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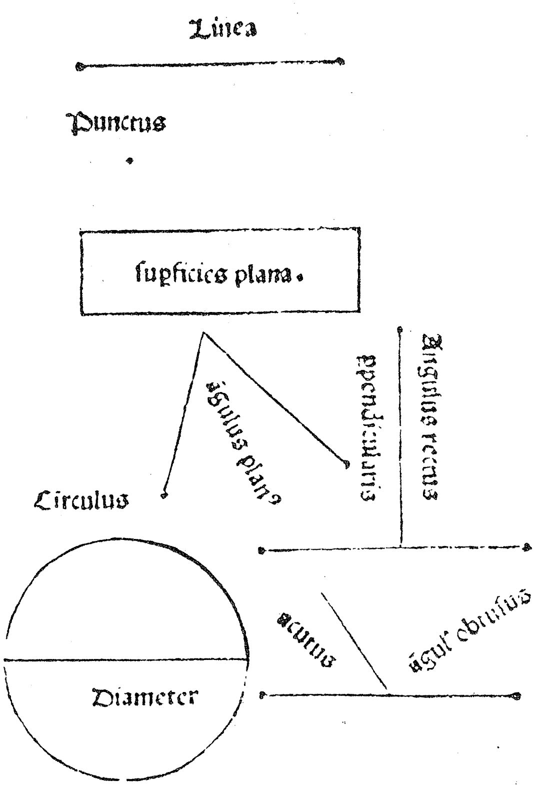

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Function is a mathematics magazine addressed principally to students in the upper forms of schools. Today mathematics is used in most of the sciences, physical, biological and social, in business management, in engineering. There are few human endeavours, from weather prediction to siting of traffic lights, that do not involve mathematics. *Function* contains articles describing some of these uses of mathematics. It also has articles, for entertainment and instruction, about mathematics and its history. Each issue contains problems and solutions are invited.

It is hoped that the student readers of *Function* will contribute material for publication. Articles, ideas, cartoons, comments, criticisms, advice are earnestly sought. Please send to the editors your views about what can be done to make *Function* more interesting for you.

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Articles, correspondence, problems (with or without solutions) and other material for publication are invited. Address them to:

The Editors,
Function,
Department of Mathematics,
Monash University,
Clayton, Victoria, 3168.

Alternatively correspondence may be addressed individually to any of the editors at the mathematics departments of the institutions shown above.

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Welcome to our 1982 readers, especially the new ones. Make sure this year's *Function* is interesting and enjoyable by sending to the editors articles and ideas to use.

Our leading article in this issue is about the publishing history of Euclid's famous book *Elements of Geometry*, which through the centuries, until very recent years, was a standard school textbook. The year 1982 is the 500th anniversary of the first printed version of Euclid's *Elements*†. We then have an article by Ian Rae explaining how those mysterious bar-line codes, that now appear on many food products, work. We finish with a review of some of the growing number of Rubik cube how-to-do-it books that have been published. This review also tells you how to concoct the best combination from those offered in the books for unscrambling your cube.

CONTENTS

†*Front Cover*: A portion of the frontispiece of this first printed edition of Euclid's *Elements*, published by Erhard Ratdolt, is our front cover for this issue.

A 500TH ANNIVERSARY OF 'EUCLID'

G. C. Smith, Monash University

This year we celebrate the 500th anniversary of the first printed book on geometry. It was, of course, an edition of Euclid's *Elements*.

Although printing from movable type was invented in the 1440's, the first forty years of printing saw little of mathematical interest. So the 1482 Euclid can be fairly considered to be the first significant printed book on mathematics.

The place where this important event in the history of mathematics occurred was Venice, and the printer was Erhard Ratdolt, a German, who learned printing in Augsburg where he was born about 1443. In 1475 he moved to Venice where he started a famous printing business. After 1486 he returned to Augsburg.

The Ratdolt edition of Euclid's *Elements* was printed in Latin and reproduces a translation from Arabic manuscripts made by Giovanni Campani (or Campanus) of Novara (1214-1294) who was chaplain to Pope Urban IV.

The Ratdolt *Elements* is a beautiful book. It is a folio with pages about 27 × 22 cm in size and the pages have large margins (about 8 cm) in which the figures are placed. In the dedication Ratdolt claimed that he was the first printer to print figures successfully (possibly he was the first one really to try!). There are hundreds of woodcut initials and the first page has an elaborate decorative border.

The diagram on the cover of this issue of *Function* shows the figures which illustrate the definitions.

The next edition of significance was that of Bartolomeo Zamberti. His edition, again in Latin, was translated from the Greek text. It was published in Venice in 1505. However the manuscript he used was one which was corrupt - this term means that the manuscript contained a lot of amendments by editors and even mistakes by the scribes who had copied it.

The Greek text first appeared in print in 1533. This text was produced by Simon Grynaeus (or Gryner), a theologian who worked in Vienna, Heidelberg, Tüburgen and Basel. The book was printed in Basel by Johann Herwagen. Grynaeus' edition remained the main source for future editions of the Greek text for many years. Unfortunately the two manuscripts principally used by Grynaeus (they have been identified and still exist in Paris and Venice) were also corrupt. However Grynaeus set the figures in the text (another 'first') rather than in the margin as Ratdolt had done.

A new Latin translation was made by Federigo Comandino (or Commandinus). This edition appeared in 1572 and was printed by Camillo Francischini, a printer working in Pesaro. Comandino and Francischini were under the influence of the Duke of Urbino. For more than 100 years the Dukes of Urbino were generous patrons to scholars and artists.

During the middle years of the 16th century translations into several European languages began to appear. The first was an Italian version by Nicolo Tartaglia - who was one of the first men to discover the method of solving cubic equations. This translation appeared in 1543. The Comandino edition was translated into Italian in 1575.

A French translation (of Books 1 - 6) appeared in Paris in 1564 and a German translation, also of Books 1 - 6, was published in Basel in 1562. Remarkably an Arabic edition was printed in Rome in 1594. This was printed from an Arabic version made by Nasir al-din in the 13th century.

British scholars were responsible for later editions of the Greek text. Henry Briggs - of logarithm fame - published an edition in 1620 which contained the first six books of *Elements* in Greek and Latin. The first complete edition of Euclid's works was that of David Gregory. It included the Greek text and Latin translation of Euclid's other geometrical and optical works as well as *Elements*. This was published in Oxford in 1703.

Let us now turn to editions in English. The first English edition of *Elements* was a translation made by Sir Henry Billingsley; it appeared in 1570. Billingsley was a London merchant who was Lord Mayor in 1597 and a member of parliament for the city in 1603. The title page is a delightful example of 16th century printing and salesmanship; it reads:

The Elements

of Geometrie

of the most Auncient Philosopher

EVCLIDE

of Megara

Faithfully (now first) translated into the English toung by H. Billingsley, Citizen of London. Whereunto are annexed certain Scholies, Annotations, and Inventions, of the best Mathematicians, both of time past, and in this our age.

Billingsley repeats a mistake, which was made by several early editors, of describing Euclid as 'Euclid of Megara'. Euclid was, of course, 'of Alexandria' and Euclid of Megara was quite another person. This confusion of two different people called Euclid was cleared up by Clavius (1537-1612) whose Latin version of *Elements* appeared in 1574.

Billingsley's edition is a large book. It contains 928 pages of folio size. But this includes a long preface by John Dee (who was probably the most prominent scientific scholar of Elizabethan England), as well as sixteen books of *Elements*. However the last three are not the work of Euclid.

Seventeenth and eighteenth century editions in English usually include only the 'geometrical books' i.e. Books 1 - 6, 11 and 12. These contain the parts of *Elements* which concern the basic ideas of plane and solid geometry - such matters as triangles, circles, tangents, lines and planes in space. The other books contain material which we would consider as the theory of numbers and algebra. However complete editions were produced by Isaac Barrow (in Latin in 1655 and in English in 1660) who was Newton's teacher; and by James Williamson in two volumes which appeared in 1781 and 1788. The most successful edition of this period was that of Robert Simson which appeared (in both Latin and English) in 1756. This was an edition of Books 1 - 6, 11 and 12 and was based upon Comandino's Latin. Simson included helpful notes in his edition and was so successful that no less than thirty editions appeared in the 100 years following its first publication. Its influence lasted well over 100 years, for many later English editors made use of Simson's edition as a starting point.

The more recent major editions have resulted from the discovery of new manuscripts - or the more careful study of manuscripts. The beginning of this phase of editing of *Elements* is the work of F. Peyrard a French scholar who published between 1814 and 1818 a three volume edition containing the Greek text and Latin and French translations using manuscripts that were finding their way to Paris as a result of Napoleon's campaigns in Italy. The authoritative edition of the Greek text was made by J.L. Heiberg and was published between 1883 and 1888. It was this work that formed the basis of the standard present-day English edition, that of T.L. Heath which was published in 1908 (and in a revised second edition in 1925). Those readers of *Function* who would like to find out more about Euclid's *Elements* are recommended to seek this edition from their library. In addition to the complete text there is a great wealth of historical information (and I acknowledge my debt to Heath for much of the information in this article) as well as discussion of the geometrical significance of the definitions, axioms and theorems that constitute the book of geometry that is the most famous of all mathematics texts - and whose 500th anniversary (in printed form) we celebrate this year.

Reference

T.L. Heath *The Thirteen Books of Euclid's Elements*. 3 volumes 1908; second edition 1925. Cambridge University Press. Reprinted 1956 (and subsequently) by Dover Publications Inc.

MACHINE READABLE CODES†

Ian D. Rae, Monash University

When I borrow a book from the Monash library, the attendant records the transaction by passing a 'light pen' over a striped tag in the book and then over one on the back of my I.D. card. This system is now used by many municipal libraries and something like it is beginning to appear in supermarkets and other large stores.

What's going on?

Well, in the library, the light pen 'reads' the coded numbers and passes the information to a small computer which notes which book has been borrowed by whom. Additional information, like the date and my staff/student classification is also taken in by the computer and each week a list is printed out showing who's got what and when it's due back. If the book doesn't come back on time, a recall or fine notice can be issued automatically.

In the supermarket, it's more complicated but a microcomputer is still an 'on line' part of the system. The reader can be a light pen or a fixed scanner over which the goods must be passed, and the information taken in can include the manufacturing company and its nationality, the nature and quantity of the goods and just when they were sold. In these applications, the computer has the price for each item registered in its memory, and this is instantly transmitted to the cash register and printed on a slip for the customer. That way, there are very few mistakes in addition but the system *can* be operated with no prices on the shelves, or, worse, a discrepancy between the shelf price and the 'computer' or real price. Consumer groups have been quick to point this out and retailers have assured them that it is not intended that the system operate, in this way, to the disadvantage of the customer.

Systems of product numbering combined with machine-readable codes are widely used in Europe, Japan and North America, but they have just begun to appear in Australia. The big advantage for the retailer is that he can tell how sales of a particular

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†Reproduced, with thanks, from *The Science Magazine*, Number 6, 1981, pp.16-21, published by the faculty of Science, Monash University.

item are going without even looking on the shelves. More important, the computer can be instructed to follow the decrease in stock and to issue a replacement order in time to keep the shelves stocked. For instance, the rate of sales of white socks would be monitored by the (correctly programmed!) computer, which would also have available to it the estimated delivery time of new stocks from the manufacturer. With normal buying patterns, it might be necessary to reorder white socks when the stock was down to one hundred pairs allowing ten days (at ten sales per day) for delivery. If the rate of sales was higher than average, then stocks would be exhausted before this time. This problem could be avoided if enough detailed information were available, and that's where the computer comes in handy. At what level of stock would it be necessary to reorder if delivery time were a constant ten days and sales were running at fifteen per day?

The Product Codes

For detailed information about the machine-readable codes we turned to the Australian Product Number Association (APNA), which has its headquarters close to Monash. Large retailers are encouraged to join the Association and adopt its APN system which is linked with the European Article Numbering (EAN) Association, the international body which supervises product numbers, including those of the earlier American Universal Product Code (UPC).

The APN standard number contains thirteen digits. Reading from left to right these numbers are

2 digits	prefix	always 93 for the APN series - allocated by EAN (Japan is 49, United Kingdom 50)
5 digits	manufacturer number	an identifier assigned by APNA
5 digits	item reference	identifying the particular product, package size, etc.
1 digit	check digit	see calculation before Fig.1.

Each number is expressed by a pattern of light and dark stripes which come in groups of seven - a bar code - and is also written as an Arabic numeral beneath the pattern. The patterns are arranged as shown in Fig.1.

In the example of Fig.1, the prefix is 93, manufacturer number 12345, item reference 67890, and the check digit 7. This check digit is an ingenious way for the computer to check that (a) this *is* a machine-readable code, and (b) it's done everything right. It is computed in the following way:

Step 1: start with digit at far right (i.e. the twelfth digit, not the check digit) and move left, summing alternate digits

Step 2: multiply the result of step 1 by 3

Step 3: sum all remaining digits

Step 4: add the result of step 2 to result of step 3

Step 5: the check digit is the smallest number which can be added to the result of step 4 to produce a multiple of 10.

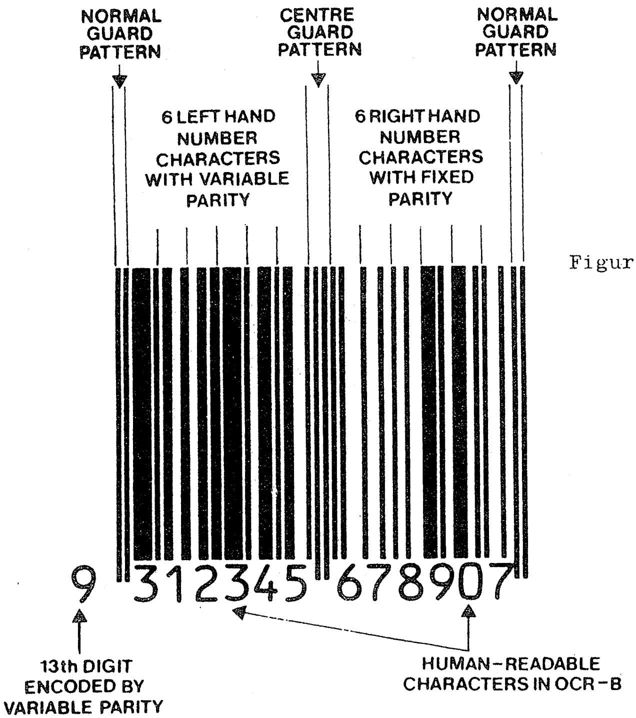


Figure 1

In the example of Fig.1, we have

$$0 + 8 + 6 + 4 + 2 + 3 = 23$$

$$23 \times 3 = 69$$

$$9 + 7 + 5 + 3 + 1 + 9 = 34$$

$$69 + 34 = 103$$

$$110 - 103 = 7 \quad \text{check digit}$$

A quick look around our food cupboards turned up a butter-vegetable oil composite with a thirteen digit code led by 'our' number 93. However, the code on my chewing gum package had only eight digits, with 93 still present at the left of the set. This is a scaled-down bar code introduced by EAN for use on small packages and consists of

2 digits	prefix	as before (Australia = 93)
5 digits	national short product number	determined by APNA
1 digit	check digit	computed as above

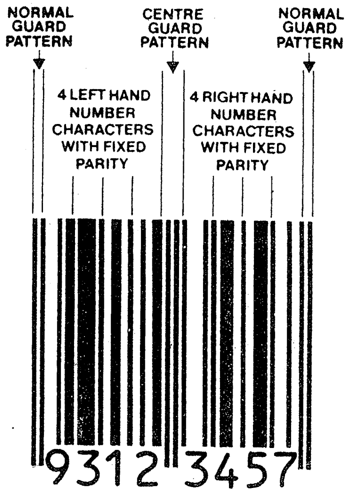


Figure 2

Which number is which?

If you study the bar codes in Figures 1 and 2, two questions will probably arise - 'just what is the code for each number?' and 'what is parity?' The answers are related and we tackle them in the following paragraphs and in Figures 3 and 4. First, in Figure 3, are three sets of bar code representations for each digit 0-9. In set A, each character (digit) is represented by a pattern with an odd number of dark stripes, i.e. set A has *odd parity*. In sets B and C the *parity is even*. Sets like this were discussed in a recent issue of *Function* (Volume 4, Part 5, October, 1980).

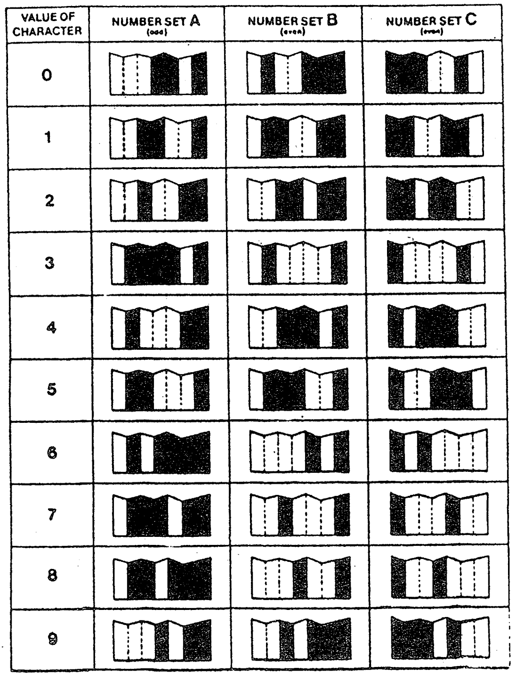
The pattern is produced by stripes, light or dark, each of which is 0.33 mm wide (total 2.31 mm for each digit). There are some minor variations of this rule which are evident upon careful scrutiny of Figures 1 and 2, but we will pass over them here. Instead, in Figure 4, we show enlarged bar codes derived from the coding table of Figure 3.

Value of character	Representation in set A	Representation in set B	Representation in set C
0	0 0 0 1 1 0 1	0 1 0 0 1 1 1	1 1 1 0 0 1 0
1	0 0 1 1 0 0 1	0 1 1 0 0 1 1	1 1 0 0 1 1 0
2	0 0 1 0 0 1 1	0 0 1 1 0 1 1	1 1 0 1 1 0 0
3	0 1 1 1 1 0 1	0 1 0 0 0 0 1	1 0 0 0 0 1 0
4	0 1 0 0 0 1 1	0 0 1 1 1 0 1	1 0 1 1 1 0 0
5	0 1 1 0 0 0 1	0 1 1 1 0 0 1	1 0 0 1 1 1 0
6	0 1 0 1 1 1 1	0 0 0 0 1 0 1	1 0 1 0 0 0 0
7	0 1 1 1 0 1 1	0 0 1 0 0 0 1	1 0 0 0 1 0 0
8	0 1 1 0 1 1 1	0 0 0 1 0 0 1	1 0 0 1 0 0 0
9	0 0 0 1 0 1 1	0 0 1 0 1 1 1	1 1 1 0 1 0 0

Figure 3

In the APN and EAN standard 13-digit numbers, such as that shown in Figure 1, the digits in the first set of six are represented by bar codes from set A or B. The combination of A and B characters is linked to the digit at the far left of the set of 13, which is itself not represented by any coding symbol. In Australia, this digit is 9, and the required sequence of characters for the next six digits is ABBABA. This sequence is determined from a standard table which we do not reproduce here, and the relationship provides yet another check that all is well with the product code. The second six-digit set is represented by characters from set C. You can work out for yourself which set of characters was used in the simpler case, the four-digit sets of Figure 2.

Figure 4



The actual size of the bar code pattern can vary over a set range, provided the proportionality is maintained. Note, from Figures 3 and 4, that no light or dark 'space' comprises more than four 'stripes', and from Figures 1 and 2, that auxiliary guard patterns are used.

Inside the light pen

These guard patterns play an essential role in the reading process, but to explain it we need to see how the light pen works. Inside is a small light-emitting diode (LED) which projects an infrared beam through a small slot on to the bar code. Reflections, from light areas, are picked up by a light-sensitive transistor placed just beside the LED, and this signal (a few hundred nano-amperes) is registered by the on-line computer as a zero. When the beam falls on a dark area, there is no signal and the computer registers a 'one'. But how does the computer know when to 'look', you ask. It's all done by sampling the line current at regular intervals, and the interval is determined by the space in the guard pattern. It is assumed that the light pen scans the pattern at a constant velocity and that, therefore, a fixed time interval is associated with each strip, be it light or dark. It rather surprised us that such an assumption of constant velocity would be realistic, but it turns out to be quite closely approximated by a hand-held scan. The time-per-strike i.e. the calibration for each scan, is determined by the time taken to cross the guard pattern. It sounds too simple to be realistic, but remember that there is the safeguard that if the velocity varies during a scan, and an incorrect reading results, it can nearly always be detected as a parity error, a discrepancy with the check digit, or simply a nonsense series which does not correspond to any of the codes in Fig.3. In this case, the computer is programmed to reject the data and to signal the operator, e.g. by means of an audible tone, to try again.

I have described the bar code used in product labelling but other codes are also in use e.g. the library code with which I introduced the article. The most recent use is for printing computer programmes themselves as long strips of bar codes. Once the computer is programmed to read the code, a light pen can be used to load other programmes cheaply and easily. Until I started my investigation it all seemed rather magical to me, but there *is* a rational basis for both code and scanner. If you come across a code which is obviously different from the ones I have described, why not try to find out how it works and write to let us know.

The latest use of machine-readable codes is for the transmission of whole computer programmes. Equipped with a light pen and the correct master programme, a computer can 'read' a new programme quickly and cheaply.

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MATRIX METHODS FOR PREDICTING AUSTRALIA'S POPULATION

James A. Koziol, University of California

Henry C. Tuckwell, Monash University

The population of Australia obtained from the 1976 census was 13 991 200. From census data, the Australian Bureau of Statistics has predicted that the total number of people in Australia will be 15 595 600 by 1986 and will rise to 18 867 300 by the year 2006. Figure 1 shows the populations up to and including 1976 obtained from census data, as well as future predictions. These future predictions, or *projections* as they are called, were obtained by what is known as the *component method*, in which the dynamics of population growth are broken up into the components of birth, death and migration.

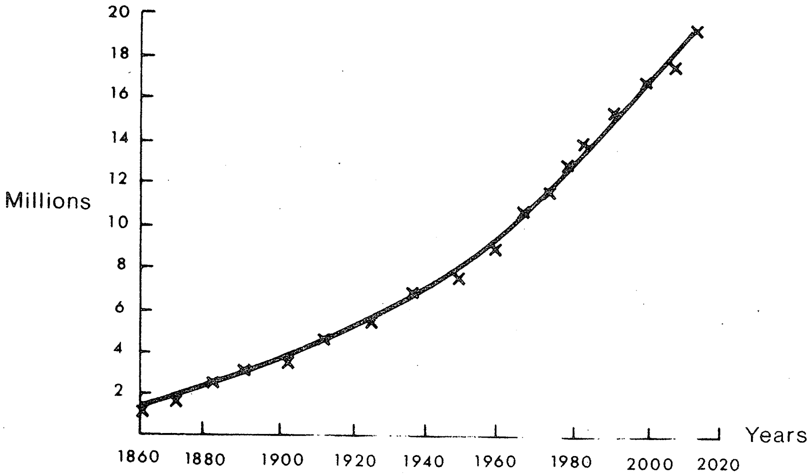

Figure 1. The population of Australia from 1860 onwards. The figures prior to 1976 are from Census data: later figures are predictions using the component method.

We shall explain in this article the basic mathematical ideas behind the component method. In principle these were first outlined in a general form by P.H. Leslie in an article for the journal *Biometrika* in 1945. Henceforth we focus attention on the females of the population, because they are the members that give birth to new individuals.

We must first subdivide the population into groups of various ages. This is because females of various ages have children at different rates and also, older females are usually more likely to die than younger ones. The rate at which females in a certain age group give birth is called an *age-specific fertility rate.* The rate at which they die is called an *age-specific mortality rate.*

Let us illustrate. In 1976, amongst 1000 Australian women between the ages of 20 and 24, there were on average 129 children born. This gives an age-specific fertility rate of ·129. By contrast, the age-specific fertility rate for women in the 35-39 year age group was ·024. Similarly, in 1976, amongst 2000 Australian women between the ages of 20 and 24 years there was on average only one death, yielding an age-specific mortality rate of ·0005. On the other hand, women between 70 and 74 years old had an age-specific mortality rate of ·03, representing a sixty-fold increase.

Births and deaths are not the only causes of changes in population size. In developing countries such as Australia, *migration* also plays a significant role. In 1976, for example, there was an increase in Australia's population of about 26 000 due to immigration, which is non-negligible in comparison with the 200 000 or so births.

Let us see how to formulate mathematically the problem of predicting the population in the future from these kinds of data. For simplicity we shall suppose there are only three age groups: up to one year; between 1 and 2 years; and between 2 and 3 years (we assume no one lives past three years). Let the age-specific fertility rates for these age groups be ·1, ·2, and ·1 respectively, and the corresponding age-specific mortality rates be ·05, ·1, and 1·0. Suppose that there are initially 300, 200, and 100 in the respective age groups. One year later, what is the age distribution of the population?

Those in the 0 - 1 age group are all from births; we therefore expect $\cdot 1 \times 300 + \cdot 2 \times 200 + \cdot 1 \times 100 = 80$ in this age group. Those in the 1 - 2 age group are survivors from the original 0 - 1 age group; we expect $(1 - \cdot 05) \times 300 = 285$ in the 1 - 2 age group. Similarly, those in the 2 - 3 year age group are survivors from the initial 1 - 2 year age group; we expect $(1 - \cdot 1) \times 200 = 180$ in the 2 - 3 year age group.

These calculations may be conveniently summarized in matrix notation as follows. Let N_0 and N_1 be the 3×1 column vectors giving the age distributions for the three age groups at times $t = 0$ and $t = 1$ respectively; for example, here

$N_0 = \begin{bmatrix} 300 \\ 200 \\ 100 \end{bmatrix}$ and $N_1 = \begin{bmatrix} 80 \\ 285 \\ 180 \end{bmatrix}$. Let M denote the 3×3 matrix, the first row of which consists of the age-specific fertility rates, the second row of which contains all zeros except for the first element, which is the age-specific survival rate (that is, one minus the age-specific mortality rate) for the first age group, and the third row of which contains all zeros except for the second element, which is the age-specific survival rate for the second age group. Thus for our example,

$$M = \begin{bmatrix} \cdot 1 & \cdot 2 & \cdot 1 \\ \cdot 95 & 0 & 0 \\ 0 & \cdot 9 & 0 \end{bmatrix} .$$

Now, our system of three linear equations,

$$\begin{aligned} \cdot 1 \times 300 + \cdot 2 \times 200 + \cdot 1 \times 100 &= 80 \\ (1 - \cdot 05) \times 300 + 0 \times 200 + 0 \times 100 &= 285 \\ 0 \times 300 + (1 - \cdot 1) \times 200 + 0 \times 100 &= 180 , \end{aligned}$$

can, by the rules of matrix multiplication, be written in mathematical shorthand notation as

$$M\ N_0 = N_1 .$$

Using the same rule, the vector N_2, whose elements are the projected numbers in each age group at time $t = 2$, can be found by multiplying the vector N_1 by the matrix M. That is,

$$M\ N_1 = \begin{bmatrix} \cdot 1 & \cdot 2 & \cdot 1 \\ \cdot 95 & 0 & 0 \\ 0 & \cdot 9 & 0 \end{bmatrix} \begin{bmatrix} 80 \\ 285 \\ 180 \end{bmatrix} = \begin{bmatrix} 88 \\ 76 \\ 256 \cdot 5 \end{bmatrix} = N_2 .$$

This procedure may be repeated indefinitely to predict the numbers in each age group in future generations so long as fertility rates and mortality rates remain fixed.

We call the matrix M a *Leslie matrix* after P.H. Leslie. Different Leslie matrices and different initial age distributions give rise to different patterns of long term population changes. An interesting example provided by Leslie occurs if

$$M = \begin{bmatrix} 0 & 0 & 6 \\ 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \end{bmatrix} .$$

Here we have a population which lives for only three years and which reproduces only in the third year of life. Let

$$N_0 = \begin{bmatrix} 1000 \\ 1000 \\ 1000 \end{bmatrix} .$$

After one year, we expect an age distribution given by

$$M\ N_0 = \begin{bmatrix} 0 & 0 & 6 \\ 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \end{bmatrix} \begin{bmatrix} 1000 \\ 1000 \\ 1000 \end{bmatrix} = \begin{bmatrix} 6000 \\ 500 \\ 333\frac{1}{3} \end{bmatrix} = N_1 \ ;$$

after two years, we expect an age distribution given by

$$M\ N_1 = M^2 N_0 = \begin{bmatrix} 0 & 0 & 6 \\ 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \end{bmatrix} \begin{bmatrix} 6000 \\ 500 \\ 333\frac{1}{3} \end{bmatrix} = \begin{bmatrix} 2000 \\ 3000 \\ 166\frac{2}{3} \end{bmatrix} = N_2 \ ;$$

and after three years, we expect an age distribution given by

$$M\ N_2 = M^2\ N_1 = M^3\ N_0 = \begin{bmatrix} 0 & 0 & 6 \\ 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \end{bmatrix} \begin{bmatrix} 2000 \\ 3000 \\ 166\frac{2}{3} \end{bmatrix} = \begin{bmatrix} 1000 \\ 1000 \\ 1000 \end{bmatrix} = N_3 .$$

Note that $N_3 = N_0$. That is, the age distribution after three years is the same as it was at the start! You should be able to see that in fact $N_6 = N_3$, $N_9 = N_6$, and so forth, so the population values repeat every three years.

Now suppose that the Leslie matrix M is given by

$$M = \begin{bmatrix} 0 & 1 & 3 \\ 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \end{bmatrix} ,$$

and that the initial population distribution is again given by

$$N_0 = \begin{bmatrix} 1000 \\ 1000 \\ 1000 \end{bmatrix} .$$

We shall not go through the calculations here, but we may show that, with these values for M and N_0, the initial population will tend toward a total population of 4000 distributed in the ratio of (6:3:1) for all subsequent generations; this age distribution will be achieved after approximately 23 generations. This is an example of a population that tends toward what is known as a *stable distribution*, that is, a population distribution in which the ratios of numbers in different age groups remain constant through generations.

As we had mentioned previously, the Australian Bureau of Statistics has made projections of Australia's population using the component method. Naturally, the calculations involved are much more complicated than the ones that we have discussed, but the ideas behind them are the same. The projections are quite useful for government and industry as a means of predicting future demand for goods and services. For example, public policy planners use population projections to allocate funds for the construction of schools, roads, and other vital services that a future population may require. Population projection can be a hazardous undertaking, however, in that we are applying assumptions in the present about future trends of fertility, mortality and the characteristics of overseas migration. If our assumptions are not borne out with time, then our projections can be grossly in error.

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EXPANDING YOUR HORIZONS†

REPORT OF A RADIO DISCUSSION ABOUT A CONFERENCE FOR GIRLS ON MATHEMATICS AND SCIENCE

"Expanding your horizons in Mathematics and Science" is the name of a sequence of conferences, initiated by Dr Susie Groves of Victoria College - Burwood, for schoolgirls. Some 600 attended the first two conferences in 1981. Two more on 20th March and 17th July are planned for 1982. Applications for the March conference closed on 26th February. Enquire now about attending the July conference.

Enquiries can be made to Dr Catherine Lassez, Footscray Institute of Technology, Ballarat Road, Footscray, 3011.

Before the first conference on July 8th, 1981, Melbourne radio station 3CR broadcast a discussion about the conference and its aims, hosted by journalist Eve Stocker. Eve interviewed two of the conference organisers, Dr Susie Groves, Lecturer in Mathematics at Victoria College - Burwood (then Burwood State College), Dr Liz Sonenberg, Research Fellow, Department of Computer Science, Melbourne University, and also Dr Gilah Leder, Lecturer in Education at Monash University, who was presenting a workshop for teachers attending the conference. The following article is based on extracts from this interview.

Eve: Susie, as the conference co-ordinator, could you tell us where the spark for this conference was kindled?

Susie: I attended the Fourth International Congress on Mathematical Education at Berkeley, California last year. I went to a session organised by a network of women in the San Francisco Bay area who have been running similar conferences there. They ran the first one in 1976 for about 200 students

† The article, *Weather Predictions*, is by one of the participants in a 1981 conference, Amanda Lynch.

and, during 1980, they ran 15 conferences. They were such a huge success there and I thought it was a really great idea, so I got together a group of women and we've been meeting regularly since last year organising this conference for July 11th.

We've had a really fantastic response. The day *after* the applications closed we received 125 applications. Altogether we've had over 500 applications from 99 schools throughout Victoria. As the conference is limited to about 200 participants we had to turn many people away. We are already planning a second conference for later this year to cater for these people.

Susie: Talking in the panel sessions in the afternoon, we have women in jobs ranging from air traffic controller to engine development engineer. Most girls don't meet many women working in jobs involving maths and science; often the only women they have met who have anything to do with mathematics have been their teachers, and so many girls think that if they're good at mathematics then what they should do is be a teacher. Not that there's anything wrong with being a teacher but there are many occupations that these girls might like to do if only they knew about them. Of course we also hope, if it's a success, that there will be spin-offs from the girls who have come talking to their friends about it. We'll be giving them some things to take home and read, that their parents may be interested in reading, also.

Eve: Gilah, as a person who is in teacher education yourself, can you comment on the awareness of the importance of girls going on in maths and science to teachers now. Is that changing, do you think?

Gilah: I think its very well documented now that mathematics qualifications at the high school level act very much as an entry filter to jobs - many courses don't need a very high proficiency of mathematics once you get into them, but to get into them you have to have a certain level of mathematics. I think with the increased publicity there, with the change in economic climate, with decreased opportunities for girls to work the traditional areas, and with the increase in the working life of so many of the girls, I think that everybody - parents and teachers - is more aware of the need to encourage girls to leave their options open as long as possible. And that may well be one of the reasons why teachers are so concerned that some of their students, both boys and girls, who are capable of continuing with mathematics, limit their options far too early.

I'd like to make one other point. Mathematics is very much a cumulative subject, and while it is not impossible to opt back into the mathematics stream once you've left it, it is very difficult. It is much easier to reach a certain level of mathematics if it's a continuing process, if you don't opt out at some stage during your high school career.

Susie: Most of our students here at Burwood are going to be primary teachers, most of them are girls, and a large number of them seem to have arrived here very surprised to find that mathematics is something which is important to a primary teacher. Of course they don't realise that they are going to be teaching mathematics just about every day of their working life. It would be especially good if girls in positions like this, a so-called traditional area, did realise how important mathematics was to them.

Eve: Do you see a need for a conference like this for boys?

Susie: Quite a number of people have said to me that they wished we were running it for boys as well as they had children who they think would have liked to have come. I think the need is there for boys. What happened in America with this network of women was that the types of activities they ran were so successful that many of the mathematics departments in various colleges and universities started to run similar activities for both boys and girls, because they saw how successful these ventures were.

Gilah: I'd really like to endorse what Susie has said. I think in some ways it is dangerous to single out either boys or girls for activities. The reason that girls have been singled out this time is to highlight the problem. But I think that in the future it would be a good idea if both boys and girls were made more aware of the fascinating ramifications of mathematics, that there really is more to mathematics than what they see in the normal course of the curriculum, and I think it's important for boys too to be aware of the very wide range of occupations that are open to them. There is a 'flight' away from the 'hard sciences' - both boys and girls are guilty of this - so I think it's important that both sexes are made aware of the importance and beauty and excitement of mathematics. But in this particular case I think that the girls are being given privileged treatment because they really are starting from behind.

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WEATHER PREDICTIONS

Amanda Lynch,†

Year 12, Lauriston Girl's School

Our whole life revolves around weather. Continually we ask; "Will it rain on my barbecue or football match?", "What will the weather be in Noosa next month?" or "How is the snow at Buller?" If we can't think of anything to say, we talk about the weather. A farmer's or fisherman's economic survival depends on the weather. The lives and homes of many Australians depend on our ability to predict floods and droughts. Whether we curse it or enjoy it, weather is an integral part of our lives.

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†This article stems from my attendance at a workshop *"Whither the Weather for Women"* as part of the conference *Expanding your horizons in Mathematics and Science*, that I attended.

Less than two hundred years ago, all weather forecasting resulted from the instincts of certain experienced individuals. They relied upon natural signs, and, as it was said, what they felt "in their bones". Since then, tremendous advances have been made so that today we rely upon computers for our weather information.

The predictions of these "sooth-sayers" of years past were sometimes quite accurate, for superstition and tradition had some roots in fact. A farmer's observation of certain clues in nature, not obvious to the untrained eye, and his or her knowledge of the area, often led to a fairly correct forecast. This, of course, could only be applied to the farmer's immediate surroundings, not to the whole state or country.

Various signs were used in this sort of prediction: colour of the sky:

> "Red sky at night, shepherd's delight,
> Red sky in the morning, shepherd's warning.";

the nature of the clouds:

> "Mackerel sky, mackerel sky
> Never long wet, never long dry.";

the wildlife of the area:

> "Seagull, seagull, stay on the sand
> It's never long fine when you're on the land.";

and even the moon:

> "Pale moon doth rain
> Red moon doth blow
> White moon doth neither rain nor snow.".

Although those who believe in such sayings could not give a reason why they work, they certainly do have some foundation in reality.

The next progression in weather prediction came late last century, when people started to use statistical methods, using measurements of temperature and rainfall. Experienced people would look for cycles in the weather, and base their predictions on what would follow the cycle. The most famous proponent of this was a man called Inigo Jones, who specialised in long term forecasting. This method was, in fact, less accurate than the previous one, but weather prediction did at least become a pseudo-science.

With the invention of the electric telegraph, data from many different places could be collected simultaneously and relayed to a central station where they could be systematized. Because of this, more variables could be taken into account, for example, pressure and humidity. Trained people interpreted these data in much the same way as the farmers did of old. The statistical method was abandoned for this more accurate interpretative method.

The way in which meteorologists are able to build up weather patterns is complicated but they do follow several basic rules. Thus, for example: where there is a low pressure area, air is rising, and so the atmosphere at that point is unstable; where there is a high pressure area, air is sinking, and so the atmosphere at that point is stable; warm air rises; cool air sinks. These rules are implemented in much the same way as the farmers' traditional sayings, and are just as numerous.

Since the behaviour of the atmosphere is so complex, the equations (or numerical model) describing it are beyond solution by analytical or exact methods. In 1918, it was realised by the meteorologist L.F. Richardson that the equations could be solved by numerical methods. He estimated, however, that it would take 64 000 people to keep up with the atmospheric changes, and even more to produce a forecast in advance. By the 1950's, electronic computers had been developed, and numerical forecasts on a regular basis became possible. However, lack of observational data meant that these forecasts were inaccurate.

Today, with advanced computing technology and communication and observation satellites, (fairly) accurate prediction of weather is now a reality.

In order to simplify the results to determine changes in temperature and humidity, meteorologists use a diagram which represents the states of certain air parcels graphically. This is called the aerological diagram, and was one of the subjects discussed at a recent conference in Melbourne - "Expanding your Horizons in Mathematics and Science".

Such a graph enables the meteorologist to determine when a cloud will form, at what height it forms, how much water it contains, the possibility of thunderstorms, and more. Unlike previous forecasting, this is an exact form of prediction within the limits of available data.

The complexity of the problem of predicting weather means that there is room for improvement in mathematical models and numerical methods used to solve the equations, in the types of instrumentation used to collect the data, and the frequency of the measurements made. Even now there is immense development in the field of computers - how fast, how efficient they are, the quality of the software.

It was clear from the conference attended that meteorology is still a developing area of science, and so there is a strong demand for sufficiently skilled people in any of the fields mentioned. It was interesting to learn that something we take for granted is an active and dynamic field of science.

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BATTLE OF THE CUBE BOOKS

John Stillwell, Monash University

Since Don Taylor's book "Mastering Rubik's Cube" became a world-wide best seller, many cube solvers have rushed to publish books of their own. And even before Don Taylor there was David Singmaster, whose book is still the most informative, though not easy to read. Altogether, the books known to me are:

Don Taylor: *Mastering Rubik's Cube*, Book Marketing Australia P/L, 1980.

David Singmaster: *Notes on Rubik's 'Magic Cube'*, 5th Edition, published by the author, London, 1980.

James Nourse: *The Simple Solution to Rubik's Cube*, Bantam Books 1981.

Patrick Bossert: *You Can Do the Cube*, Puffin Books, 1981.

Bridget Last: *A Simple Approach to the Fantasy Block*, John Engelander & Co., Melbourne, 1981.

Robert Groom: *Conquer the Cube*, Conflict Simulations of Australia, 1981.

David and Stephen Chang: *Conquer the Cube in Six Steps*, Wen Chun-Lin, Taiwan, 1981.

Czes Kosnowski: *Conquer that Cube*, Cambridge University Press, 1981.

John and Lyn St Clair-Thomas and Steve Shackel: *Cube*, Alpha Creations, Goulburn, N.S.W., 1981.

Don Taylor and Leanne Rylands: *Cube Games*, Greenhouse Publications, Collingwood, 1981,

and there is also a book on the related Pyraminx puzzle:

Ronald Turner-Smith: *The Amazing Pyraminx*, Meffert Novelties Ltd, Hong Kong, 1981.

These cube solvers have one characteristic in common - they do not take any notice of each other's methods. Thus, Taylor is very easy for the first two layers, but lousy for the last; Singmaster is nice for the last layer but ridiculous for the second; Bossert is hopeless even for the first layer, and so on.

In contrast, I write as a complete cube idiot. Before I looked at the books, I couldn't get even one face right, but now I can fix a cube from memory in about two minutes. The secret is not to try to be original, but to simply pick out the best features of the various published methods. At this stage it is probably a waste of time trying to invent new routines for the basic processes. This article is a guide to the books, with suggestions on the best selection of routines. At the end, I give my selection, which is easier to remember than any of the published methods.

I assume that readers know the cube basics, such as the meaning of corner pieces, edge pieces and layers; a good reference is Leo Brewin's article on the cube in *Function* Volume 5, Part 5.

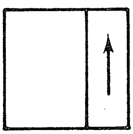
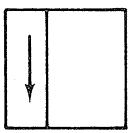
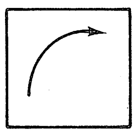
Notation

Most authors use Singmaster's notation:

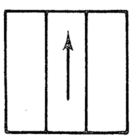
U for up	R for right	F for front
D for down	L for left	B for back

where each letter denotes a clockwise $\frac{1}{4}$ turn of the face in question. One wants to say "top" and "bottom", rather than "up" and "down", but this leads to the same initial letter, B, for both back and bottom. St Clair-Thomas and Shackel, for no particular reason, use T = top and D = down, while Nourse rescues T = top, B = bottom by calling the back the *posterior* and writing it P. But isn't "posterior" going to be confused with "bottom"?

The 13-year-old Bossert has what seems to be a good idea for people unfamiliar with mathematical symbols. Each move is shown by a picture of the front face of the cube, with an arrow showing the layer being moved. Thus

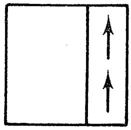
R =  L =  F = 

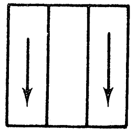
It gets a bit awkward to show B (a dotted arrow), but what really fouls things up is his insistence on using slice moves like



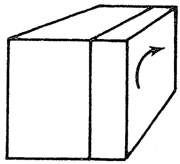
Yes, he really expects you to turn the centre slice and hold the right and left layers steady! This is impossible on a stiff cube, though I must admit that Bossert includes instructions on greasing. This is probably the most useful information in his book.

Last has a similar pictorial notation, but a bit better. Where possible, two moves are shown on the one picture, e.g.

R^2 = 

$R^{-1}L$ = 

(The latter is her, more sensible, handling of the slice move.) Chang has even more literal pictures which show three faces of the cube, e.g.

R = 

and Turner-Smith uses analogous pictures for the Pyraminx. It is not clear to me how helpful pictorial rotation is for beginners (primary school children, for example, seem quite able to learn symbolic notation) and whatever charm it has certainly wears off when one tries to read and memorise long sequences. You can't exactly recite it under your breath, can you?

I therefore plump for the U,D,R,L,F,B notation. Next there is the question of whether to write

$$RF^2U^{-1} \quad \text{or} \quad R + 2F - U \quad \text{or} \quad R^+F2U^-$$

for R (clockwise) followed by

F (twice clockwise) followed by

U (anticlockwise).

The first ("multiplicative notation") is in line with mathematical practice, and most authors use it. The second is used by Brewin and the third by Nourse; they could be called "additive notation". I argue against additive notation, not only because it may lead to bad mathematical habits; it is also less concise (all those superfluous + signs), and less amenable to punctuation. For example, Brewin's routine

$$F + U - F + U + F + 2U - F - U + F + U - F + U + F + 2U - F$$

is more easily read and remembered as

$$FUF^{-1}UFU^2F^{-1}\circ U^{-1}\circ FUF^{-1}UFU^2F^{-1},$$

with the punctuation ∘ used to mark off the subroutine $FUF^{-1}UFU^2F^{-1}$.

Brackets can also be used as punctuation. Kosniowski does this, but only Taylor and Rylands take full advantage of brackets by using exponents to indicate repetitions, e.g. $R^2F^2B^2 \cdot R^2F^2B^2 = (R^2F^2B^2)^2$. Special symbols for common combinations of moves are also helpful, but only Singmaster uses them. His "slice" and "antislice", e.g. $R_s = RL^{-1}$ and $R_a = RL$, and the standard notation [A,B] for the "commutator" $ABA^{-1}B^{-1}$ of any two elements make many routines easier to remember and compare.

The algorithms for solving the cube

All except Bossert and Last proceed by layers, doing the first two rather simply and bringing out their big guns for the last one. As already mentioned, Bossert has a hard time even with the first layer (24 pages of his book!) and his method gets even worse after that, when he tries to do two layers at once. Last does not believe in using layers at all, and tries to discourage this approach in her introduction. Her method is to place the 8 corner pieces first, then orient them, then place and orient the 12 edge pieces. The number of routines to be learned is very small, and they are quite short.

The catch is that only two routines are stated for moving the 12 edge pieces, each of which permutes 3 pieces, and no guidance is given on how to use them. If Bossert's book is any indication, large scale rearrangements of edge pieces are nasty, because he takes 30 pages to rearrange just 8. Also, it is diabolically difficult to see where *any* of Last's routines applies, because the cube appears scrambled right up to the last moment. The Bossert and Last methods seem suited only to people with amazing visual ability, and so I turn to the layer-by-layer methods.

First layer. Since most people understand the first layer, I will not go over the methods for it. All the remaining authors do it quite simply.

Second layer. Taylor, Groom and Brewin have the simplest method for the second layer; it depends only on the subroutine FU^2R. One follows this with U or U^{-1}, depending on the situation, then concludes with its inverse $R^{-1}U^2F^{-1}$. The method is fun to use and seems resilient to mistakes. The commonest errors, such as doing the wrong one of U, U^{-1} in the middle, do not damage the work which has already been done.

Nourse, Chang and Kosniowski all use a routine which is a product of two commutators. It is short, but harder to memorise in my opinion. St Clair-Thomas and Shackel's routine is rather similar, while Singmaster's method is quite eccentric and best ignored.

Last layer. This is where the Taylor book has caused a lot of mischief. Taylor himself is unhappy with it - he thought of some improvements, but too late to include them in the book before printing. He begins the last layer with routines for permuting 3 corners, but fails to point out that they are superfluous, because it is always possible to get 2 corners in place in his setup! Even if the reader notices this, there are still too many long routines to remember, and one faces the embarrassment of having to say "Excuse me, I have to get the book now" in the middle of an attempt to solve the cube in front of admiring friends.

I found the way out when a neighbour borrowed my copy of Taylor one day and I was forced to look at Singmaster. In the middle of all his mathematics, there is a 2-page solution of the cube which *can* be memorised! Singmaster has fewer routines, and they are mostly shorter or more repetitive than Taylor's. This is partly because Singmaster does things in a different order, for example, flipping edges first instead of last, and partly because Taylor's routines often do more than is required, conserving the positions of pieces which have not yet been correctly placed.

Brewin's solution in *Function* Volume 5, Part 5 is even simpler than Singmaster's in some ways, and by combining the best bits of both one obtains the solution given by Kosniowski. Kosniowski's book is beautifully illustrated and generally clear, but he also includes superfluous routines, and a very simple method for twirling corners (Singmaster's) is made to look difficult by writing it as a long sequence without explanation. The same routines for flipping edges and twirling corners appear in Chang, but the routines for placing edges and corners are more complicated. Groom's method is similar in complexity to Taylor's, though he does have some remarkably short routines for twirling corners before the edges have been placed. St Clair-Thomas and Shackel is a mish-mash. They flip edges like Singmaster, twirl corners rather like Groom and place edges rather like Taylor. The result actually takes less moves in the average case than any of the other methods, but requires more thought and memory. Finally, you can forget about Nourse. One of his routines has 32 moves!

In the appendix I give a method for the last layer which is basically Kosniowski, minus all unnecessary routines. Only four essentially different routines are needed, of 6, 9, 8 and 8 moves respectively.

Understanding the cube

Why does one never end up with a single corner to be twirled in the last layer? Why can repetitions of $FDF^{-1}D^{-1}$ be used to twirl corners? Why on earth does it take 105 repetitions of RU to get back to start? The only book which gives the understanding needed to answer these questions is Singmaster's. His book is a mine of information on special purpose routines, pretty patterns, and mathematical questions arising from the cube. Unfortunately, the book is exasperating to read, because Singmaster wrote it in five stages, as his ideas developed, and was too lazy to revise the early stages in the light of later simplifications and changes in notation. It is interesting to see, however, that his earliest and crudest solution is along the lines of Bossert and Last!

Taylor and Rylands' "Cube Games" is largely a series of illustrations for Singmaster's routines for pretty patterns and special purposes, nevertheless they perform a great service by organising the material and making it clear by means of colour pictures. If a routine in Singmaster is used to produce a pattern, one forgets immediately which way the cube was held while the routine was being executed, so it is futile to use the inverse routine to get back to start. Taylor and Rylands' picture tells which way the cube must be held. In a few cases they have also found shorter routines than Singmaster Perhaps the only well-known pattern they omit is the one known to schoolchildren as "Fish", which is given by $(R^2U^2)^3$. Singmaster is very fond of this routine, to the extent of using it when simpler methods are available (e.g. for doing the second layer).

Related puzzles

Kosniowski and Taylor and Rylands are the only books to deal with the barrel (octagonal prism) puzzle and cubes with pictures. The barrel has some "trimmed" edge pieces with only one colour on them, so these little devils can get flipped without being noticed. The problems this causes can be handled by standard edge flipping routines on the cube, but Kosniowski and Taylor and Rylands give faster methods which are specific to the barrel.

A more interesting problem arises with the ordinary cube when the faces have pictures (or symbols) on them, so that the centre squares have to be correctly oriented. Some routines which rotate centre squares are given by Singmaster, and the two essential ones are explained by Kosniowski. Taylor and Rylands also have these two, and they give a number of optional ones of their own. To these I would like to add my favourite: $(RUR^{-1}U)^5$ gives a half turn to the centre square of the upper face. This is not the shortest way to do it, but it is the easiest to remember.

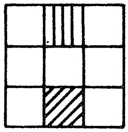
These methods also apply to the octahedral puzzle known as the "Star puzzle", because it is abstractly just a cube with the corners missing and centre squares (at the vertices of the octahedron) which have to be oriented.

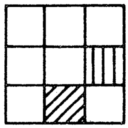
The Pyraminx puzzle is easier than the cube, though not so trivial that one can't write a book about it. Turner-Smith manages to spin it out for 192 pages, about 30 of which contain useful information (solution and mathematical appendix). The rest of the book is padded to bursting with testimonials from cube celebrities, shots of the factory where the puzzles are made, and other trivia. Worst of all, Turner-Smith deliberately gives routines for pretty patterns which are longer than necessary. The patterns called "Jewel" and "Cat's Paws", which are the keys to the solution of the Pyraminx, are given in the patterns section by 26 moves and 16 moves respectively. Only by carefully searching the appendix does one discover 8 move sequences for each of these patterns. "Jewel", incidentally, is a vertex rotation analogous to rotation of a centre square on the cube, and it can be done by a similar routine: $(RUR^{-1}U)^2$.

Appendix: Condensed Kosniowski method for last layer.

Stage 1: Flip edges.

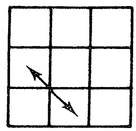
Since edges are not correctly placed yet, "flip" just means get the right colour facing up. (Edge pieces are in fact permuted by these routines.) Colours which have to be corrected are shown shaded.

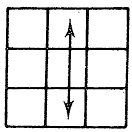
To correct  $F \cdot RUR^{-1}U^{-1} \cdot F^{-1}$

To correct  $F \cdot URU^{-1}R^{-1} \cdot F^{-1}$

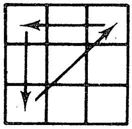
Stage 2: Place edges.

Turn the upper layer until at least two edges are in place. Then

To swap  $RUR^{-1}U \cdot RU^2R^{-1}U$

To swap  first do $RUR^{-1}U \cdot RU^2R^{-1}U$, then turn upper layer to get the first kind of swap

Stage 3: Place corners.

To cycle  $L \cdot FR^{-1}F^{-1} \cdot L^{-1} \cdot FRF^{-1}$

To cycle in the other direction, do it twice, or use the inverse. If no corner is in place, use this routine anyway. It will put one corner in place, then repeat as above to cycle the other three.

Stage 4: Twirl corners.

Turn the upper layer until the corner to be twirled is in the URF position, then twirl it

Clockwise by $(FDF^{-1}D^{-1})^2$

Anticlockwise by $(DFD^{-1}F^{-1})^2$.

A single twirl scrambles the two lower layers of the cube, but don't panic! Rotate the upper layer again to bring the next corner to be twirled into the URF position, and repeat. When all twirls have been completed, the whole cube will be restored, to the amazement of onlookers.

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From the book, BE MY GUEST *by Conrad Hilton, Prentice Hall, N.J., 1957.*

I'm not out to convince anyone that calculus, or even algebra and geometry, are necessities in the hotel business. But I will argue long and loud that they are not useless ornaments pinned onto an average man's education. For me, at any rate, the ability to formulate quickly, to resolve any problem into its simplest, clearest form, has been exceedingly useful. It is true that you do not use algebraic formulae but in those three small brick buildings at Socorro I found higher mathematics the best possible exercise for developing the mental muscles necessary to this process.

In later years, I was to be faced with large financial problems, enormous business deals with as many ramifications as an octopus has arms, where bankers, lawyers, consultants, all threw in their particular bit of information. It is always necessary to listen carefully to the powwow, but in the end someone has to put them all together, see the actual problem for what it is, and make a decision - come up with an answer.

A thorough training in the mental disciplines of mathematics precludes any tendency to be fuzzy, to be misled by red herrings, and I can only believe that my two years at the School of Mines helped me to see quickly what the actual problem was - and where the problem is, the answer is. Any time you have two times two and know it, you are bound to have four.
(Reproduced by permission of the Mathematical Assoc. of America.)

LETTERS TO THE EDITOR

Dear Editors,

In connexion with your remarks on the etymology of the words duellum and bellum I would like to present a different view. (See *Function*, Volume 5, Part 5, page 32.)

Duellum is not a version of *bellum*, but rather *bellum* is a late form of Old Latin *duellum* which is, indeed, derived from *duo*. Compare:

bis = twice, which is derived from *duis* (Greek δις), or *bonus* which is Old Latin *duonus*, *duenos* for "good".

(Reference: Der Kleine Stowasser, Latin-German dictionary for Austrian high schools.)

Yours faithfully,
Hans Lausch, Monash University

Dear Editors,

Lest it be thought that there is something God-given, fixed, or immutable about the rule for multiplying matrices, which for the purposes of this letter can be stated as "row-into-column", the little examples which follow show that the above-stated rule is a mere convention, perhaps a little more substantial than that conventional map brought by the bellman on board the ship in which they hunted the Snark, but nonetheless convention. Everything can be done equally well according to the rule "column-into-row". It need hardly be added that "row-into-row" or "column-into-column" will not work for matrices in general even though it can be used for determinants. The examples are hardly necessary, for to work the "column-into-row" rule one merely has to, temporarily, translate the *word* "row" into the *word* "column" and vice versa. For the doubting Thomases, however, the examples follow. Explanations are not needed, all one has to do is look and think.

$$\begin{aligned} x &= a_{11}u + a_{12}v \\ y &= a_{21}u + a_{22}v \\ z &= a_{31}u + a_{32}v \end{aligned} \quad \text{is} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}, \quad \text{say } \underset{\sim}{x} = \underset{\sim}{u}\underset{\sim}{A}$$

$$\begin{aligned} u &= b_{11}\alpha + b_{12}\beta + b_{13}\gamma \\ v &= b_{21}\alpha + b_{22}\beta + b_{23}\gamma \end{aligned} \quad \text{is} \quad \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}, \quad \text{say } \underset{\sim}{u} = \underset{\sim}{\alpha}\underset{\sim}{B}.$$

Now $\underset{\sim}{x} = \underset{\sim}{u}\underset{\sim}{A} = (\underset{\sim}{\alpha}\underset{\sim}{B})\underset{\sim}{A} = \underset{\sim}{\alpha}(\underset{\sim}{B}\underset{\sim}{A})$.

$x = (a_{11}b_{11} + a_{12}b_{21})\alpha + (a_{11}b_{12} + a_{12}b_{22})\beta + (a_{11}b_{13} + a_{12}b_{23})\gamma$

$y = \ldots$

$z = \ldots$

$$BA = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

$$= \begin{bmatrix} b_{11}a_{11} + b_{21}a_{12}, & b_{12}a_{11} + b_{22}a_{12}, & b_{13}a_{11} + b_{23}a_{12} \\ b_{11}a_{21} + b_{21}a_{22}, & b_{12}a_{21} + b_{22}a_{22}. & \ldots \\ \ldots & \ldots & \ldots \end{bmatrix}$$

Yours faithfully,
John Barton, Melbourne University

We had several letters pointing out that the solution to Problem 5.3.2 given in part 5 of Volume 5 of *Function* (p.30) was wrong. Both the argument and the answer are wrong. For a correct solution provided by David Shaw and the pupils of Year 11, Maths A students at Geelong West Technical School see the Problem Section below.

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PROBLEM SECTION

We begin with some corrections.

PROBLEM 5.3.2.

An incorrect solution to the problem was given in the last issue of *Function* of 1981, on page 30. The incorrect statement is:

Similarly

$$5555^{2222} = 4 + \text{a multiple of } 7.$$

[Always beware when you see the word 'similarly' in a mathematical argument. It often indicates a relaxation of attention. Even more dangerous are the phrases "it is obvious that", or "it follows trivially": these too often indicate that the author has ceased thinking.]

Colin A. Wratten, of Highett, pointed out the error and also offered a much better solution. We publish the solution (slightly modified) of the students of Year 11, Maths A, at Geelong West Technical School, as transmitted by David Shaw, the Head of their Mathematics Department.

First a convention, that they use in their letter. We write $a \equiv b \pmod 7$ to mean that a differs from b by a multiple of 7 (read "a is congruent to b, mod 7"). Thus $2 \equiv 9 \pmod 7$ and $(317 \times 7 + 3)^{5555} \equiv 3^{5555} \pmod 7$.

Recall that the problem is:

Is $2222^{5555} + 5555^{2222}$ divisible by 7?

Since $2222 = 317 \times 7 + 3$,

$$2222^{5555} \equiv 3^{5555} \pmod 7.$$

Since $3^6 = 7 \times 104 + 1$, and $5555 = 5 + 6 \times 925$,

$$\begin{aligned} 3^{5555} &= 3^5 \times (3^6)^{925} \\ &\equiv 3^5 \pmod 7 \\ &= 243 = 7 \times 34 + 5 \\ &\equiv 5 \pmod 7. \end{aligned}$$

Similarly,

$$\begin{aligned} 5555^{2222} &= (7 \times 793 + 4)^{2222} \\ &\equiv 4^{2222} \pmod 7 \\ &= (4^3)^{740} \times 4^2 \\ &\equiv 4^2 \pmod 7 = 16 \\ &\equiv 2 \pmod 7, \end{aligned}$$

using the fact that $4^3 = 7 \times 9 + 1 \equiv 1 \pmod 7$.

Thus

$$\begin{aligned} 2222^{5555} + 5555^{2222} &\equiv 5 + 2 \pmod 7 \\ &\equiv 0 \pmod 7; \end{aligned}$$

in other words the expression on the left is divisible by 7.

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PROBLEM 5.5.3

Police Witness: Your worship, the defendant had to brake suddenly while travelling up a 30° slope. His skid marks measured 30m. I later tested the defendant's car on the level road outside the police station. Both roads are paved with the same material. I slammed on the brakes at 60 km/h and skidded to a halt in exactly 30m. obviously he was travelling faster to require 30m to stop on the steep grade.

Magistrate (after consulting a pocket calculator):
Case dismissed.

Explain the magistrate's reasons for his decision.

SOLUTION. If the magistrate thought he had good reasons for his solution then it must have been either because his calculator was not working or because he did not understand the mathematics of the situation. Perhaps he was just carrying on a personal feud with the policeman concerned.

It is always dangerous to accept without question the result of a calculation that flies in the face of common sense. In fact here the data presented by the policeman show that the motorist was travelling at a little over 80km/h.

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Again we apologise for the imprecision of the statement of

PROBLEM 5.4.2

We have had a number of problems from Russian problem-books. This one, we are told, is from a recent Chinese examination.

Ten women in a village go to collect water at the village pump. Some take only a short time, others rather more and yet others even more. How should one schedule them in order to minimize the total woman-hours spent at the pump?

Now do the same problem for the case in which two pumps are available.

SOLUTION. The obvious solution is to arrange that each woman arrives at the pump just when the woman ahead of her finishes. There are no waiting times. So the total time taken is just the sum of the individual times taken by each woman. This is clearly the minimum.

For those who perhaps interpreted the problem differently we now offer a solution for the case when all the women turn up simultaneously.

Suppose the first woman to use the pump takes time t_1 to fill her bucket, the second woman takes time t_2, and so on. Thus the second woman has to wait time t_1 before starting to fill her bucket, the third woman has to wait time $t_1 + t_2$ before she starts, and so on. The total time spent by all the women is

$$t_1 + (t_1 + t_2) + (t_1 + t_2 + t_3) + \dots + (t_1 + t_2 + \dots + t_{10})$$
$$= 10t_1 + 9t_2 + 8t_3 + \dots + 2t_9 + 1t_{10}.$$

This will be smallest if we make the term with the largest coefficient (10) be the smallest time, etc. Thus arrange the women so that

$$t_1 \leqslant t_2 \leqslant t_3 \leqslant \ldots \leqslant t_{10},$$

letting the quick ones get away early and leave the slowest till last. We leave the two pump problem for our readers again.

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PROBLEM 6.1.1

Mr B.W. Harridge of Melbourne High School tells us of the following problem posed to students at the MAV Mathematics Camp at Glenaladale this year.

"Represent each of the integers from 1 to 100, using the digits of 1982 *in that order*, and any of the following mathematical operations:

$+$, $-$, $\div$ (or $/$), $\times$, raising to a power

$\sqrt{\ }$, !, brackets, decimal points.

Adjacent digits such as 98 are understood to be decimal numbers (ninety-eight here).

For example $23 = 19 + 8/2$

$51 = 1 + (\sqrt{9})! \times 8 + 2$."

There was one number which defied all our efforts, namely 52.

We leave it to *Function* readers either to find a representation of 52 in the required form or to prove that such a form does not exist.

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Drawing by Dana Fradon © The New Yorker Magazine, Inc

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MONASH SCHOOLS' MATHEMATICS LECTURES, 1982

Monash University Mathematics Department invites secondary school students studying mathematics, particularly those in years 11 and 12 (H.S.C.) to a series of lectures on mathematical topics.

The lectures are free, and open also to teachers and parents accompanying students. Each lecture will last for approximately one hour and will not assume attendance at other lectures in the series.

Location: Monash University, Rotunda Lecture Theatre R1. The Rotunda shares a common entry foyer with the Alexander Theatre. For further directions please enquire at the Gatehouse in the main entrance of Monash in Wellington Road, Clayton. Parking is possible in any car park at Monash.

Time: Friday evenings as below; 7.00 p.m. to 8.00 p.m. (approx.).

Program:

March 26	"Probability for Pleasure and Profit". Professor W.J. Ewens.
April 2	"Having Fun with Irrational Numbers - some of the remarkable applications of number theory to Stonehenge, Computing and Statistics". Dr J.J. Monaghan.
April 16	"Mathematical Paradoxes". Professor G.B. Preston.
April 30	"Stonehenge and Ancient Egypt - the mathematics of radiocarbon dating". Dr R.M. Clark.
June 4	"How Aeroplanes Fly". Mrs B.L. Cumming
June 18	"The Mathematics of the Rubik Cube". Dr J.C. Stillwell.
July 2	"Chaos - Fluctuations in Populations". Dr G.A. Watterson.
July 16	"Formation of the Solar System". Dr A.J. Prentice.
July 30	"Two Circles Intersect at Four Points!". Dr C.F. Moppert.

Enrolment: There will be no enrolment formalities or fees. Just come along!

Further Information: Dr C.B.G. McIntosh: (03) 541 2607.

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