. However, this is not so. Try the puzzle yourself, before turning the page and reading the answer.

Here is the answer.

Because there are three hats, and two possibilities per hat, there are eight possible arrangements; in two of these, all the contestants have hats of the same colour; in the other six cases, two hats match and the third is different. Thus the probability of there being one odd hat is $\frac{3}{4}$. The strategy to adopt makes use of this fact. Suppose that Ada is wearing a red hat and that Bet and Col both have blue hats. Then Ada will see two blue hats, but Bet and Col will each see two different-coloured hats. Ada can now reason thus: "if my hat is blue, then there will be three blue hats, and so the hats will all match; the likelihood of this is $\frac{1}{4}$, so I will do better to guess that my hat is red". Bet however sees two different hats, and so can only argue that her own hat could be red or blue with equal likelihood. The same is true for Col. So, rather than risk getting the answer wrong (and thus forfeiting the prize), Bet and Col pass.

This then is the strategy.

> A player seeing two similar hats (say blue) guesses the opposite (here red); a player seeing two dissimilar hats passes.

With this strategy, the award of the prize rests entirely on the (one) player actually venturing a guess. In our example, if Ada guesses right, then the prize will be awarded, if not, then it won't. But it will not be jeopardised by the chance of a mistake by either Bet or Col.

Three quarters of the time, the hats will be dissimilar like this and the strategy will work; one quarter of the time, the hats will be identical and all three will give incorrect answers, and so they lose (badly!). It is of course impossible for all three to pass or to give contradictory answers.

It is important to notice that Ada has a chance of $\frac{1}{2}$ of getting her own hat-colour right. When she guesses she does not know what responses Bet and Col are entering simultaneously. The pre-arranged strategy means that *if she is right* then the others will not destroy her good work. Of course if she is wrong the three had no chance anyway. The laws of probability are not violated; it is the "rules of the game" that make the existence of such a winning strategy possible.

HISTORY OF MATHEMATICS

The Talmud and e

Hans Lausch and Michael A B Deakin, Monash University

In an earlier column (*Function*, *Vol* **21**, *Part* **2**), we discussed the suggestion that the Bible contained coded information on the value of π. While it is widely held that the biblical text gives only the crude approximation $\pi = 3$, there is a theory that the original Hebrew text actually reveals (to those in the know) the much better approximation

$$\pi = 333/106 = 3.1415\ldots .$$

We ended up arguing against this theory, but the discussion revealed much of the mathematical sophistication that Jewish culture has long encouraged. Here we turn to another such expression of that interest and describe an early account that involves the other famous transcendental number e.

The number e is often defined as the limit as $n \to \infty$ of the expression $\left(1+\frac{1}{n}\right)^n$ and its value is 2.71828..... . For an account of some of the properties of this number, see *Function*, *Vol* **24**, *Part* **1**, p 2. Here we discuss how this limit appeared early in Jewish tradition. But first a few mathematical preliminaries.

The equation

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$

is actually a special case of a more general result:

$$\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^x,$$

and this has the corollary (found by setting $n = -1$):

$$\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n = \frac{1}{e}$$

This last has the value 0.3678.... , a little over 1/3. (This was one of the forms used in the earlier *Function* article.)

Now to the Talmud. This name is given to both of two separate compendia of Jewish legal opinions, interpretations and annotations on earlier Jewish oral laws including the Mishna, the first such collection. Two separate groups of scholars worked on this project, one of them in Palestine, the other in Babylonia. Although they used as their bases the same Mishna, and although there was some collaboration between the two groups, their work resulted in two separate collections of material.

The Palestinian group completed their work in about 400 CE, but the Babylonian scholars continued their labours for another century. The earlier compilation is called the Palestinian Talmud (Talmud Yerushalmi), the later one is known as the Babylonian Talmud (Talmud Bavli). This latter is, in consequence of the extra time taken over its preparation, the more extensive and so it is regarded as the more authoritative.

There are several passages in the Talmud that bear on our topic. The first passage comes from Bavli Kethuboth 68a, b, that is to say, Chapters 68a, 68b of the book of Kethuboth in the Babylonian Talmud. (In this context, "Kethuboth" means "marriage contracrs".) The matter at issue is a problem of inheritance. We pick up the story late in Chapter 68a.

> "Rabbi said, A daughter who is maintained by her brothers is to receive a tenth of [her father's] estate. They said to Rabbi: According to your statement, if a man had ten daughters and one son the son would receive no share at all on account of the daughters? He replied: What I mean is this: The first [daughter] receives a tenth of the estate, the second [receives a tenth] of what [the first] had left, the third [gets a tenth] of what [the second] had left, and then they divide again [all that they had received] into equal shares."

The story continues in Chapter 68b.

"But did not each receive what was hers? – It is this that was meant: If all of them wish to marry at the same time they are to receive equal shares. This provides support for [the opinion] of R. Mattena; for R. Mattena said: If all of them wish to marry at the same time they are to receive one tenth. 'One tenth'! Can you imagine [such a ruling]? The meaning must consequently be that they are to receive their tenths at the same time."

The passage continues, but the Mathematics involved is all covered in what we have just quoted.

The quotations reproduced here come from the translation by Rabbi Dr I Epstein, who also includes some notes in explanation. These were supplied by the Rev Dr Israel Slotki. According to these notes, "the first daughter" is to be taken to mean "the first of the daughters to marry", etc, and it is obvious that, if with each marriage there was to be a redistribution of wealth, then the result (with ten possible marriages) would be likely chaos. Thus the calculation is to be performed once and at the outset.

Dr Slotki carries out the calculation explicitly for the case of three daughters, but the principle is the same whatever the number of daughters. He gives the following illustration for the simpler case. [We here correct a minor misprint in the note.]

"One daughter would be allowed one tenth of the estate; the other [second] $\frac{1}{10}\times\frac{9}{10}$; and the third $\frac{1}{10}\times\frac{81}{100}$. The son would, therefore, receive $1-\left(\frac{1}{10}+\frac{9}{100}+\frac{81}{1000}\right)=\frac{729}{1000}$ of the entire estate."

What is involved here is a geometric series with a common ratio of 0.9. If there were n daughters, then the son's share would be

$$1-\left(\frac{1}{10}+\frac{9}{100}+.....+\frac{9^{n-1}}{10^n}\right)=\left(\frac{9}{10}\right)^n$$

[Each daughter, by the way, receives $\frac{1}{n}\left(1-\left(\frac{9}{10}\right)^n\right)$ because they are to redivide the total dowry equally.]

Thus, with ten daughters, the son would receive $\left(1-\frac{1}{10}\right)^{10}$. Because $\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{n}=\frac{1}{e}$, we should therefore expect the value of the son's inheritance to be close to $\frac{1}{e}$ of the total value of the estate. This proves to be the case. The exact value of the son's inheritance works out to be 0.348... of the total; $\frac{1}{e}\approx 0.368$.

However, this example does not use a particularly large value of *n*. Nor is it any more than coincidence that the number of daughters is taken as ten. The figure of 10% is independent of the number of daughters, as Dr Slotki's note shows. The number ten is chosen because of the naïve expectation that the entire inheritance would then go to the daughters, with nothing left for the son.

There is another similar passage in Bavli Nedarim 39b. ("Nedarim" means "vows".)

> "R. Abba son of R. Hanina said: He who visits an invalid takes away a sixtieth of his pain. Said they to him: If so, let sixty people visit him and restore him to health? – He replied: the sixtieth is as the tenth spoken of in the school of Rabbi ... ."

And the text continues by referring to the text we have just discussed. Rabbi Epstein's translation here includes notes by Rabbi Dr H Freedman, who writes: "... the first visitor ... takes away a sixtieth of the sickness; the second a sixtieth of the remainder, and so on. Hence he would not be completely cured."

This time, the number n is considerably larger than in the previous case. After receiving sixty visitors, the invalid experiences a residue of $\left(1-\frac{1}{60}\right)^{60}$ of the initial pain. This works out to be 0.364... , and this time we are closer to the value of $\frac{1}{e}$ as we would expect. Again notice that there is no necessity for the number of visitors to be sixty. Yet more visitors would reduce the pain still further. Once more the number sixty is chosen in order to counter a naïve and incorrect expectation.

There is yet another passage of similar character, this time in Yerushalmi Talmud, in the book of Ketubbot (as this transliteration has it) Chapter 6, Section 6. This is an alternative version of the inheritance question.

> [II.A] "R. Zeira asked R. Nahman bar Jacob and R. Amni bar Papi, 'Who is the Tanna who stands by the view that a tenth of the man's possessions [are assigned to the daughter for her dowry]?'
>
> [B] "Said to him R. Zeira in the name of R. Jeremiah, 'The requirement of giving a tenth of one's possessions accords with the view of Rabbi.'
>
> [C] "They asked before Rabbi, 'Lo, if there were ten daughters and one son, if the first should take a tenth of the property of the father, and the second a tenth, and the third, fourth, fifth, and so through to the tenth, then nothing at all will be left for the son!'
>
> [D] "He said to them, 'The first takes a tenth of the property and leaves. The second takes her tenth of the property from what is left. The third takes her tenth from what is left. The fourth takes her tenth from what is left, onwards to the tenth. It turns out that the daughters take two shares less a mite, and the son takes one share and a mite more.' "

The fact that the daughters then divide their total into equal shares is overlooked in this version, but the overall effect of the division between the son and the daughters is here given explicitly. The son's share, as we saw earlier, is to be $\left(1-\frac{1}{10}\right)^{10}$ or 0.348... . The remaining 0.651... is shared between the ten daughters. Paragraph [D] above compares this with a division into a one third share for the son and a two thirds share for the daughters. Such a division would give the son 0.333... of the property, so that, over and above this third, the son receives 0.0153... of the wealth. This extra is what the translator (Jacob Neusner) has called the *mite*, which we might render colloquially as a *tad*, or a *smidgin*. The girls have their approximate two-thirds (0.666...) reduced by a corresponding amount, so that each receives one-thirtieth (0.0333...) of the total less her share (0.00153...) of this mite.

COMPUTERS AND COMPUTING

Storing Numbers: Round-Off Error

J C Lattanzio, Monash University

Computers have finite memory, and each number must fit into a pre-determined format and length. Hence a computer simply cannot store irrational numbers like π or $\sqrt{2}$ exactly. Likewise, numbers like $\frac{1}{3}$ can only be stored to finite accuracy, because $\frac{1}{3} = 0.\dot{3}$.

Numbers are divided into a mantissa (the fractional part) and the exponent. A certain number of binary digits (or "bits") is used for each exponent. For example, a common format for a 32 bit number is:

- 1 bit for the sign (+ or –)
- 7 bits for the exponent (usually stored in base 16)
- 24 bits for the mantissa (stored in base 2).

Thus for the exponent, the largest number that can be stored is

$$1111111 = 1\times 2^6 + 1\times 2^5 + 1\times 2^4 + 1\times 2^3 + 1\times 2^2 + 1\times 2^1 + 1\times 2^0 = 127 .$$

Likewise the smallest number is

$$0000000 = 0 .$$

Thus the exponent range is from 16^0 to 16^{127}, or 1 to 10^{127}. But because we are usually interested in storing very small numbers also, the convention is to subtract 64 from the exponent. Thus the range becomes

$$16^{-64} \text{ to } 16^{63}, \text{ or } 10^{-77} \text{ to } 10^{76} .$$

For example, consider the number stored in a computer as

0 10000101011001100000010000000000.

This is a 32-bit number, and the first bit tells us the sign of the number (0 for positive, 1 for negative). So this is a positive number. The exponent is next, so the next 7 bits tell us the exponent is 1000010. Now

$$1000010 = 1\times 2^6 + 1\times 2^1 = 66 .$$

So the exponent is $16^{66-64} = 16^2 = 256$. Moving to the mantissa, we have

$$101100110000010000000000 = 1\times 2^{-1} + 1\times 2^{-3} + 1\times 2^{-4} + 1\times 2^{-7} + 1\times 2^{-8} + 1\times 2^{-14}$$

and hence the number represented is

$$\left[\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^4+\left(\frac{1}{2}\right)^7+\left(\frac{1}{2}\right)^8+\left(\frac{1}{2}\right)^{14}\right]\times 16^{66-64} = 179.015625 .$$

The important point to note is that the next larger number which can be stored is obtained by changing the last bit in the mantissa from a 0 to a 1, resulting in an increment of $1\times 2^{-24}\times 16^2 \approx 1.5\times 10^{-5}$. So all numbers are quantised by an amount which depends on the exponent! This leads to errors in the representation of real numbers, which are collectively known as **round-off error**.

There is a scaling convention in binary numbers, which says that the first bit of the mantissa is unity (1). If there were a few zeroes in the mantissa before the first 1, then one could alter the exponent so that the first bit of the mantissa was again a 1.

The **smallest** positive (non-zero) number that can be stored is therefore

$$0\ 0000000100000000000000000000000 = \left(1\times 2^{-1}\right)\times 16^{0-64} = 4.3\times 10^{-78} .$$

Any attempt to store a number smaller than this will generate an "underflow", and the number is usually stored as **zero**.

The **largest** (positive) number which can be stored is

$$0\ 1111111111111111111111111111111 = 16^{127-64} \approx 10^{76} .$$

Any attempt to evaluate a number larger than this will generate an "overflow", which usually stops the calculation. In most compilers there are options for particular actions to be taken when an overflow is obtained. Note that many Unix compilers will store the result as "NaN", for "not a number", and then continue the calculation (!) although compiler options usually allow you to override this.

Often both underflow and overflow can be avoided by forcing the order of arithmetic operations. For example, in calculating the gravitational potential energy of a globular cluster of stars, we use the formula

$$E_{grav} = \frac{GM^2}{R}$$

where G is the Gravitational Constant, M is the cluster mass, and R is its radius. For the cluster known as Messier 3, we have

$$M = 6\times10^5 M_\odot = 1.2\times10^{39}\,\mathrm{g},\ R = 128\ \mathrm{pc} = 4\times10^{20}\,\mathrm{cm}.$$

[Here $M_\odot$ is the mass of the sun (2×10^{33} g), and 1pc is one parsec, 3.1×10^{18}cm.] Thus

$$E_{grav} = \frac{6.7\times10^{-8}\times\left(1.2\times10^{39}\right)^2}{4\times10^{20}}$$

which will overflow because of the $\left(10^{39}\right)^2 = 10^{78}$ in the numerator. We can avoid this by writing the formula as

$$E_{grav} = (GM)\times\left(\frac{M}{R}\right) = \left(6.67\times10^{-8}\times1.2\times10^{39}\right)\times\left(\frac{1.2\times10^{39}}{4\times10^{20}}\right).$$

Note that the previous discussion was for real numbers, but a similar one holds for integers as well, the only difference being that there is no exponent. A 24-bit integer may be stored as 1 bit for the sign and then 23 bits for the magnitude. The smallest (non-negative) integer is, of course 0 and the largest is then $2^{23}-1 = 8388607$.

A Note on Double Precision

Most computers allow the user to double the number of bits assigned to a number. This actually more than doubles the precision. For example, a single 32-bit number has 1 bit for the sign, 7 bits for the exponent and 24 bits for the mantissa. These 24 bits equate to $2^{24} = 1.6\times10^7$, or about 7 digits of accuracy in the stored number. (Note that this is 7 **digits**, not 7 **decimal places**!) If we go to double-precision then the convention is still to use the 1 bit for the sign and 7 bits for the exponent, but now to expand the mantissa to 56 bits. Thus the mantissa now stores up to $2^{56} = 7.2\times10^{16}$, which corresponds to about 16 digits of precision in the stored number.

In some machines there may be extra bits assigned to the exponent in double-precision. This depends on the machine being used, and is detailed in the manuals. Often there are different formulations for you to choose from.

Double-precision is usually the best way to minimise round-off errors, but it has its costs:

- it doubles the memory requirements for the job
- it slows the speed of the computation.

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OLYMPIAD NEWS

Hans Lausch, Special Correspondent on Competitions and Olympiads

The XLII International Mathematics Olympiad

Between July 1st and 14th, the US capital Washington, DC, was the venue for the 42nd IMO. There was a record participation of 473 students from 83 countries. As usual, students had to contend with six problems during nine hours spread equally over two successive days.

This year's papers were among the hardest of the last decade. Here are the problems. Each problem was worth seven points.

Problem 1

Let ABC be an acute-angled triangle with circumcentre O. Let P on BC be the foot of the altitude from A.

Suppose that $\angle BCA \geq \angle ABC + 30^{\circ}$.

Prove that $\angle CAB + \angle COP < 90^{\circ}$.

Problem 2

Prove that

$$\frac{a}{\sqrt{a^2+8bc}}+\frac{b}{\sqrt{b^2+8ca}}+\frac{c}{\sqrt{c^2+8ab}}\geq 1$$

for all positive real numbers *a*, *b* and *c*.

Problem 3

Twenty-one girls and twenty-one boys took part in a mathematical contest.

Each contestant solved at most six problems

For each boy and girl, at least one problem was solved by both of them.

Prove that there was a problem solved by at least three girls and at least three boys.

Problem 4

Let n be an odd integer greater than 1, and let $k_1, k_2, ..., k_n$ be given integers. For each of the $n!$ permutations $a=\{a_1, a_2, ..., a_n\}$ of 1, 2, ..., n, let

$$S(a)=\sum_{i=1}^{n} k_i a_i .$$

Prove that there are two permutations b and c, $b \neq c$, such that $n!$ is a divisor of $S(b)-S(c)$.

Problem 5

In the triangle *ABC*, let *AP* bisect $\angle BAC$, with *P* on *BC*, and let *BQ* bisect $\angle ABC$, with *Q* on *CA*.

It is known that $\angle BAC = 60^{\circ}$ and that $AB + BP = AQ + QB$.

What are the possible angles of the triangle ABC?

Problem 6

Let a, b, c, d be integers with $a > b > c > d$. Suppose that

$$ac + bd = (b + d + a - c)(b + d - a + c).$$

Prove that $ab + cd$ is not prime.

Following the Australian Mathematical Olympiad in February and the Asian Pacific Mathematics Olympiad in March, members of the Australian team were selected after two more tests held in April during the IMO Selection School. The successful students were:

David Chan, NSW, year 11, Sydney Grammar School,
Peter McNamara, WA, year 12 Hale School,
Hugh Miller, NSW, year 12, Barker College,
Bobbi Ramchen, Vic, year 10, Melbourne Girls' Grammar School,
Stewart Wilcox, NSW, year 11, North Sydney Boys' High School,
Yiying Zhao, Vic, year 11 Penleigh and Essendon Grammar School.
Reserve: Nicholas Sheridan, Vic, year 11, Scotch College.

In this year's IMO, ustralia came twenty-fifth, with 97 points out of a possible 252. The top ten teams were: China (225 points), Russia (196), USA (196), Bulgaria (185), Korea (185), Kazakhstan (168), India (148), Ukraine (143), Taiwan (141), Vietnam (139).

Further placings were Japan (14^{th} placing), Germany (15^{th}), Israel (18^{th}), Hong Kong (20^{th}), Hungary (21^{st}), Argentina (22^{nd}), Thailand (23^{rd}), Canada (24^{th}), Cuba (26^{th}), France (28^{th}), Singapore (29^{th}), United Kingdom (31^{st}), South Africa (36^{th}), New Zealand (44^{th}), Indonesia (59^{th}), Malaysia (59^{th}), Sri Lanka (71^{st}), Phillipines (75^{th}).

Our team won one gold and four bronze medals. Most remarkably, Peter McNamara became the first Australian to win a gold medal twice at International Mathematical Olympiads. He scored 36 points out of a possible 42, achieving four complete solutions, and tenth place in the world! Another first was the participation of two girls in the Australian team. Bobbi Ramchen and Yiyang Zhao both hail from Melbourne and both won bronze medals. The other bronze medallists were David Chan and Stewart Wilcox.

Congratulations to our excellent team!

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BOOLE ON THOUGHT AND ITS LAWS

"The object of science, properly so called, is the knowledge of laws and relations. To be able to distinguish what is essential to this end, from what is only accidentally associated with it, is one of the most important conditions of scientific progress. I say, to *distinguish* between these elements, because a consistent devotion to science does not require that the attention should be altogether withdrawn from other speculations, often of a metaphysical nature, with which it is not unfrequently connected. Such questions, for instance, as the existence of a sustaining ground of phænomena, the reality of cause, the propriety of forms of speech implying that the successive states of things are connected by *operations*, and others of a like nature, may possess a deep interest and significance in relation to science, without being essentially scientific. It is indeed scarcely possible to express the conclusions of natural science without borrowing the language of these conceptions. Nor is there necessarily any practical inconvenience arising from this source. They who believe, and they who refuse to believe, that there is more in the relation of cause and effect than an invariable order of succession, agree in their interpretation of the conclusions of physical astronomy. But they only agree because they recognise a common element of scientific truth, which is independent of their particular views of the nature of causation.

"If this distinction is important in physical science, much more does it deserve attention in connexion with the science of the intellectual powers. Fir the questions which this science presents become,, in expression at least, almost necessarily mixed up with modes of thought and language, which betray a metaphysical origin. The idealist would give the laws of reasoning one form of expression; the sceptic, if true to his principles, another. They who regard the phænomena with which we are concerned in this inquiry as mere successive *states* of the thinking subject devoid of any causal connexion, and who refer them to the *operations* of an active intelligence, would, if consistent, equally differ in their modes of statement. Like difference would also result from a classification of the mental faculties. Now the principle I would here assert, as affording us the only ground of confidence and stability amid so much of seeming and of real diversity, is the following, viz., that if the laws in question are really deduced from observation, they have a real existence as laws of the human mind, independently of any metaphysical theory which may seem to be involved in their mode of statement. They contain an element of truth which no ulterior criticism upon the nature, or even the reality, of the mind's operation, can easily affect. Let it even be granted that the mind is but a succession of states of consciousness, a series of fleeting impressions uncaused from without or within, emerging out of nothing, and returning into nothing again, — the last refinement of the sceptic intellect, — still, as the laws of succession, or at least of a past succession, the results to which observation had led would remain true. They would require to be interpreted into a language from whose vocabulary all such terms as cause and effect, operation and subject, substance and attribute, had been banished; but they would still be valid as scientific truths.

"Moreover, as any statement of the laws of thought, founded on actual observation, must contain scientific elements which are independent of metaphysical theories of the nature of the mind, the practical application of such elements to the construction of a system or method of reasoning must also be independent of metaphysical distinctions. For it is upon the scientific elements involved in the statement of the laws, that any practical application will rest, just as the practical conclusions of physical astronomy are independent of any theory of the cause of gravitation, but rest only on a knowledge of its phænomenal effects. And, therefore, as respects both the determination of the laws of thought, and the practical use of them when discovered, we are, for all really scientific ends, unconcerned with the truth or falsehood of any metaphysical speculations whatever."

From Chapter III of *An Investigation of the Laws of Thought on which are founded the mathematical theories of logic and probabilities*, George Boole's major work.

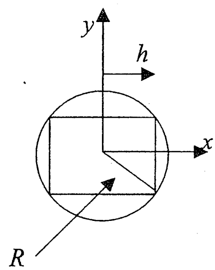
PROBLEMS AND SOLUTIONS

We begin with solutions to problems posed earlier.

PROBLEM 25.2.1 (Submitted by David Halprin)

If a hole of length 6 cm is drilled right through the centre of a solid sphere, what is the volume of the remaining material?

SOLUTION. Solutions were received from Keith Anker, J A Deakin, Carlos Victor, Julius Guest and the proposer. We here print a composite of their solutions. Refer to the diagram below, which depicts the situation in cross-section.



It is to be understood that the "length of the hole" is the length of the cylindrical surface of the remaining material. This length is given as 6cm, but more generally denote it by $2h$. If R is the radius of the original sphere, then by Pythagoras the radius of the cylindrical hole is $\sqrt{R^2 - h^2}$. The *total volume* (original sphere plus cylindrical hole) contained between the planes $x = \pm h$ is given by $2\pi \int_0^h \left(R^2 - x^2\right) dx = 2\pi\left(R^2 h - \frac{h^3}{3}\right)$. From this amount deduct the volume of the cylinder: $2\pi h\left(R^2 - h^2\right)$ to find the result $\frac{4\pi h^3}{3}$. In the case given, h = 3cm, and so we find a residual volume of $36\pi\text{cm}^3$.

The result is quite independent of the original radius of the sphere, as several of the solvers pointed out. The proposer indicates that if (from the wording of the problem) we assume this from the outset, then we may suggest a formula $V = kh^3$, because the volume will necessarily be measured in units of length-cubed and h is the only length given. Now consider the special case in which $h \to \infty$. For very large h, $h \approx R$ and the volume left behind after drilling the hole is almost the entire volume of the sphere, namely $\frac{4\pi R^3}{3}$, and this suggests that $k = \frac{4\pi}{3}$, which is correct.

PROBLEM 25.2.2 (Submitted independently by J A Deakin and Julius Guest)

Sum the infinite series

$$1 - \frac{1}{5} + \frac{1}{9} - \frac{1}{13} + \ldots\ldots\ldots .$$

SOLUTION. Solutions were received from Keith Anker, David Halprin and the proposers. Here is Halprin's solution.

Assume the series to be convergent, for if it is not, then this will emerge during the course of the working. Write S for the sum of the series, and note that $S = f(1)$, where

$$f(x) = 1 - \frac{x^5}{5} + \frac{x^9}{9} - \frac{x^{13}}{13} + \frac{x^{17}}{17} - \ldots\ldots\ldots .$$

Then

$$f'(x) = -x^4 + x^8 - x^{12} + x^{16} - \ldots\ldots ,$$

and this series (a geometric series) has the known sum $-1 + \frac{1}{1+x^4}$.

Therefore

$$f(x) = -x + \int \left((1+x^4)\right)^{-1} dx$$
$$= -x + \frac{1}{4\sqrt{2}} \ln\left(\frac{x^2 + x\sqrt{2} + 1}{x^2 - x\sqrt{2} + 1}\right) + \frac{1}{2\sqrt{2}}\left(\arctan(x\sqrt{2}+1) + \arctan(x\sqrt{2}-1)\right) + K$$

where K is a constant of integration. (Deakin's solution included a detailed derivation of this difficult integral.) However, since $f(0) = 1$, we deduce that $K = 1$. Thus we further deduce that

$$f(1) = S = \frac{1}{4\sqrt{2}}\ln\left(\frac{2+\sqrt{2}}{2\sqrt{2}}\right) + \frac{1}{2\sqrt{2}}\left(\arctan(\sqrt{2}+1) + \arctan(\sqrt{2}-1)\right)$$
$$= \frac{\pi\sqrt{2}}{8} + \frac{\sqrt{2}\ln\left(\sqrt{2}+1\right)}{4} = 0.8669.... .$$

PROBLEM 25.2.3 (Submitted by Claudio Arroncher)

ABC is a triangle, right-angled at *A*. *H* is the foot of the perpendicular drawn from *A* to *BC*; *J* is the mid-point of *BC* and *M* is the point where the angle-bisector of *A* meets *BC*. Construct the triangle.

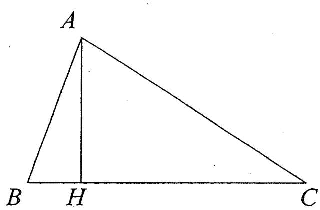
SOLUTION (from Julius Guest, another solution was sent by Carlos Victor)

As *ABC* is a right-angled triangle we need only two further pieces of data, not three, in order to construct the triangle. I elected to ignore the point *M*. Let $BC = a$, $CA = b$, $AB = c$, $BJ = p$, $BH = q$, with $a = 2p$. Because the triangle *AHC* is right-angled at *H*, $b^2 = (2p-q)^2 + AH^2$.
From this same triangle, we also have $AH = q\tan B$. From the triangle *ABC*, $b^2 + c^2 = a^2 = 4p^2$. We thus deduce that $q^2\sec^2 B = 4pq - q^2\sec^2 B$, which in turn simplifies to $\cos B = \sqrt{\frac{q}{2p}}$. We therefore wind up with

$$a = 2p,\ b = \sqrt{2p(2p-q)}\ ,\ c = \sqrt{2pq}\ .$$

Thus, with p, q both given, the three sides a, b, c are all constructible and hence the triangle *ABC* can be constructed.

[*The diagram below helps us to follow the argument, but because the task is to* ***construct*** *the diagram from the given data, it has no use beyond this. It should be noted that if the point M as given is not collinear with H, B, C (as constructed), then the task is impossible. Eds.*]



PROBLEM 25.2.4 (From *Mathematical Digest*)

Prove that if both the sides and the angles of a triangle are in arithmetic progression, then the triangle is equilateral.

SOLUTION. Solutions were received from Keith Anker, Julius Guest and Carlos Victor. This is Anker's solution.

Since the three angles add to 180°, their mean must be 60°. Suppose for definiteness that $A \le B \le C$, with $B - A = C - B$, i.e. $A + C = 2B$. Then $B = 60°$.

Now by the sine rule

$$\frac{c+a}{2b} = \frac{\sin C + \sin A}{2\sin B} = \frac{2\sin\left(\frac{C+A}{2}\right)\cos\left(\frac{C-A}{2}\right)}{2\sin B} = \frac{2\cos\frac{B}{2}\cos\left(\frac{C-A}{2}\right)}{4\sin\frac{B}{2}\cos\frac{B}{2}} = \frac{\cos\left(\frac{C-A}{2}\right)}{2\cos\left(\frac{C+A}{2}\right)}$$

But, as $2\cos\left(\frac{C+A}{2}\right) = 2\cos 60^\circ = 2\times\frac{1}{2} = 1$, we have

$$\frac{c+a}{2b} = \cos\left(\frac{C-A}{2}\right) \le 1, \text{ with equality only if } C = A.$$

so either the sides are not in arithmetic progression, or else

$$C = A = B = 60°,$$

and the triangle is equiangular and thus equilateral.

Guest's solution proceeded along different lines. After showing as above that $B = 60°$, he noted that the convention on the angles implied that $a \le b \le c$, and thus he set $a = b - d,\ c = b + d$. He then used the cosine rule with $B = 60°$ to find $d = 0$.

And indeed there are other possible ways to the result. Yet another could proceed along these lines. Let the angles be $B - D, B, B + D$ and let the opposite sides be (respectively) $b - d, b, b + d$. Then by the sine rule

$$\frac{\sin(B-D)}{b-d} = \frac{\sin B}{b} = \frac{\sin(B+D)}{b+d} .$$

It follows that

$$(b \pm d)\sin B = b(\sin B \cos D \pm \cos B \sin D).$$

Then from these two equations:

$$d \sin B = b \cos B \sin D$$

$$\cos D = 1 .$$

The result follows. This is a modification of Victor's solution.

Here are the new problems for this issue.

PROBLEM 25.4.1

A man dies and leaves behind one son and N daughters to share his estate. This is to be divided up according to the rabbinical law of inheritance described in the History of Mathematics section. Each of the daughters receives a fraction d of the total, while the residue s goes to the son. Can it happen that $d > s$, and if so, in what circumstances?

PROBLEM 25.4.2 (Submitted by Julius Guest)

In one of his letters to his friend and fellow mathematician Mersenne, Fermat stated the following theorem: "No prime number of the form $10s + 1$ (where s is a positive integer) is a divisor of any number of the form $5^n + 1$ (where n is a positive integer)". Was Fermat right?

PROBLEM 25.4.3 (From *Math-Jeunes*)

A number, written in decimal notation, consists of 2001 6's followed by a 7. What is its square?

PROBLEM 25.4.4 (Submitted by J A Deakin)

Find the primitive (indefinite integral) of the function $\dfrac{x+1}{x(1+xe^x)}$.

Function is a refereed mathematics journal produced by the Department of Mathematics & Statistics at Monash University. The journal was founded in 1977 by Prof G B Preston. *Function* is addressed principally to students in the upper years of secondary schools, and more generally to anyone who is interested in mathematics.

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Articles, correspondence, problems (with or without solutions) and other material for publication are invited. Address them to:

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